

Does the Presence of a Public Firm Facilitate Merger Approvals?

Kiriti Kanjilal*, Ana Espinola-Arredondo†, and Felix Munoz-Garcia‡

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Abstract

We study mergers between two or more private firms operating in mixed oligopolies, where we allow for the merger to produce cost-reduction effects. We identify that the presence of public firms can facilitate the approval of welfare-improving mergers, even when their cost-reducing effect is relatively low.

KEYWORDS: Mergers, Cost-reduction effects, Public firm.

JEL CLASSIFICATION: L13, L32, L44.

*Address: Indraprastha Institute of Information Technology, Address: B-208, Research and Development Block, Delhi 110020, India, E-mail: kanjilal@iiitd.ac.in.

†Address: 101B Hulbert Hall, Washington State University, Pullman, WA 99164. E-mail: anaespinola@wsu.edu.

‡Address: 103H Hulbert Hall, Washington State University, Pullman, WA 99164. E-mail: fmunoz@wsu.edu.

1 Introduction

Mergers often produce cost-reduction effects (e.g., better management practices, avoiding cost duplicities, generally known as “efficiencies”), resulting in lower prices for consumers.¹ While these mergers have been extensively studied in industries where all firms are private, they received little attention in markets where a public firm is present.²

Our paper examines the incentives of private firms to merge in this type of industry and under which conditions the competition authority (CA) approves the merger.³ We identify that the approval of profitable mergers is facilitated by the presence of public firms. Private firm managers could be concerned that their merger request will be blocked because of the presence of a public company. Our results indicate, however, that these concerns are unfounded. Alternatively, mergers are more likely to be blocked when the public firm is privatized, suggesting an unintended consequence from privatizations: merger requests are more likely to be blocked in equilibrium.

Our model follows Fujiwara’s (2007) analysis of mergers in mixed oligopolies, but allows for the merger to produce cost-reduction effects. When the merger generates small cost-reduction effects, we confirm that mergers are welfare reducing, but otherwise demonstrate that the merger can be both profitable and welfare improving under certain conditions. More interestingly, we identify that this result is more likely to arise when the public firm assigns a larger weight on welfare.⁴

2 Model

Consider an industry with $n \geq 3$ firms (one public and $n - 1$ private firms) competing à la Cournot, facing inverse demand function $p(Q) = 1 - Q$, where $Q \equiv q + \sum_{i=1}^{n-1} q_i$ denotes aggregate output, q represents the public firm’s output, and q_i is the output of private firm $i = 1, 2, \dots, n - 1$. Every private firm maximizes its profits

$$\pi_i = p(Q)q_i - cq_i$$

¹For empirical studies estimating cost-reduction effects in the airline industry, see Gayle and Le (2013) for the US and Yan et al. (2019) for China. Similarly, Ganapati (2021) studies a large data set of US industries for 1972-2012. For a review of empirical studies see, for instance, Kolaric and Schiereck (2014).

²Nakamura and Inoue (2007) assume that the merging entity enjoys the costs of the most efficient merging firm. While this assumption reduces the cost of the merged entity, it does not allow for an explicit analysis of how merger efficiencies affect the profitability of the merger or its welfare effects. In addition, they consider mergers between only two firms and increasing marginal costs; as opposed to our paper.

³Examples of private firms merging in industries where a public firm is also present include, for instance, FedEx merging with Genco in 2015 and with TNT Express in 2016, where USPS also operates and is publicly governed (holding around 19 percent of market share in US package delivery). FedEx had a 25 percent market share in the domestic courier and delivery services industry, increasing to 34 percent in 2020. Another example is that of Swiss telecommunications provider Sunrise merging with UPC in 2020, where Swisscom is a 51 percent state-owned enterprise. While Swisscom still has the highest market share (59.7 percent), the merged company Sunrise/UPC is now the second largest (25 percent), followed by Salt (16 percent). Finally, Czech brewery Radegast merging with Velke Popovice and Plzensky Prazdroj, approved in 2002, provides us with another example, where Budweiser Budvar Brewery is state-owned, resulting in a single joint stock company called Plzeňský Prazdroj.

