EconS 503 - Microeconomic Theory II
Homework #8 - Due date: April 27th, in class.

1. **Exercises from MWG:** Chapter 23 (mechanism design): Exercise 23.C.10.

2. **Procurement auctions under complete information.** Consider a town mayor inviting N firms to bid in a procurement contract that will allocate to the selected firm the right of water distribution for town residents. The efficiency in implementing the project is $\theta_i \in [0, 1]$, so bidders are regarded as more efficient when their efficiency parameter, θ_i, increases. In this exercise, we consider that all players can observe every bidder's efficiency while in the next exercise we relax this assumption, allowing bidder i to privately observe his efficiency parameter.

   The cost of bidder i to implement the contract is $C_i(q_i, \theta_i)$, which is increasing and convex in output q_i, decreasing and convex in bidder i's efficiency θ_i, and satisfies $\frac{\partial^2 C_i(q_i,\theta_i)}{\partial q_i \partial \theta_i} \leq 0$. Each bidder has a quasilinear utility function,

   $$U(q_i, \theta_i) = t_i(q_i) - C_i(q_i, \theta_i),$$

   where $t_i(q_i)$ represents the transfer that the bidder receives from the procurer when the bidder produces q_i units of output (e.g., gallons of water). For simplicity, assume that bidders earn a zero reservation utility if they choose to not participate in the auction.

   The procurer's welfare function is

   $$V(q_i) - (1 + \lambda) t_i(q_i)$$

   where $V(q_i)$ denotes the value that the procurer assigns to q_i units of output, while λ captures the shadow cost of raising public funds (as the procurer needs to raise distortionary taxes in order to pay the transfer $t_i(q_i)$ to bidder i).

   (a) Interpret the sign of the cross partial derivative, $\frac{\partial^2 C_i(q_i,\theta_i)}{\partial q_i \partial \theta_i}$.

   (b) Setup the procurer's program that induces participation and revelation of the bidders.

   (c) Solve for the socially optimal output of bidder i.

   (d) *Parametric example.* Let us now assume a parametric form for the value and cost functions in a setting with two bidders. In particular, assume that the cost function of bidder i is

   $$C_i(q_i, \theta_i) = \frac{q_i^2}{1 + 2\theta_i}$$

   Furthermore, the value that the procurer assigns to the output of bidder i is $V(q_i) = q_i$, and $\lambda = \frac{1}{10}$. Solve for the optimal output and transfer of bidder i.

3. **Procurement auctions under incomplete information.** Consider the procurement auction in the previous exercise, but assume that every bidder i's efficiency of implementing the project, θ_i, is privately observable to bidder i. Efficiency θ_i is uniformly distributed, $U[0, 1]$, which is common knowledge among all players.

(a) Setup the procurer's program that induces participation and revelation of the bidders.

(b) Solve for the optimal output and transfer of bidder i. [*Hint*: Apply Myerson's Characterization Theorem to rewrite the incentive compatibility condition, and note that the individual rationality condition must hold with equality.]

(c) *Comparison.* Compare your results against those in the complete information setting of the previous exercise (part c). Interpret.

(d) *Parametric example.* Consider the same parametric forms as in the previous exercise (part d). Solve for the optimal output and transfer of bidder i. Compare your results with those in the previous exercise.

4. **Stone-Geary utility function in a pure exchange economy.** Consider a pure exchange economy with two individuals, A and B, whose utility functions are

$$u^A\left(x_1^A, x_2^A\right) = \left(x_1^A - b_1\right)^{\frac{1}{2}}\left(x_2^A - b_2\right)^{\frac{1}{2}}$$
$$u^B\left(x_1^B, x_2^B\right) = x_1^B x_2^B$$

where $b_1, b_2 > 0$ represent the minimal amounts of goods 1 and 2 that individual A must consume in order to remain alive (such as water and shelter). Individuals A and B have endowments of $\omega^A = \left(\omega_1^A, \omega_2^A\right) = (4, 2)$ and $\omega^B = \left(\omega_1^B, \omega_2^B\right) = (2, 4)$, respectively.

(a) Set up the Lagrangian and find the individuals' Walrasian demand functions.

(b) Find the set of Pareto efficient allocations (PEAs). (*Hint*: Your answer should be in terms of b_1 and b_2).

(c) Find the Walrasian equilibrium allocation (WEA). (*Hint*: Your answer should be in terms of b_1 and b_2).

(d) Evaluate the contract curve and WEA at the following three different subsistence levels: (i) $(b_1, b_2) = (4, 2)$, (ii) $(b_1, b_2) = (3, 3)$, and (iii) $(b_1, b_2) = (2, 4)$. In which case(s) is individual A unable to survive?

(e) Consider now a tax transfer so individual A survives in the case(s) you identify in part (b) where he suffers from a negative utility at the WEA. Identify the tax/transfer that the government can impose, and the resulting WEA. (For compactness, let us normalize $p_2 = 1$ so that $p \equiv p_1 = \frac{p_1}{p_2}$.)