Endogenous Horizontal Product Differentiation in a Mixed Duopoly

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Contents

Introduction

Mixed oligopoly

- Studies of mixed markets have received significant attentions by economists.
  - Matsumura and Matsushima 2004; Ghosh and Mitra 2010; Heywood and Ye 2010; and Matsumura and Ogawa 2012.
- Industries: airlines, telecommunication, railways, electricity, oil and gas, iron and steel.
- Introducing welfare-maximizing public firms yields results that are often strikingly different from the standard oligopoly.

Objectives

- Following the approach by Brander and Spencer (2015a):
  - Allow firms to undertake costly investments to differentiate products rather than choosing locations as in the Hotelling model.
  - Study the strategic role of a welfare-maximizing public firm in a horizontal product differentiation model.
  - Extend our basic model in two directions: one with a foreign private firm, and the other with asymmetric costs.

Literature Review

Endogenous horizontal product differentiation

- **Brander and Spencer (2015a)**: a new and different model of endogenous horizontal product differentiation
  - An exogenous parameter β: measure the effectiveness of differentiation and compare the threshold of β which induces firms to invest under two modes of competition.
  - Result: a higher threshold of differentiation effectiveness in Cournot firms than in Bertrand firms, implying that Bertrand firms are more likely to invest in differentiation than Cournot firms.

Mixed duopoly

- Cremer et al. (1991) showed that, in a mixed duopoly, the equilibrium locations are optimal for social welfare.
- Matsumura and Matsushima (2004) incorporated endogenous cost-reducing activities into the model of Cremer et al. (1991), and showed that their result is robust with endogenous cost differentials.

Mixed duopoly

- Kitahara and Matsumura (2013) introduced elastic demand into the standard location-price model and showed that reduced product differentiation may lessen competition, which is contrary to Cremer et al. (1991).
- Matsumura and Tomaru (2015) considered the shadow cost of public funding in the model and found that product differentiation increases with the shadow cost.

- In these papers, products are less differentiated with the presence of a public firm.
- Our results indicate that mixed duopoly yields a higher level of product differentiation than private duopoly is thus new to the literature.

Model Setup

Model

- Firm 1 (the public firm that maximizes social welfare) produces good x_1 and firm 2 produces good x_2.

$$U = a\left(x_1 + x_2\right) - \frac{x_1^2 + 2sx_1x_2 + x_2^2}{2} + m \tag{1}$$

  - $s \in [0, 1]$, measuring the degree of substitutability between goods x_1 and x_2.
  - If $s = 1(v = 1 - s = 0)$, goods x_1 and x_2 are homogeneous.
- This utility function generates the system of linear inverse demand functions:

$$p_i = a - x_i - sx_j, \text{ where } i, j = 1, 2, i \neq j, \tag{2}$$

which (imposing $s < 1$) can be inverted to obtain the system of linear demand functions:

$$x_i = \frac{a - p_i - s\left(a - p_j\right)}{1 - s^2}, \text{ where } i, j = 1, 2, i \neq j \tag{3}$$

Model

- **Stage 1:** Firms i simultaneously and independently choose its investment in product differentiation, k_i. Following Brander and Spencer (2015a), the effect of investment on the degree of product differentiation is measured by:

$$v = 1 - s \text{ where } s = 1/e^{\beta K} = e^{-\beta K} \tag{4}$$

  - $K = k_1 + k_2$, and β measures the effectiveness of differentiation investment. A large β means investment has substantial effect in product differentiation.
- **Stage 2:** Observing (k_1, k_2), firms plays the Cournot or Bertrand game by choosing quantities and prices simultaneously to maximize their objectives.

