ECONS 424 – STRATEGY AND GAME THEORY

MIDTERM EXAM #2 (TAKE-HOME EXAM)

DUE DATE: MONDAY, APRIL 11TH 2022, IN CLASS.

Instructions:

- Please read the exam and let me know if you have any questions. I will only accept clarifying questions until Friday, April 8th, at midnight.
- Write your answers to each exercise in a different page.
- Show all your work, and be as clear as possible in your answer. You can work in groups, but each student must submit a copy of his/her exam.
- The due date of this take-home exam is Monday, April 11th, at 11:10am, in class. You can submit it earlier via email if you prefer, attaching your responses in a single PDF file.
- I strongly recommend you work a few exercises every day, rather than trying to solve all exercises in one day.
- Because this is a take-home exam, late submission will be subject to significant grade reduction.

Exercise 1 – Simultaneous Public good game

Consider two consumers (1, 2), each with income M to allocate between two goods. Good 1 provides 1 unit of consumption to its purchaser and α units of consumption to the other consumer, where $0 \le \alpha \le 1$. Each consumer *i, i* = 1,2, has the utility function $U^i = \log(x_1^i) + x_2^i$ where $x_1^i = y_1^i + \alpha y_1^j$ is his consumption of good 1 (that takes into account the amount of good 1 purchased by individual *i,* y_1^i, and *j,* y_1^j), and x_2^i is his consumption of good 2.

a) Provide a verbal interpretation of α
b) Suppose that good 2 is a private good. Find the Nash equilibrium levels of consumption when both goods have a price of 1.
c) By maximizing the sum of utilities, show that the equilibrium is Pareto-efficient if $\alpha = 0$ but inefficient for all other values of α.
d) Now suppose that good 2 also provides 1 unit of consumption to its purchaser and α units of consumption to the other consumer, where $0 \le \alpha \le 1$. For the same preferences, find the Nash equilibrium.
e) Show that it is efficient for all values of α.
f) Explain the conclusion in part (d).

Exercise 2 - Stackelberg game with three firms acting sequentially

Consider an industry with *three* firms, each firm *i* having the same cost function $C(q_i) = 5 + 2q_i$, where q_i denotes firm *i*'s output. Industry demand is given by the inverse demand function $P(Q) = 18 - Q$, where Q denotes aggregate output, i. e., $Q = q_1 + q_2 + q_3$. The production timing is as follows:

1) Firm 1 produces its output first.
2) Observing firm 1's output, firm 2 chooses its output.
3) Observing both firm 1's and firm 2's output, firm 3 then produces its own output.

This timing of production is common knowledge among all three firms. The industry demand and cost functions are also known to each firm.

Find the values of output level q_1, q_2 and q_3 in the SPNE of the game.

Exercise 3 – Cournot under incomplete information about market demand

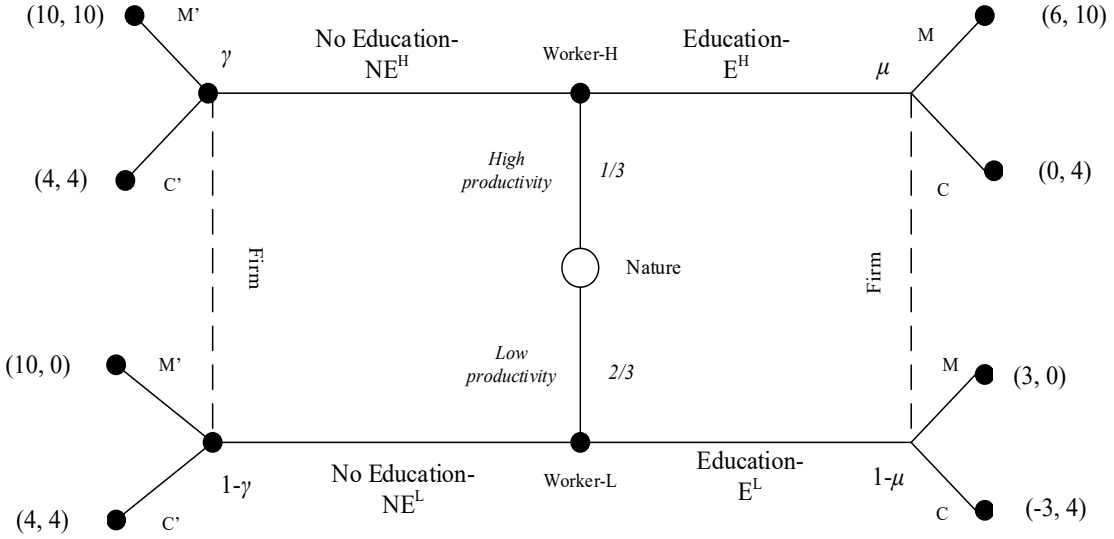
Consider a Cournot duopoly game where two firms compete in quantities. Both firms have the same marginal costs, MC=$2, but they are asymmetrically informed about the actual state of market demand. In particular, firm 2 does not know what the actual state of demand is, but knows that it is p(Q)=20-Q with probability 2/3, and p(Q)=8-Q with probability 1/3.

On the other hand, firm 1 knows the actual state of market demand, and firm 2 knows that firm 1 knows this information (i.e., it is common knowledge among the players).

a) *Informed firm.* Let us first focus on firm 1, the informed player in this game, as we usually do when solving for the BNE of games of incomplete information. Find firm 1's best response function when the firm operates in a high-demand market, and its best response function when the firm operates in a low-demand market.
b) *Uninformed firm.* Let us now turn to firm 2, the uninformed player in this game. Write the expected profits of this firm. Find firm 2's best response function.
c) Use your above results in parts (a) and (b) to find the equilibrium output (a triplet). Summarize the BNE of the game.