⁴Bárcena-Ruiz and Garzón (2003) and Méndez-Naya (2008) also consider mergers in mixed oligopoly markets. However, they assume increasing marginal costs, unlike standard merger models such as Salant et al. (1983), and do not allow for cost-reduction effects.

while the public firm maximizes a combination of social welfare and profits

$$V = \alpha W + (1 - \alpha)\pi$$

where social welfare is given by $W = \int_0^Q p(y)dy - cQ$, and its profit is $\pi = p(Q)q - cq$. Parameter α represents the weight that the manager of the public firm assigns to welfare, while $1 - \alpha$ is the weight she assigns to profits; as in Matsumura (1998) and Fujiwara (2007), among others.⁵

Every firm's initial marginal cost is c , where $1 > c \geq 0$. After a merger between k firms, the marginal cost of each merging firm (the insiders) decreases from c to $c - x$, where parameter x denotes the cost-reduction effect of the merger, as in Perry and Porter (1985), where k satisfies $n \geq k \geq 2$. In contrast, the marginal cost of the remaining $n - k$ firms (outsiders) remains at c . Therefore, after the merger, $n - k + 1$ firms operate in the industry.⁶ All information is common knowledge.

The time structure of the game is the following:

1. In the first stage, the k firms that seek to merge choose whether to submit, as an entity, a merger approval request to the CA.
2. In the second stage, the CA responds approving or blocking the merger request.
3. In the third stage, firms observe the CA's decision, and compete à la Cournot.

We solve this complete-information game by backward induction, first finding output levels in the third stage.

3 Equilibrium Analysis

3.1 Third stage - No merger

If a merger does not ensue, either because firms do not request it or the CA blocks it, firms compete à la Cournot, yielding the following output level and profits.

Lemma 1. *In the third stage, if the merger is blocked, the equilibrium output of every private firm i is $q_i^{NM} = \frac{(1-c)(1-\alpha)}{(n-1)(1-\alpha)+(2-\alpha)}$ and that of the public firm is $q^{NM} = \frac{1-c}{(n-1)(1-\alpha)+(2-\alpha)}$. Output q_i^{NM} is decreasing in c , n , and α ; while q^{NM} is increasing in α , decreasing in c , and also decreasing in n if and only if $c < \frac{1}{2-\alpha}$.*

As expected, all firms' output are decreasing in their marginal cost c , but the public (private) firm increases (decreases) its output when it assigns a larger weight on welfare, as captured by α .

⁵ Alternatively, parameter α can be interpreted as the government's share in the public firm. In this interpretation, $1 - \alpha$ denotes the degree of privatization of the public firm, with $\alpha = 1$ indicating the firm that the firm is not privatized at all while $\alpha = 0$ implies that the firm is fully privatized.

⁶ We consider that $x < 1 - c$; otherwise, the cost-reduction effect can lead to the merged entity to monopolize the market.

The public firm's output increase, however, dominates, yielding an increase in aggregate output, $Q^{NM} = q^{NM} + (n-1)q_i^{NM}$, as the next corollary identifies.

Lemma 2. *Aggregate output under no merger, $Q^{NM} = \frac{(1-c)[(n-1)(1-\alpha)+1]}{1+n(1-\alpha)}$, is increasing in α and n but decreasing in c .*

For intuition consider two extreme cases. When $\alpha = 0$, all firms are private, and aggregate output coincides with that in a standard Cournot model with $n \geq 2$ firms, $Q^{NM} = \frac{n(1-c)}{n+1}$. In contrast, when $\alpha = 1$, the public firm only cares about welfare, increasing Q^{NM} , which now coincides with the perfectly competitive output, $Q^{NM} = 1 - c$.

