Endogenous Horizontal Product Differentiation under Bertrand and Cournot Competition: Revisiting the Bertrand Paradox

James A. Brander and Barbara J. Spencer

NBER Working Paper No. 20966

February 2015

Contents

Introduction

Bertrand Paradox

- **Bertrand model**: in which symmetric price-setting duopoly firms produce a homogenous product at constant marginal cost.
- **Bertrand paradox**: it is hard to believe that firms in industries with few firms never succeed in manipulating the market price to make profits (Tirole, 1988).
- **Empirical Bertrand paradox**: Homogeneous product oligopolies are much more likely to be well approximated by the Cournot than the Bertrand model.

Objectives

- Develop a model that can explain the empirical Bertrand paradox.
  - why homogeneous product oligopoly is rarely
  - why homogeneous product Cournot models are likely to be empirically relevant
- Determine whether Bertrand industries are necessarily more competitive than corresponding Cournot industries once we allow for endogenous horizontal product differentiation.
- Develop the welfare comparison of Bertrand and Cournot models

Endogenous horizontal product differentiation

- The model nests the two extremes of homogeneous goods and unrelated goods.
- An exogenous parameter β: measure the effectiveness of differentiation.
  - Allow for the important possibility that there is some sufficiently low level of differentiation effectiveness at which firms choose not to differentiate their products.

Literature Review

Horizontal product differentiation

- **Hotelling (1929)**:
    - Firms choose locations of the product on a line or a circle.
    - When firms choose product locations but do not set prices, they offer the same product (no differentiation).
- **âAspremont, Gabszewicz, Thisse (1979)**:
    - Firms select locations first and then prices in the second stage.
    - Firms wish to be as far apart as possible in order to maximize product differentiation by solving subgame perfect equilibria.
- We require firms to make costly investments to differentiate their products rather than simply choosing their location.

Bertrand paradox

- Vertical product differentiation
  - Shaked and Sutton, 1982, 1983; Motta, 1993; Boccard and Wauthy, 2010
- Horizontal product differentiation:
  - Bertrand firms can avoid extreme price competition.
  - Explain the choice of Cournot firms to produce homogeneous products

Comparative competitiveness properties of Bertrand and Cournot models

- The basic finding:
  - If Bertrand and Cournot duopolies face the same demand and cost conditions, the Bertrand industry would generate lower profits, lower prices, and more consumer surplus (Cheng, 1985; Singh and Vives, 1984; Vives, 1985; Hsu and Wang, 2005).
- Singh and Vives (1984) relies on an assumed ability of firms to sign binding price contracts or binding quantity contracts.
  - If firms know that they will produce homogeneous products, they would opt for quantity-setting behavior rather than price-setting behavior.

Comparative competitiveness properties of Bertrand and Cournot models

- Qiu (1997) provides an important extension in which cost is made endogenous through the introduction of investment in RD.
- Our model differs from Qiu (1997) because investment causes product differentiation not cost reduction.

Model of Horizontal Product Differentiation

Model

- Firm 1 produces good x_1 and firm 2 produces good x_2.

$$U = a\left(x_1 + x_2\right) - \frac{x_1^2 + 2sx_1x_2 + x_2^2}{2} + m \tag{1}$$

  - $s \in [0, 1]$, measuring the degree of substitutability between goods x_1 and x_2.
  - If $s = 1$ ($v = 1 - s = 0$), goods x_1 and x_2 are homogeneous.
- The aggregate utility function generates the following inverse demand functions:

$$\begin{aligned} p_1 &= \frac{\partial U}{\partial x_1} = a - x_1 - sx_2 \\ p_2 &= \frac{\partial U}{\partial x_2} = a - sx_2 - x_1 \end{aligned} \tag{2}$$

Model

- **Stage 1:** Firms i simultaneously and independently choose its investment in product differentiation, k_i. The effect of investment on the degree of product differentiation is measured by:

$$v = 1 - s \text{ where } s = 1/e^{\beta K} = e^{-\beta K} \tag{3}$$

  - $K = k_1 + k_2$, and β measures the effectiveness of differentiation investment. A large β means investment has substantial effect in product differentiation.
  - Assumption: the differentiation investment affects only the degree of differentiation with no effects on other aspects of demand.
- **Stage 2:** Observing (k_1, k_2), firms plays the Cournot or Bertrand game by choosing quantities and prices simultaneously to maximize their objectives.