Exercise 4 – Finding BNEs

Consider the labor market signaling game we discussed in class, and reproduced in the figure below. Recall that we found two PBEs: one separating, in which the high-productivity worker is the only type acquiring education; and one pooling, in which both types of workers choose no education when the firm's off-the-equilibrium beliefs satisfy $\mu<2/5$. You can read more about these equilibria in the handout of the labor market signaling game posted on the course website.



As we know from other sequential-move games with incomplete information, the set of PBEs is usually a subset of all BNEs. In other words, the set of BNEs is normally large, containing equilibria that are not necessarily sequentially rational (some of these equilibria can be crazy!). Let's check if this is also the case in the labor market signaling game by identifying its BNEs in the following steps, and comparing them with the two PBEs listed above.

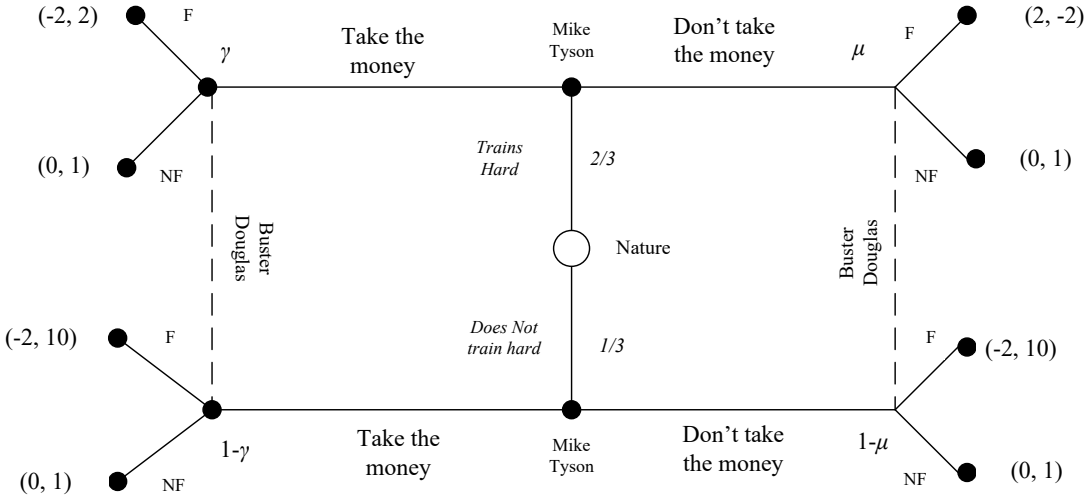
a) Identify the strategy space of the worker and that of the firm. (Hint: each should contain four different strategies.)
b) Construct the Bayesian normal form representation of the game, i.e., a 4x4 matrix. Fill out the expected payoffs of each player in every cell of the matrix.
c) Underline best response payoffs in order to find the set of BNEs in this game.
d) Which of the BNEs you found in step (c) do not coincide with the two PBEs listed at the beginning of this exercise? Why would you say that they predict sequentially irrational responses by the firm?

Exercise 5 – Mike Tyson vs. Buster Douglas

Consider the following sequential move game with incomplete information. The first player to move is Mike Tyson, who privately knows whether he trained hard, or he didn't. Let us assume that the type of training that Mike Tyson receives before a fight is not something he can strategically decide, but instead, it depends on his state of mind between the time he signed up the contract for a fight and the time of the fight. In particular, assume that the probability that Mike Tyson trains hard is given by Nature, and it is 2/3, as depicted in the figure.

Knowing what kind of physical training he had, Mike Tyson decides whether to offer player 2 (Buster Douglas) $1 million dollars if he gives up his right to fight Mike Tyson. We will denote Mike Tyson's strategies as "Take the money" (that is, offering the bribe) or "Don't take the money" (don't offering any bribe to Douglas).

After observing whether Mike Tyson has offered him any money, Buster Douglas must decide whether to Fight (F) or Not Fight (NF), without knowing whether Mike Tyson has previously trained hard or not.



a) Show that there is no Separating PBE where Mike Tyson makes the offer "Take the money" when he has trained hard, but does not make such an offer (he chooses "Don't take the money") when he has not trained hard. In order to show this, follow the usual steps for finding PBE.
   1. Find Buster Douglas' beliefs in this Separating PBE (use Bayes' rule).
   2. Find Buster Douglas' optimal action (whether to Fight or Not Fight) after observing that Mike Tyson offers him "Take the money". In addition, find Buster Douglas' optimal action (whether to Fight or Not Fight) after observing that Mike Tyson does not offers him any bribe (Douglas observes the action "Don't take the money").
   3. Find Mike Tyson's optimal action when he has trained hard, and when he has not trained hard.
   4. Can this separating PBE be supported from your answer in c)? Obviously, you should obtain that it cannot be supported, but you have to show why from your answers in part c).

b) Find a Pooling PBE where Mike Tyson makes the offer "Take the money" when he has trained hard, and he also makes this offer "Take the money" when he has not trained hard. In order to show this, follow the usual steps for finding PBE.
   1. Find Buster Douglas' beliefs in this Pooling PBE (use Bayes' rule).
   2. Find Buster Douglas' optimal action (whether to Fight or Not Fight) after observing that Mike Tyson offers him "Take the money". In addition, find Buster Douglas' optimal action (whether to Fight or Not Fight) after observing that Mike Tyson does not offers him any bribe (Douglas observes the action "Don't take the money").
   3. Find Mike Tyson's optimal action when he has trained hard, and when he has not trained hard.
   4. Can this pooling PBE be supported from your answer in c)?

Exercise 6 – Repeated games

Exercise 1 from Chapter 14 in Harrington's textbook. You can use the first or second edition, as their numbering coincides.