3.2 Third stage - Merger

After the merger, $n - k + 1$ firms operate in the industry: (i) the merged entity, with k "insider" firms benefiting from a lower production cost, $c - x$; and (ii) the unmerged private firms, $(n - 1) - k$, "outsiders" along with the public firm, all of them with marginal cost c .

Lemma 3. *In the third stage, if the merger is approved, the equilibrium output of the private insiders (as a group) is $q_I^M = \frac{(1-\alpha)(1-c)+[1+(1-\alpha)(n-k)]x}{1+(1-\alpha)(n-k+1)}$, that of every private outsider is $q_O^M = \frac{(1-\alpha)(1-c-x)}{1+(1-\alpha)(n-k+1)}$, and that of the public firm is $q^M = \frac{1-c-x}{1+(1-\alpha)(n-k+1)}$. All output levels are decreasing in c and n , but q^M is increasing in α while q_I^M and q_O^M are decreasing in α .*

As in the no merger setting, a higher weight on welfare (higher α) increases the public firm's output while decreasing that of private insiders and outsiders. The next lemma confirms that, as without a merger, aggregate output in this context also increases in α .

Lemma 4. *Aggregate output under the merger, $Q^M = \frac{(1-c)[1+(1-\alpha)(n-k)]+(1-\alpha)x}{1+(1-\alpha)(n-k+1)}$, is decreasing in k and c , but increasing in α and n .*

As expected, Q^M decreases in the number of merging firms, k , but increases in the weight on welfare, α . Evaluating Q^M at extreme values of α , we find that, when all firms are private ($\alpha = 0$), aggregate output coincides with that in standard merger models, $Q^M = \frac{(1-c)(n-k+1)+x}{n-k+2}$, see Perry and Porter (1985). When the weight on welfare is positive, however, aggregate output increases, reaching the perfectly competitive output, $Q^M = 1 - c$, when $\alpha = 1$.

3.3 Second stage

The CA anticipates the equilibrium output, both with and without the merger, and approves a merger if it improves consumer surplus. For compactness, we use $\theta \equiv \frac{x}{1-c}$ to capture how intense the cost-reduction effect of the merger is relative to initial costs.

Lemma 5. *In the second stage, the CA approves a k -firm merger if and only if $\theta > \bar{\theta}$, where $\bar{\theta} \equiv \frac{1}{1-\alpha} \left[\frac{[1+(1-\alpha)(n-k+1)][(n-1)(1-\alpha)+1]}{1+n(1-\alpha)} - [1 + (1-\alpha)(n-k)] \right]$.*

The next corollary evaluates cutoff $\bar{\theta}$ at different values of α . We discuss our results below.

Corollary 3. *When $\alpha = 0$, cutoff $\bar{\theta}$ simplifies to $\bar{\theta} = \frac{k-1}{n+1}$; when $\alpha = 1$, it collapses to $\bar{\theta} = 0$.*

When all firms are private, $\alpha = 0$, we confirm the results in Perry and Porter (1985), where the merger improves consumer surplus if it represents a sufficiently large market share, i.e., $\theta > \frac{k-1}{n+1}$. Figure 1 depicts this cutoff, as a function of k on the horizontal axis, considering $n = 9$ firms. As α increases, cutoff $\bar{\theta}$ shifts downwards, indicating that when the weight on welfare increases, the CA approves mergers with smaller cost-reducing effects. This occurs because the CA anticipates that the output reduction stemming from the merger will be corrected by the public firm in the subsequent stage, either partially when $\alpha \in (0, 1)$, or fully when $\alpha = 1$. Indeed, when $\alpha = 1$, cutoff $\bar{\theta}$ shifts downwards, overlapping with the horizontal axis. In this setting, all mergers are approved regardless of θ , since the CA anticipates the public firm producing the socially optimal output.