Bertrand Competition

Second Stage: Pricing Decisions

Each firm maximizes variable profit

$$V_i \equiv (p_i - c)\, x_i \tag{4}$$

We rewrite the inverse demand functions (2) as $x_1 + sx_2 = a - p_1$ and $sx_1 + x_2 = a - p_2$ and solve to obtain:

$$\begin{aligned} x_1 &= \frac{(a - p_1) - (a - p_2)\, s}{(1 - s^2)} \\ x_2 &= \frac{(a - p_2) - (a - p_1)\, s}{(1 - s^2)} \end{aligned} \tag{5}$$

Second Stage: Pricing Decisions

Maximize variable profit (4) using (5) to solve for the Bertrand equilibrium prices and quantities:

$$\begin{aligned} p &= p^B(s) = \frac{(a-c)(1-s)}{(2-s)} + c \\ x &= x^B(s) = \frac{(a-c)}{(2-s)(1+s)} \\ V &= V^B(s) = \left(1-s^2\right)\left(x^B(s)\right)^2 \end{aligned} \tag{6}$$

Proposition 1: Under Bertrand competition, an increase in product differentiation, v, causes
i) prices to rise,
ii) outputs to fall if $0 \leq v < 1/2$, reach a minimum at $v = 1/2$, and then rise for $1/2 < v < 1$.
iii) variable profits to rise.

First Stage: Investments in Product Differentiation

Firm 1 chooses k_1 and firm 2 chooses k_2 to maximize profit taking the investment of the other firm as given.

- The partial effect of each firm's investment on s:

$$\frac{\partial s}{\partial k_1} = \frac{\partial s}{\partial k_2} = \frac{\mathrm{d}s}{\mathrm{d}K} = -\beta e^{-\beta K} = -\beta s < 0 \tag{7}$$

- The first stage profit for firm i can be written as:

$$\pi_i = V^B(s) - k_i = \left(1 - s^2\right)\left(x^B(s)\right)^2 - k_{\mathrm{i}} \tag{8}$$

- Firm i's profit maximization problem is therefore:

$$\frac{\partial \pi_{\mathrm{i}}}{\partial k_i} = \left(\frac{\mathrm{d}V^B}{\mathrm{d}s}\right)\left(\frac{\partial \mathrm{s}}{\partial \mathrm{k}_{\mathrm{i}}}\right) - 1 =$$

$$\frac{\beta 2s\left(x^B(s)\right)^2\left(1 - s + s^2\right)}{(2 - s)} - 1 = 0 \tag{9}$$

First Stage: Investments in Product Differentiation

Proposition 2:Under Bertrand competition, both firms choose to differentiate their products at stage 1 if and only if $\beta > \frac{2}{(a-c)^2}$. If $\beta \leq \frac{2}{(a-c)^2}$ then no differentiation investment takes place and products are homogeneous at stage 2.

Proof: No differentiation $(s = 1)$ takes place if and only $k_1 = k_2 = 0$, which occurs if and only if

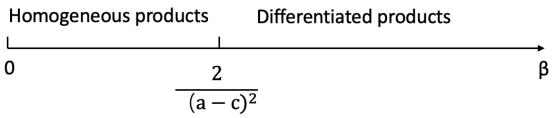
$$\frac{\partial \pi_i}{\partial k_i} \leq 0 \text{ at } k_i = 0 \text{ and } s = 1$$

Substituting $s = 1$ into (9) and using $x = \frac{a-c}{2}$ from (6) shows that

$$\frac{\partial \pi_i}{\partial k_i} = \frac{\beta(a-c)^2}{2} - 1 \leq 0 \text{ if and only if } \beta \leq \frac{2}{(a-c)^2}$$

First Stage: Investments in Product Differentiation

Proposition 2: Under Bertrand competition, both firms choose to differentiate their products at stage 1 if and only if $\beta > \frac{2}{(a-c)^2}$. If $\beta \leq \frac{2}{(a-c)^2}$ then no differentiation investment takes place and products are homogeneous at stage 2.