Analysis of Horizontal Product Differentiation

Cournot Competition: Second Stage

In the second stage, firm 1 chooses x_1 to maximize social welfare SW given by

$$\begin{aligned} SW &= CS + V_1 + V_2 \\ CS &= \frac{x_1^2 + 2sx_1x_2 + x_2^2}{2} \end{aligned} \tag{5}$$

Firm i's stage 2 profit:

$$V_i = (a - x_i - sx_j - c)\, x_i, i, j = 1, 2, i \neq j \tag{6}$$

The first order conditions

$$\begin{aligned} a - x_1 - sx_2 - c = 0 \\ a - sx_1 - 2x_2 - c = 0 \end{aligned} \tag{7}$$

Cournot Competition: Second Stage

Equations (6) and (7) yield the following Cournot equilibrium quantities:

$$\begin{cases} x_1^C = \frac{(a-c)(2-s)}{2-s^2} \\ x_2^C = \frac{(a-c)(1-s)}{2-s^2} \end{cases} \quad (8)$$

Then we obtain equilibrium prices

$$\begin{cases} p_1^C = c \\ p_2^C = \frac{(a-c)(1-s)}{2-s^2} + c, \end{cases} \quad (9)$$

From (8), if $s = 1$, we obtain that $x_2^C = 0$ and $p_2^C = c$: Without product differentiation, firm 2 chooses to stop production, and therefore firm 1 is the only supplier in the market.

Lemma 1: Under Cournot competition, the public firm (firm 1) always prices lower and produces more compared to the private firm (firm 2).

Cournot Competition: Second Stage

Profits in terms of s

$$\begin{cases} V_1^C = 0 \\ V_2^C = \left(x_2^C\right)^2. \end{cases} \tag{10}$$

The stage 2 social welfare is given by

$$SW^C = \frac{(a-c)^2\left(7 - 6s - 2s^2 + 2s^3\right)}{2\left(2 - s^2\right)^2}. \tag{11}$$

Cournot Competition: Second Stage

Lemma 2: Under Cournot competition, an increase in product differentiation v (i.e., a decrease in s) causes both the social welfare and the profit of firm 2 to rise.

An increase in product differentiation v creates two effects:

- First, it raises the price charged by the private firm, which yields the private firm a higher profit margin.
- Second, under elastic total demand, consumers gain from variety since $\frac{\partial U}{\partial s} < 0$.

Cournot Competition: First Stage

The partial effect of each firm's investment on s:

$$\frac{\partial s}{\partial k_1} = \frac{\partial s}{\partial k_2} = \frac{\partial s}{\partial K} = -\beta s < 0 \tag{12}$$

Firm 1 chooses k_1 to maximize social welfare

$$W = SW^C - k_1 - k_2. \tag{13}$$

We further calculate the derivative of social welfare W with respect to k_1 :

$$\frac{\partial W}{\partial k_1} = \beta(a-c)^2 g_1(s) - 1, \tag{14}$$

where

$$g_1(s) = \frac{s\left(6 - 10s + 3s^2 + 2s^3 - s^4\right)}{\left(2 - s^2\right)^3}.$$

Cournot Competition: First Stage

Firm 2's stage-one profit is given by

$$\pi_2 = V_2^C - k_2. \tag{15}$$

We have

$$\frac{\partial \pi_2}{\partial k_2} = \beta(a-c)^2 g_2(s) - 1, \tag{16}$$

where

$$g_2(s) = \frac{2s(1-s)\left(2-2s+s^2\right)}{\left(2-s^2\right)^3}.$$

Cournot Competition: First Stage

Lemma 3: The functions $g_1(s)$ and $g_2(s)$ satisfy $g_1(s) > g_2(s) > 0$ for all $s \in (0, 1)$.

- Lemma 3 implies that $\frac{\partial W}{\partial k_1} > \frac{\partial \pi_2}{\partial k_2}$ for any given $s \in (0, 1)$.
- Firm 1 possesses a stronger willingness to invest until the products are completely differentiated.
- Firm 2 will free ride by not investing in product differentiation.