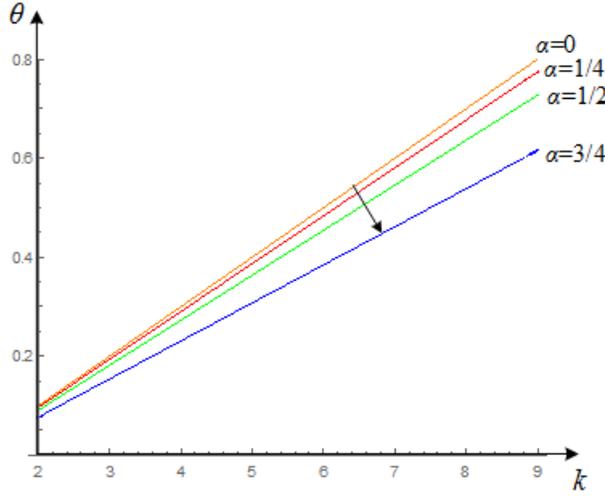


Figure 1. Cutoff $\bar{\theta}$, welfare-improving mergers.

3.4 First stage

In the first stage, the merging entity anticipates the CA's decision, approving the merger if and only if $\theta > \bar{\theta}$, and submits a request if its profits satisfy $\pi_I^M \geq k\pi_i^{NM}$ which, solving for θ , yields the following condition.

Proposition 1. *The merging entity submits a merger request if and only if $\theta > \hat{\theta}$, where*

$$\hat{\theta} \equiv \frac{1}{1+(1-\alpha)(n-k+1)} \left[\sqrt{k \frac{(1-\alpha)[1+(1-\alpha)(n-k+1)]}{1+n(1-\alpha)}} - (1-\alpha) \right].$$

Figure 2 depicts cutoff $\hat{\theta}$ considering the same parameter values as in figure 1; and evaluating how it is affected by an increase in α . When all firms are private, $\alpha = 0$, cutoff $\hat{\theta}$ simplifies to

$\widehat{\theta}(0) \equiv \frac{\sqrt{k}}{n+1} - \frac{1}{n-k+2}$, exhibiting an inverted-U shape.⁷ Intuitively, this concavity originates from Salant et al.'s (1983) results: a merger between k firms produces a positive effect on their profits (as they can coordinate their output decisions) but a negative effect (as the $n - k$ unmerged firms increase their output levels). Salant et al. (1983) show that, when k is relatively low, the negative effect dominates the positive effect, but our findings highlight the presence of three regions: (i) if k is extremely low, the negative effect dominates, but it increases faster than the positive effect, *raising* cutoff $\widehat{\theta}$, making the merger less attractive; (ii) when k is intermediate, the negative effect still dominates, but the positive effect now increases faster than the negative effect, *decreasing* cutoff $\widehat{\theta}$, making the merger more attractive; and (iii) when k is sufficiently large (i.e., market share $\frac{k}{n}$ above 80%), the negative effect dominates the positive effect for all values of θ , implying that the merger becomes profitable regardless of its cost-reduction effect.⁸

When α increases, figure 2 illustrates that region (i) expands, while regions (ii) and (iii) shrink. Specifically, this occurs because an increase in α steepens the public firm's best response function. Hence, for a given output decrease in the private firms' aggregate output, the public firm responds with an increase in its own output, which is increasing in α . In addition, the decrease in private firms' aggregate output after the merger is relatively small when the number of merging firms, k , is low, but becomes large otherwise, entailing that the public firm responds with a small (large) output increase to the merger when k is low (high, respectively).⁹ Ultimately, this implies that, when k is low (high), the presence of the public firm attenuates (emphasizes) the negative effect of the merger, making it attractive under larger (more restrictive) parameter conditions. Figure 2 illustrates this result with a downward (upward) shift in cutoff $\widehat{\theta}$.

