EconS 503 - Microeconomic Theory II
Homework #6 - Due date: April 18th, in class.

1. **Nonlinear pricing.** A monopolist faces demand function $Q(p) = 1 - p$, and constant marginal cost c, where $1 > c \geq 0$.

   (a) *Uniform pricing.* Suppose that the monopolist cannot discriminate in any way among the consumers and has to charge a uniform price, p^U. Calculate both the price that maximizes profits and the profits that correspond to this price.

   (b) *Single two-part tariff.* Suppose now that the monopolist can charge a two-part tariff (F, p), where F is the fixed fee and p is the price per unit. Expenditure then is $F + pq$. Calculate the two-part tariff that maximizes profits and the profits that the monopolist earns from this tariff. Compare p^U and F and comment briefly. Compare the situation with a uniform price and a two-part tariff in terms of welfare (a verbal argument is sufficient).

   (c) *Two two-part tariffs.* Assume now instead that there are two types of consumer. The consumers of type 1 have demand $Q_1(p) = 1 - p$ and the consumers of type 2 have demand $Q_2(p) = 1 - \frac{p}{2}$. The population is of size 1 and both consumer types are equally likely and assume, for simplicity, that $c = 1/2$. Calculate the two-part tariff that maximizes the profits of the monopolist. Compare the two-part tariffs found in part (b) and (c), both of them evaluated at $c = 1/2$, and comment briefly.

2. Exercises 12 and 13 from Bolton and Dewatripont.

3. **Hidden information in financial contracts, based on Freixas and Laffont (1990).**[1] Asymmetric information significantly affects behavior in financial markets. In Freixas and Laffont (1990), the principal is a lender who provides a loan of size k to a borrower. The cost of capital is Rk to the lender since it could be invested elsewhere to earn the risk-free interest rate R. The lender has, thus, a utility function $V = t - Rk$. The borrower makes a profit $U = \theta f(k) - t$ where $\theta f(k)$ denotes the borrower's production with k units of capital (e.g., completing a project) and t represents the borrower's repayment to the lender. We assume that $f' > 0$ and $f'' < 0$. Parameter θ denotes a productivity shock, which is either high, θ_H, or low, θ_L, with probabilities p and $1 - p$, respectively.

   (a) *Complete information.* If the lender can observe the borrower's type (the realization of θ), which contract does he offer?

   (b) *Incomplete information.* If the lender does not observe the borrower's type (the realization of θ), find the contract that he offers.

[1] Freixas, Xavier and Laffont, Jean-Jacques (1990) "Optimal Banking Contracts." In: *Essays in Honor of Edmond Malinvaud*. MIT Press. pp. 33-61.

4. **Moral hazard with multiple tasks.**[2] Let us consider a moral hazard problem between a principal and an agent. However, let us now allow the agent to take two effort levels e_1 and e_2. This represents, for instance, a salesman choosing how much effort to exert visiting potential customers, how much time to spend creating a more attractive website for online sales, investigating new sales strategies, etc. In this exercise we seek to understand how the multidimensionality in the agent's effort affects our results in the standard moral hazard problem analyzed in this chapter.

   Assume that the cost of exerting effort levels e_1 and e_2 is

   $$c\left(e_1, e_2\right) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2$$

   These effort levels produce output y with output function

   $$y = f_1 e_1 + f_2 e_2 + \varepsilon$$

   with performance $p = g_1 e_1 + g_2 e_2 + \phi$. Random shocks in output, ε, and performance, ϕ, follow distributions of $G\left(\phi\right)$ and $H\left(\varepsilon\right)$, respectively, with zero expectations, that is, $E(\varepsilon) = E(\phi) = 0$.

   For simplicity, assume that both principal and agent are risk neutral with payoff functions of $\pi = y - w$ for the principal (e.g., firm), where w denotes the salary she pays to the agent; and $U = w - c\left(e_1, e_2\right)$ for the agent (e.g., worker). Consider that the principal offers a salary $w = F + bp$ where F is fixed component of the contract and b is the bonus which provides a higher salary to the agent as his performance p increases. In particular, the timing of the game is as follows:

   - The principal and agent sign a contract $w = F + bp$.
   - The agent takes effort levels e_1 and e_2 which are unobservable to the principal.
   - Random shocks ε and ϕ, are realized, affecting the agent's output and performance, respectively.
   - Output y and performance p are observed by the principal and agent.
   - The agent receives wage $w = F + bp$.

   Answer the following questions.

   (a) Find the agent's optimal efforts and indirect utility as a function of the bonus parameter b.

   (b) Find the principal's optimal contract w^* and his equilibrium profits.

   (c) *Comparative Statics.* How is the optimal contract you found in part (b) affected by the output rates f_1 and f_2? How is it affected by the performance rates g_1 and g_2? Explain.

   (d) Given the optimal contract found above, what are the principal's expected payoff, the agent's expected utility, and the expected social welfare in equilibrium?

[2] For a more general presentation, see Bolton and Dewatripont (2005), pp. 216-28.

(e) What is the socially optimal contract? Compare it against the contract that emerges in the subgame perfect equilibrium of the game you found in part (b).

(f) What is the deadweight loss in this contractual setting?

(g) *Numerical example.* Consider output rates $f_1 = \frac{1}{2}$ and $f_2 = \frac{1}{3}$, and performance rates $g_1 = \frac{2}{3}$ and $g_2 = \frac{1}{4}$. In this context, evaluate the equilibrium bonus b^*, efforts e_1^* and e_2^*, wage w^*, the agent's expected utility $U(w^*)$, the principal's expected profit $\pi(w^*)$, and social welfare SW^*. Then, evaluate the socially optimal bonus b^{**}, social welfare SW^{**} at b^{**}, and deadweight loss due to unobservability of effort.