Proof: In equilibrium, we must have $\frac{\partial W}{\partial k_1}|_{K^*} = 0$. Note that at $K = K^*$, $\frac{\partial \pi_2}{\partial k_2} < 0$, which implies that firm 2 will always reduce its investment if $k_2^* > 0$. As a result, $K^* = k_1^*$ in equilibrium. In other words, the one with a stronger willingness to invest will be the only one to undertake investments in equilibrium.

Cournot Competition

Proposition 1 : Under Cournot competition, if $\beta \leq \frac{6.07}{(a-c)^2}$, no firm invests in product differentiation and firm 1 is the only supplier in the market 1. If $\beta > \frac{6.07}{(a-c)^2}$, only firm 1 invests in product differentiation, and both firms are active in equilibrium.

Proof:

$$\left\{ \begin{array}{l} \partial W / \partial k_1 = \beta (a-c)^2 g_1\left(s^*\right) - 1 = 0 \\ SW^C\left(s^*\right) - k_1^* = SW^C (s=1) \end{array} \right.$$

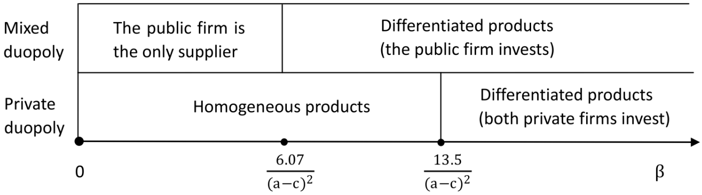
The above two equations can be rewritten as

$$\left\{ \begin{array}{l} \beta (a-c)^2 g_1\left(s^*\right) - 1 = 0 \\ \frac{\left(7-6s^*-2s^{*2}+2s^{*3}\right)(a-c)^2}{2\left(2-s^{*2}\right)^2} + \frac{\ln s^*}{\beta} = \frac{(a-c)^2}{2} \end{array} \right.$$

By solving the two equations, we obtain that $s^* = 0.3945$ and $\hat{\beta} \leq \frac{6.07}{(a-c)^2}$

Cournot Competition

Proposition 2: Under Cournot competition, the products are more differentiated with the presence of a public firm, as compared to a private duopoly.



Bertrand Competition: Second Stage

The equilibrium prices as solutions to the second-stage game:

$$\begin{cases} p_1^B = \frac{(a-c)(1-s)s}{2-s^2} + c \\ p_2^B = \frac{(a-c)(1-s)}{2-s^2} + c \end{cases} \tag{17}$$

The equilibrium quantities in terms of s,

$$\begin{cases} x_1^B = \frac{a-c}{1+s} \\ x_2^B = \frac{a-c}{(1+s)(2-s^2)} \end{cases} \tag{18}$$

Lemma 4: Under Bertrand competition, the public firm (firm 1) always prices lower and produces more compared to the private firm (firm 2).

Bertrand Competition: Second Stage

We obtain the firms' profits

$$\begin{cases} V_1^B = \frac{s\left(1-s^2\right)\left(x_1^B\right)^2}{\left(2-s^2\right)} \\ V_2^B = \left(1-s^2\right)\left(x_2^B\right)^2 \end{cases} \tag{19}$$

and the social welfare

$$SW^B = \frac{(a-c)^2\left(7+s-7s^2-s^3+2s^4\right)}{2(1+s)\left(2-s^2\right)^2}. \tag{20}$$

Lemma 5: Under Bertrand competition, an increase in product differentiation v (i.e., a decrease in s) causes both the social welfare and the profit of firm 2 to rise.