Cournot Competition

Second Stage: Quantity Decisions

We obtain the first order conditions:

$$\begin{aligned} \frac{\partial V_1}{\partial x_1} &= a - c - 2x_1 - sx_2 = 0 \\ \frac{\partial V_2}{\partial x_2} &= a - c - sx_1 - 2x_2 = 0 \end{aligned} \tag{10}$$

Second Stage: Quantity Decisions

We express the common output, price and profit at the stage 2 Cournot equilibrium as follows:

$$
\begin{aligned}
x &= x^{C}(s) = \frac{(a-c)}{(2+s)} \\
p &= p^{C}(s) = \frac{(a-c)}{(2+s)} + \mathrm{c} \\
V &= V^{C}(s) = \left(x^{C}(s)\right)^{2}
\end{aligned}
\tag{11}
$$

Proposition 3: Under Cournot competition, an increase in product differentiation v, causes outputs to rise, prices to rise, and variable profits to rise.

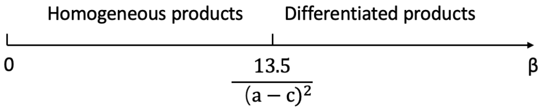
First Stage: Investments in Product Differentiation

The associated first order condition for an interior solution for firm i is given by

$$\frac{\partial \pi_{\mathrm{i}}}{\partial k_i} = \left(\frac{\mathrm{d}V^C}{\mathrm{d}s}\right)\left(\frac{\partial \mathrm{s}}{\partial \mathrm{k_i}}\right) - 1 = \tag{12}$$

$$\frac{2\beta s\left(x^C(s)\right)^2}{(2+s)} - 1 = 0 \tag{13}$$

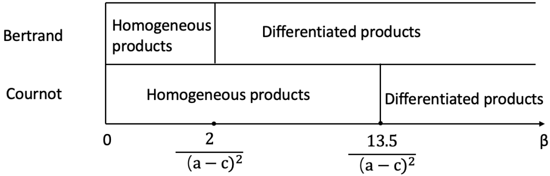
Proposition 4: With Cournot competition, firms choose to differentiate their products at stage 1 if and only if $\beta > \frac{13.5}{(a-c)^2}$. If $\beta \leq \frac{13.5}{(a-c)^2}$ then no investment takes place and products are homogeneous at stage 2.



Comparing the Cournot and Bertrand Models

The Empirical Bertrand paradox

Proposition 5:
(i) If $\beta \leq \frac{2}{(a-c)^2}$, then products are homogeneous under both Bertrand and Cournot competition.
(ii) If $\frac{2}{(a-c)^2} < \beta \leq \frac{13.5}{(a-c)^2}$, then products are differentiated under Bertrand competition and are homogenous under Cournot competition: $v^B > v^C = 0$.
(iii) If $\beta > \frac{13.5}{(a-c)^2}$, then products are differentiated under both Bertrand and Cournot competition, but are more differentiated under Bertrand than Cournot competition: $v^B > v^C > 0$.



The Empirical Bertrand paradox

- The stronger is demand, the less effective differentiation investments needs to be to justify differentiation.
- There is a very wide range of parameter values under which products are homogeneous under Cournot competition, but differentiated under Bertrand competition.