⁷This cutoff originates at $k = 2$, with a vertical intercept of $\frac{\sqrt{2}}{n+1} - \frac{1}{n}$, which is unambiguously positive since $(\sqrt{2} - 1)n > 1$ holds given that $n \geq 3$ by definition. When k increases, cutoff $\widehat{\theta}(0)$ initially increases since $\left. \frac{\partial \widehat{\theta}(0)}{\partial k} \right|_{k=2} = \frac{1}{2\sqrt{2}(n+1)} - \frac{1}{n^2} > 0$ for all $n \geq 3.6$, but then decreases since $\left. \frac{\partial \widehat{\theta}(0)}{\partial k} \right|_{k=n} = \frac{1}{2\sqrt{n}(n+1)} - \frac{1}{4} < 0$ for all admissible values. Therefore, cutoff $\widehat{\theta}(0)$ exhibits an inverted U-shape, as depicted in figure 2, but its concavity is subtle, due to the low values of θ on the vertical axis. Figure 3a confirms this point by depicting cutoffs $\widehat{\theta}$ and $\bar{\theta}$ together to illustrate our equilibrium results.

⁸When all firms are private and cost-reduction effects are absent, $\alpha = \theta = 0$, the inequality in Proposition 1 simplifies to $0 \geq \frac{\sqrt{k}}{n+1} - \frac{1}{n-k+2}$ or, after solving for k , $k > \frac{2n+3-\sqrt{5+4n}}{2} \equiv \widehat{k}$, where ratio $\frac{\widehat{k}}{n}$ satisfies $\frac{\widehat{k}}{n} \geq 0.8$, yielding the so-called "80% rule" in mergers, after Salant et al. (1983). as a special case of our analysis where $\alpha = \theta = 0$.

⁹Formally, the private firms' aggregate output after the merger, $Q^M - q^M = q_I^M + (n-k)q_O^M$, satisfies $\frac{\partial(Q^M - q^M)}{\partial k} < 0$ and $\frac{\partial^2(Q^M - q^M)}{\partial k^2} < 0$, while that of the public firm, q^M , satisfies $\frac{\partial q^M}{\partial k} > 0$ and $\frac{\partial^2 q^M}{\partial k^2} > 0$.

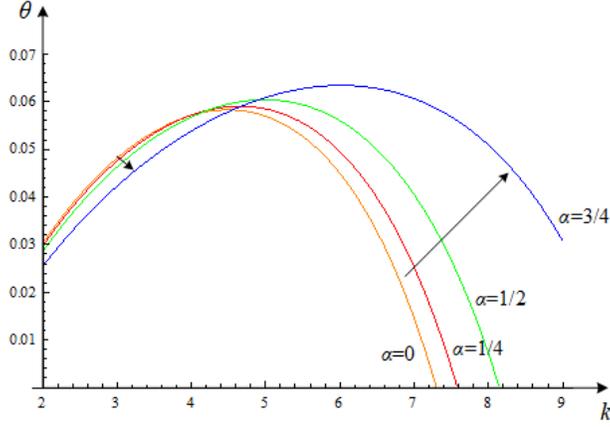


Figure 2. Cutoff $\hat{\theta}$, profit-improving mergers.

4 Comparisons

Comparing cutoffs $\bar{\theta}$ and $\hat{\theta}$, we obtain that $\hat{\theta} < \bar{\theta}$ under all admissible parameter values, giving rise to three regions in the (θ, k) -quadrant, depicted in figure 3a. In region *A*, θ is relatively low, $\theta < \hat{\theta}$, and firms do not submit a merger request, as it would be unprofitable. In region *B*, θ is intermediate, $\hat{\theta} < \theta < \bar{\theta}$, and firms find the merger profitable, since $\hat{\theta} < \theta$. However, firms anticipate the merger being declined by the CA, because $\theta < \bar{\theta}$, implying that they do not submit the request in equilibrium. Finally, in region *C*, θ satisfies $\theta > \bar{\theta}$, implying that the merger is both profitable and welfare improving, so firms submit the request as they anticipate it being approved.