Bertrand Competition: First Stage

In the first stage, the firms decide on their product differentiation investments.
For firm 1 , the stage 1 social welfare, W, can be written as:

$$W = SW^B - k_1 - k_2 \tag{21}$$

Differentiating W with respect to k_1 gives:

$$\frac{\partial W}{\partial k_1} = \beta(a-c)^2 f_1(s) - 1, \tag{22}$$

where

$$f_1(s) = \frac{s\left(6 - 9s^2 - s^3 + 5s^4 + s^5 - s^6\right)}{(1+s)^2\left(2-s^2\right)^3}.$$

Bertrand Competition: First Stage

Firm 2's profit in stage 1 can be written as:

$$\pi_2 = V_2^B - k_2 \tag{23}$$

Differentiating π_2 with respect to k_2 yields that

$$\frac{\partial \pi_2}{\partial k_2} = \beta(a-c)^2 f_2(s) - 1, \tag{24}$$

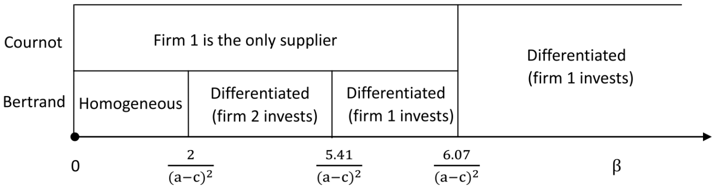
where

$$f_2(s) = \frac{2s\left(2 - 2s - s^2 + 2s^3\right)}{(1+s)^2\left(2-s^2\right)^3}.$$

Lemma 6: The functions $f_1(s)$ and $f_2(s)$ have the following properties:
(i) both $f_1(s)$ and $f_2(s)$ monotonically increase in $s \in [0,1]$;
(ii) $f_1(s) > f_2(s)$ for $s < 0.84$; $f_1(s) \leq f_2(s)$ for $s \geq 0.84$.

Bertrand Competition

Proposition 3: Under Bertrand competition:
(i) if $\beta \leq \frac{2}{(a-c)^2}$, no firm chooses to invest in differentiation and therefore they produce homogeneous products in the second stage;
(ii) if $\frac{2}{(a-c)^2} < \beta < \frac{5.41}{(a-c)^2}$, firm 2 invests but firm 1 does not;
(iii) if $\beta \leq \frac{5.41}{(a-c)^2}$, firm 1 invests but firm 2 does not.

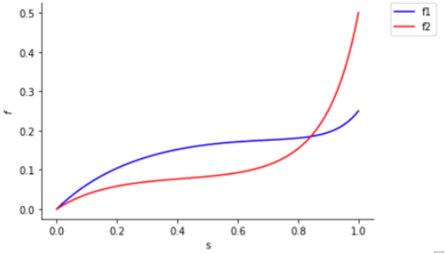


Bertrand Competition

Proof: Suppose that firm 2 makes the investment but firm 1 does not. We have that $k_1^* = 0, K^* = k_2^*$, and $s^* = e^{-\beta K^*}$. It occurs if and only if

$$\begin{cases} \partial W / \partial k_1|_{s=s^*} = \beta(a-c)^2 f_1(s^*) - 1 < 0, \\ \partial \pi_2 / \partial k_2|_{s=s^*} = \beta(a-c)^2 f_2(s^*) - 1 = 0. \end{cases}$$

To satisfy the above two conditions, we need that $f_1(s^*) < f_2(s^*)$ and $\beta = \frac{1}{(a-c)^2 f_2(s^*)}$. By setting $f_1(s^*) < f_2(s^*)$, we obtain that $s^* > 0.84$. Thus, we have that $\beta < \frac{5.41}{(a-c)^2}$.



Bertrand Competition

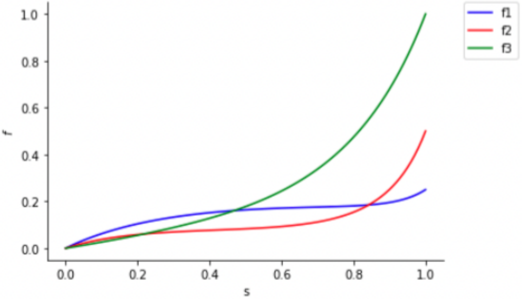
Proposition 4: Under Bertrand competition, the products are less differentiated in a mixed duopoly when $\frac{2}{(a-c)^2} < \beta \leq \frac{5.74}{(a-c)^2}$; when $\beta > \frac{5.74}{(a-c)^2}$, the products are more differentiated.