Table 1: Critical Values of Differentiation Effectiveness, β

Demand $(a - c)$	Bertrand (β^B)	Cournot (β^C)
2	0.50	3.38
4	0.13	0.84
8	0.031	0.21
12	0.014	0.094
16	0.0078	0.053
20	0.0050	0.034
30	0.0022	0.015
50	0.00080	0.0054

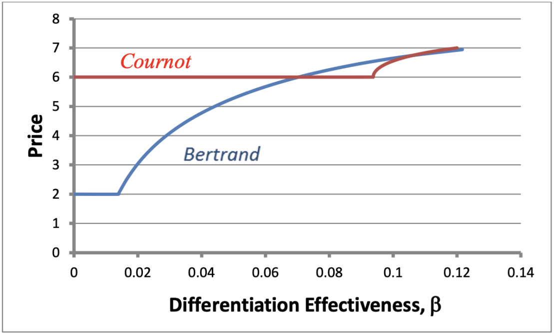
Endogenous Product Differentiation and Comparative Competitiveness

Proposition 6:
i) Output is higher under Bertrand than Cournot competition: $x^B > x^C$.
ii) For the same level of product differentiation, both price and profit are strictly lower under Bertrand than Cournot competition: $p^B < p^C$ and $\pi^B < \pi^C$.
iii) With endogenous product differentiation, it is possible for Bertrand firms to charge higher prices and earn more profit than corresponding Cournot firms.

Endogenous Product Differentiation and Comparative Competitiveness

$a = 14$, $c = 2$, $a - c = 12$
$\beta^B = 0.014$, $\beta^C = 0.094$

Figure 1: The effect of differentiation effectiveness, β, on price



Endogenous Product Differentiation and Comparative Competitiveness

Figure 2: Product differentiation and output per firm

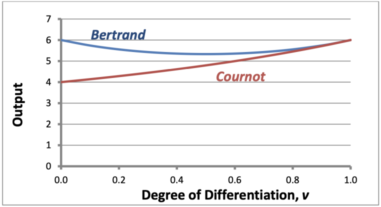
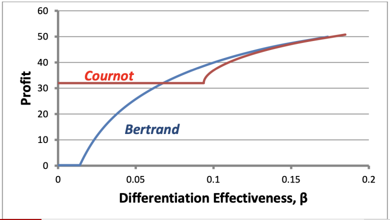


Figure 3: The effect of differentiation effectiveness, β, on profits



Endogenous Product Differentiation and Consumer Surplus

Letting $G \equiv U - (p_I x_1 - p_2 x_2 - M)$ denote consumer surplus, we obtain

$$G = 2(a - p)x - (1 + s)x^2 = (1 + s)x^2 \tag{14}$$

Proposition 7:

i) For any given level of product differentiation, v, an increase in v :

(a) reduces consumer surplus under Bertrand competition.

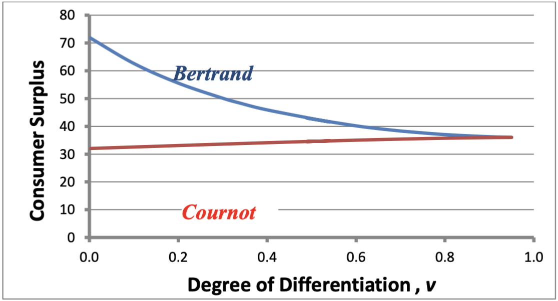
(b) increases consumer surplus under Cournot competition.

ii) Whatever the levels of product differentiation, v^B and v^C, other than unrelated products $(v = v^B = v^C = 1$), consumer surplus is always higher under Bertrand than Cournot competition.

Endogenous Product Differentiation and Comparative Competitiveness

$a = 14$, $c = 2$, $a - c = 12$
$\beta^B = 0.014$, $\beta^C = 0.094$

Figure 4: Product Differentiation and Consumer Surplus



Concluding Remarks

Conclusion

- Even though product differentiation is costly, Bertrand firms have a much stronger incentive to undertake product differentiation than Cournot firms.
- For sufficiently high values of differentiation effectiveness, Bertrand firms charge higher prices and earn larger profits than Cournot firms.

Conclusion

- For any given level of product differentiation short of completely unrelated products, variable profits are lower under Bertrand competition than Cournot competition.
- Regardless of differences in product differentiation across the two modes of competition, consumer surplus is always higher under Bertrand competition than Cournot competition.

Extension

- Consider sequential move games in which entry deterrence or, at least, entry manipulation becomes important.
- Consider multi-product firms (as in Chen and Chen (2014))

Thank you!