A natural question is how these three regions are affected by an increase in α . Cutoff $\hat{\theta}$ may increase or decrease in α , entailing that regions *A* and *B* may expand or shrink. To identify the overall effect of α on these regions, define the size of region *B* as $\text{Reg}_B \equiv \int_2^{n-1} (\bar{\theta} - \hat{\theta}) dk$. Figure 3b evaluates Reg_B at our ongoing parameter values, showing that, as the weight on welfare is larger (higher α), region *B* shrinks. Finally, the size of region *C* is given by $\text{Reg}_C \equiv \int_2^{n-1} (1 - \bar{\theta}) dk = \frac{(n-3)[3-\alpha+n(1-\alpha)]}{2+2n(1-\alpha)}$, which satisfies $\frac{\partial \text{Reg}_C}{\partial \alpha} = \frac{n(n-4)+3}{2[n(1-\alpha)+1]^2} \geq 0$ for all $n \geq 3$, which holds by definition.

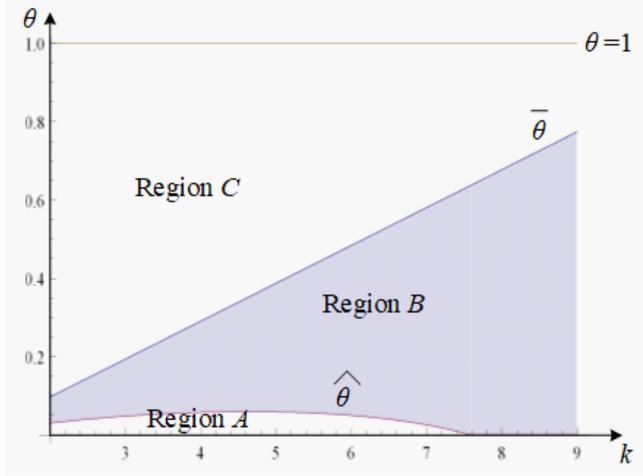


Fig. 3a. Equilibrium regions.

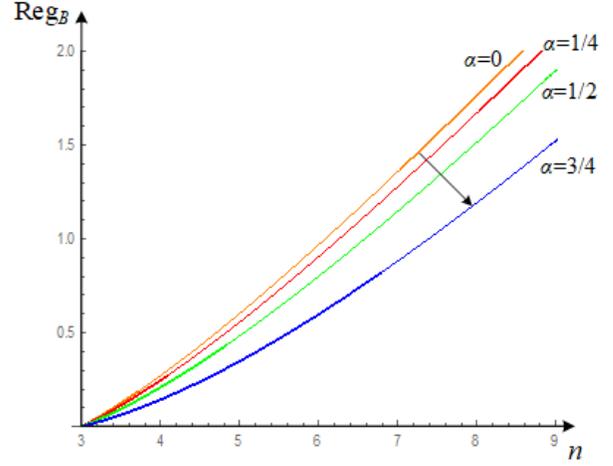


Fig. 3b. Area of region B for different values of α .

Our results entail that, as α increases, welfare-improving mergers, approved by the CA, become more likely to arise in equilibrium, i.e., region C can be sustained under more parameter combinations. This holds even when the cost-reducing effect, θ , is not substantial. Furthermore, welfare-reducing mergers, blocked by the CA, are less likely to occur, i.e., region B shrinks. Intuitively, the presence of a public firm assigning a larger weight on welfare makes it less likely that firms and CA end up in a situation where their interests about approving the merger differ or, in other words, helps align their preferences. Alternatively, a more privatized public company (lower α) expands region B , leading to more profitable mergers being blocked.

5 Appendix

5.1 Proof of Lemma 1

Each private firm i solves

$$\max_{q_i \geq 0} \pi_i^{NM} = \left[1 - \left(q^{NM} + q_i^{NM} + \sum_{j \neq i}^{n-2} q_j^{NM} \right) \right] q_i^{NM} - c q_i^{NM}$$

which yields

$$q_i^{NM}(q^{NM}) = \frac{1-c}{n} - \frac{1}{n} q^{NM}.$$

The public firm solves

$$\max_{q^{NM} \geq 0} V^{NM} = \alpha W^{NM} + (1 - \alpha) \pi^{NM}$$

which yields

$$q^{NM} (q_i^{NM}) = \frac{1 - c}{2 - \alpha} - \frac{n - 1}{2 - \alpha} q_i^{NM}$$