Bertrand Competition

Proof: In Brander and Spencer (2015a), firms will differentiate their products from rival's product until

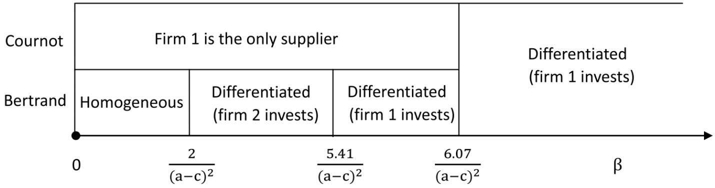
$$\beta(a-c)^2 f_3(s) - 1 = 0$$

where $f_3(s) = \frac{2s(1-s+s^2)}{(2-s)^3(1+s^2)}$. We have that $f_3(s) < f_1(s)$ when $s < 0.70$ and otherwise $f_3(s) > \max\{f_1(s), f_2(s)\}$. The corresponding β for $s = 0.70$ in equilibrium is $\beta = \frac{5.74}{(a-c)^2}$.



Cournot Versus Bertrand Under Mixed Duopoly

Proposition 5:



- Firms have more incentives to differentiate their products under Bertrand competition than under Cournot competition.
- The public firm generally has more incentives to differentiate products than does the private firm.

Two extensions

Mixed Duopoly with a Foreign Private Firm

The objective of the public firm (firm 1) now does not include firm 2's profit: $SW = CS + V_1$

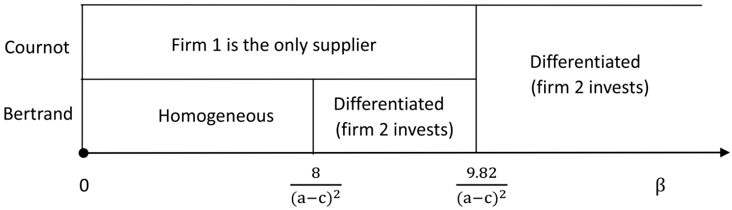


Fig. 3 Comparison of Cournot and Bertrand Equilibria in Mixed Duopoly with a Foreign Firm

- Firms are less likely to differentiate products under both Cournot and Bertrand.
- The public firm will never undertake differentiation investment

Mixed Duopoly with Asymmetric Costs

The private firm is more efficient in production, i.e., $c_2 < c_1$ (Matsumura and Sunada, 2013; Han et al, 2017)

- Cournot competition:
  - When the effectiveness of investment is low, neither firm invests in product differentiation and two firms produce homogeneous products.
  - When the effectiveness of investment is sufficiently high, the public firm invests in product differentiation.
  - Compared to the basic model, the products are less differentiated when the public firm is less efficient.

Mixed Duopoly with Asymmetric Costs

The private firm is more efficient in production, i.e., $c_2 < c_1$ (Matsumura and Sunada, 2013; Han et al, 2017)

- Bertrand competition:
  - When the effectiveness of investment is low, neither firm invests in product differentiation and the private firm monopolizes the market.
  - When the effectiveness of investment is sufficiently high, the public firm invests in product differentiation.
  - Compared to the basic model, the products are less differentiated when the public firm is less efficient.

Conclusion

Conclusion

- Product differentiation arises provided investment in product differentiation is sufficiently effective.
- Cournot competition: the public firm always has a greater incentive for product differentiation, and the products are more differentiated in the mixed duopoly than in the private duopoly.
- Bertrand competition: as long as differentiation investments are sufficiently effective, the public firm undertakes the differentiation of the products, and the products are more differentiated in the mixed duopoly.

Future directions

- The two firms sequentially choose whether or not to invest in product differentiation in the first stage.
- A public firm competing against two or more private firms (see Haraguchi and Matsumura 2016).

Thank you!