EconS 501 - Micro Theory I
Assignment #3 - Due date: September 28th, in class.

1. **Exercises from Nicholson and Snyder (Chapter 6):**

   (a) Exercise 6.10 (Separable utility).

2. **Compensating and equivalent variation - An application.** An individual consumes only good 1 and 2, and his preferences over these two goods can be represented by the utility function

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta} \quad \text{where } \alpha, \beta > 0 \text{ and } \alpha + \beta \gtrless 1$$

   This individual currently works for a firm in a city where initial prices are $p^0 = (p_1, p_2)$, and his wealth is w.

   (a) Find the Walrasian demand for goods 1 and 2 of this individual, $x_1(p, w)$ and $x_2(p, w)$.

   (b) Find his indirect utility function, and denote it as $v(p^0, w)$.

   (c) The firm that this individual works for is considering moving its office to a different city, where good 1 has the same price, but good 2 is twice as expensive, i.e., the new price vector is $p' = (p_1, 2p_2)$. Find the value of the indirect utility function in the new location, i.e., when the price vector is $p' = (p_1, 2p_2)$. Let us denote this indirect utility function $v(p', w)$.

   (d) This individual's expenditure function is

$$e(p, u) = (\alpha + \beta) \left(\frac{p_1}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{p_2}{\beta}\right)^{\frac{\alpha}{\alpha+\beta}} u^{\frac{1}{\alpha+\beta}}$$

   Find the value of this expenditure function in the following cases:

   1. Under initial prices, p^0, and maximal utility level $u^0 \equiv v(p^0, w)$, and denote it by $e(p^0, u^0)$.
   2. Under initial prices, p^0, and maximal utility level $u' \equiv v(p', w)$, and denote it by $e(p^0, u')$.
   3. Under new prices, p', and maximal utility level $u^0 \equiv v(p^0, w)$, and denote it by $e(p^1, u^0)$.
   4. Under new prices, p', and maximal utility level $u' \equiv v(p', w)$, and denote it by $e(p', u')$.

   (e) Find this individual's equivalent variation due to the price change. Explain how your result can be related with this statement from the individual to the media: "I really prefer to stay in this city. In fact, I would accept a reduction in my wealth if I could keep working for the firm staying in this city, instead of moving to the new location"

(f) Find this individual's compensating variation due to the price change. Explain how your result can be related with this statement from the individual to the media: "I really prefer to stay in this city. The only way I would accept to move to the new location is if the firm raises my salary".

(g) Find this individual's variation in his consumer surplus (also referred as area variation). Explain.

(h) Which of the previous welfare measures in questions (e), (f) and (g) coincide? Which of them do *not* coincide? Explain.

(i) Consider how the welfare measures from questions (e), (f) and (g) would be modified if this individual's preferences were represented, instead, by the utility function $v(x_1, x_2) = \alpha \ln x_1 + \beta \ln x_2$.

3. [**Comprehensive exam, August 2011**] Consider a representative consumer in an economy with J goods, $j = 1, 2, ..., J$. Since we are mainly interested in this individual's consumption of goods 1 and 2, we group all the remaining goods $j = 3, 4, ..., J$ as good zero. The price of good zero is $p_0 = 1$ (the numeraire). The prices of goods 1 and 2 are p_1 and p_2, and income is $m > 0$. This consumer's preferences are represented by utility function

$$u(q_1, q_2, q_0) = q_1^{\frac{1}{4}} q_2^{\frac{1}{4}} + q_0$$

(a) Find the Walrasian demands and the associated indirect utility function. Invert the indirect utility function to obtain the expenditure function.

(b) Consider that the price vector increases from $\mathbf{p^0} = (p_1^0, p_2^0) = (1, 1)$ to $\mathbf{p}^1 = (p_1^1, p_2^1) = (2, 1)$, i.e., only the price of good 1 doubles. Let us next use the equivalent variation (EV) to evaluate the loss in welfare that the consumer suffers from the increase in the price of good 1.

1. What is the EV when income satisfies $m > \frac{1}{8}$, i.e., the consumer is relatively rich?
2. What is the EV when income satisfies $\frac{1}{8} > m > \frac{1}{8\sqrt{2}}$, i.e., the consumer is moderately rich?
3. What is the EV when income satisfies $\frac{1}{8\sqrt{2}} > m$, i.e., the consumer is poor?

4. **Exercises from Jehle and Reny (3rd edition):**

(a) Chapter 3: Exercises 3.6, 3.7, and 3.21.