Simultaneously solving for q^{NM} and q_i^{NM} , we obtain

$$\begin{aligned} q^{NM} &= \frac{1 - c}{(n - 1)(1 - \alpha) + (2 - \alpha)} \quad \text{and} \\ q_i^{NM} &= \frac{(1 - c)(1 - \alpha)}{(n - 1)(1 - \alpha) + (2 - \alpha)}. \end{aligned}$$

Output q_i^{NM} satisfies $\frac{\partial q_i^{NM}}{\partial \alpha} - \frac{1 - c}{[1 + n(1 - \alpha)]^2} < 0$, $\frac{\partial q_i^{NM}}{\partial c} = -\frac{1 - \alpha}{1 + n(1 - \alpha)} < 0$, and $\frac{\partial q_i^{NM}}{\partial n} = -\frac{(1 - c)(1 - \alpha)^2}{[1 + n(1 - \alpha)]^2} < 0$; while q^{NM} satisfies $\frac{\partial q^{NM}}{\partial \alpha} - \frac{1 - c}{[1 + n(1 - \alpha)]^2} < 0$, $\frac{\partial q^{NM}}{\partial c} = -\frac{1}{1 + n(1 - \alpha)} < 0$, and $\frac{\partial q^{NM}}{\partial n} = -\frac{1 - \alpha - c(1 - \alpha)}{[1 + n(1 - \alpha)]^2} < 0$, which holds if and only if $c < \frac{1}{2 - \alpha}$.

5.2 Proof of Lemma 2

Using Lemma 1, we obtain

$$\begin{aligned} Q^{NM} &= q^{NM} + (n - 1) q_i^{NM} \\ &= \frac{(1 - c)[(n - 1)(1 - \alpha) + 1]}{1 + n(1 - \alpha)}. \end{aligned}$$

which satisfies $\frac{\partial Q^{NM}}{\partial \alpha} = \frac{1 - c}{[1 + n(1 - \alpha)]^2} > 0$, $\frac{\partial Q^{NM}}{\partial c} = -\frac{N - \alpha(N - 1)}{1 + n(1 - \alpha)} < 0$, and $\frac{\partial Q^{NM}}{\partial n} = \frac{(1 - c)(1 - \alpha)^2}{[1 + n(1 - \alpha)]^2} > 0$.

5.3 Proof of Lemma 3

After the merger is approved, the merging entity's solves

$$\max_{q_I^M \geq 0} \pi_I^M = \left[1 - \left(q^M + q_I^M + \sum_{i=1}^{n-1-k} q_{O,i}^M \right) \right] q_I^M - (c - x) q_I^M$$

which yields

$$q_I^M (q^M, q_O^M) = \frac{1 - (c - x)}{2} - \frac{q^M + (n - k - 1) q_O^M}{2}$$

Every unmerged private firm solves

$$\max_{q_O^M \geq 0} \pi_O^M = \left[1 - \left(q^M + q_I^M + \sum_{i=1}^{n-1-k} q_{O,i}^M \right) \right] q_O^M - c q_O^M$$

which yields

$$q_O^M(q^M, q_I^M) = \frac{1-c}{n-k} - \frac{q^M + q_I^M}{n-k}$$

The public firm solves

$$\max_{q^M \geq 0} V^M = \alpha W^M + (1-\alpha)\pi^M$$

where $W^M = \int_0^{Q^M} (1-y)dy - cQ^M$, $\pi^M = [1 - Q^M]q^M - cq^M$, and $Q^M = q^M + q_I^M + (n-k-1)q_O^M$. Differentiating and solving for q^M , we obtain

$$q^M(q_I^M, q_O^M) = \frac{1-c}{2-\alpha} - \frac{q_I^M + (n-k-1)q_O^M}{2-\alpha}$$

Solving simultaneously for q^M , q_I^M , and q_O^M in the above best response functions yields

$$q^M = \frac{1-c-x}{1+(1-\alpha)(n-k+1)}, \quad q_I^M = \frac{(1-\alpha)(1-c) + [1+(1-\alpha)(n-k)]x}{1+(1-\alpha)(n-k+1)}, \quad \text{and}$$

$$q_O^M = \frac{(1-\alpha)(1-c-x)}{1+(1-\alpha)(n-k+1)}.$$

Output q^M and q_O^M are decreasing in n , and so is q_I^M because $\frac{\partial q_I^M}{\partial n} = -\frac{(1-c-x)(1-\alpha)^2}{[1+(1-\alpha)(n-k+1)]^2} < 0$. Output q^M is increasing in α , while q_I^M and q_O^M are decreasing in α because $\frac{\partial q_O^M}{\partial \alpha} = -\frac{1-c-x}{[1+(1-\alpha)(n-k+1)]^2} < 0$ and $\frac{\partial q_I^M}{\partial \alpha} = -\frac{(1-c-x)}{[1+(1-\alpha)(n-k+1)]^2} < 0$.

5.4 Proof of Lemma 4

The aggregate output is

$$\begin{aligned} Q^M &= q^M + (n-k-1)q_O^M + q_I^M \\ &= \frac{(1-c)[1+(1-\alpha)(n-k)] + (1-\alpha)x}{1+(1-\alpha)(n-k+1)} \end{aligned}$$

which is unambiguously decreasing in c . In addition, Q^M satisfies $\frac{\partial Q^M}{\partial \alpha} = \frac{1-c-x}{[1+(1-\alpha)(n-k+1)]^2} > 0$, $\frac{\partial Q^M}{\partial n} = \frac{(1-c-x)(1-\alpha)^2}{[1+(1-\alpha)(n-k+1)]^2} > 0$, and $\frac{\partial Q^M}{\partial k} = -\frac{(1-c-x)(1-\alpha)^2}{[1+(1-\alpha)(n-k+1)]^2} < 0$.

5.5 Proof of Lemma 5

The CA approves the merger only if it improves consumer surplus, which is equivalent to aggregate output being higher, that is,

$$\frac{(1-c)[1+(1-\alpha)(n-k)] + (1-\alpha)x}{1+(1-\alpha)(n-k+1)} > \frac{(1-c)[(n-1)(1-\alpha) + 1]}{1+n(1-\alpha)}$$

which yields

$$\theta \equiv \frac{x}{1-c} > \frac{1}{(1-\alpha)} \left[\frac{[1 + (1-\alpha)(n-k+1)][(n-1)(1-\alpha)+1]}{1+n(1-\alpha)} - [1 + (1-\alpha)(n-k)] \right] \equiv \bar{\theta}$$

5.6 Proof of Proposition 1

Using outputs from Lemma 1, we find the equilibrium profits for the public firm, $\pi^{NM} = \frac{(1-c)(1-\alpha)^2}{[1+n(1-\alpha)]^2}$, and each private firm, $\pi_i^{NM} = \left(\frac{(1-c)(1-\alpha)}{1+n(1-\alpha)} \right)^2$. The equilibrium profits for merging firms (insiders) is

$$\left(\frac{(1-\alpha)(1-c) + [1 + (1-\alpha)(n-k)]x}{1 + (1-\alpha)(n-k+1)} \right)^2$$

Hence, firms merge if and only if

$$\left(\frac{(1-\alpha)(1-c) + [1 + (1-\alpha)(n-k)]x}{1 + (1-\alpha)(n-k+1)} \right)^2 \geq k \left(\frac{(1-c)(1-\alpha)}{1+n(1-\alpha)} \right)^2$$

which entails,

$$\theta \equiv \frac{x}{1-c} \geq \frac{1}{1 + (1-\alpha)(n-k+1)} \left[\sqrt{k} \frac{(1-\alpha)[1 + (1-\alpha)(n-k+1)]}{1+n(1-\alpha)} - (1-\alpha) \right] \equiv \hat{\theta}.$$

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