EconS 424 - Strategy and Game Theory
Homework #8 - Due date: Friday, April 22nd, in class.

1. **Exercises from Watson:**

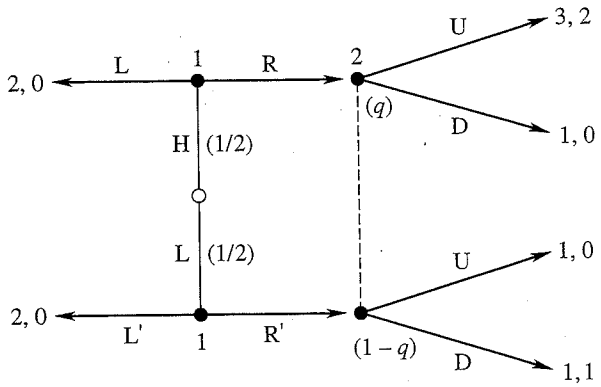
   (a) **Chapter 28:** Exercises 3 and 6 (see scanned pages at the end of this assignment).

2. **Exercises from Harrington**:

   (a) **Chapter 11:** Exercise 4 and 7.

   (b) **Chapter 12:** Exercises 5 and 7.

3. Consider the following game of incomplete information.



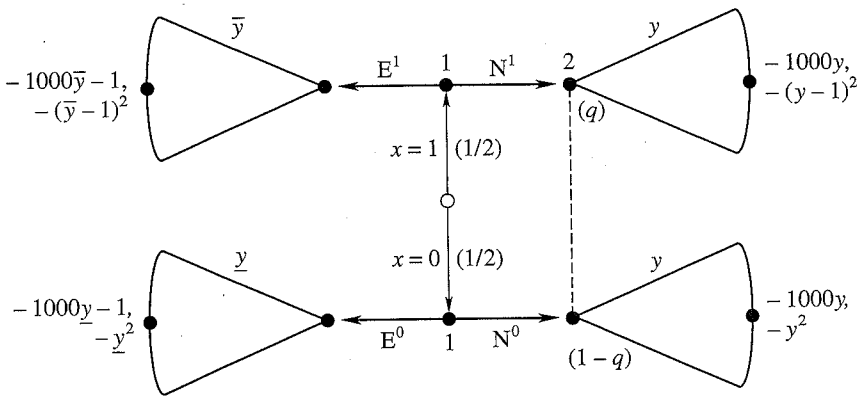
(a) Does this game have a *separating* perfect Bayesian equilibrium? If so, fully describe it.

(b) Does this game have a *pooling* perfect Bayesian equilibrium? If so, fully describe it.

4. Consider an extensive-form game in which player 1 is one of two types: A and B. Suppose that types A and B have *exactly* the same preferences; the difference between these types has something to do with the payoff of another player. Is it possible for such a game to have a separating PBE, where A and B behave differently?

5. A defendant in a court case appears before the judge. Suppose the actual harm to the plaintiff caused by the defendant is equal to $1000x$ dollars, where either $x = 0$ or $x = 1$. That is, the defendant is either innocent ($x = 0$) or guilty of 1000 dollars of damage ($x = 1$). The defendant knows x and has evidence to prove it. The judge does not observe x directly; she only knows that $x = 1$ with probability $1/2$ and $x = 0$ with probability $1/2$.

The judge and defendant interact as follows: First, the defendant has an opportunity to provide his evidence of x. He freely chooses whether or not to provide the evidence; the court cannot force him to do it. Providing evidence to the court costs the defendant one dollar (for photocopying). If the defendant chooses to provide the evidence, then it reveals x to the judge. Whether or not evidence is provided, the judge then decides the level of damages y (in thousands of dollars) that the defendant must pay. The judge prefers to select y "fairly"; she would like y to be as close as possible to x. The defendant wishes to minimize his monetary loss. These preferences and the players' interaction are summarized by the extensive-form diagram shown here. Note that "E" stands for "provide evidence" and N stands for "do not provide evidence."



(a) This game has a unique perfect Bayesian equilibrium. Find and report it. (Hint: Start by showing that it is optimal for the judge to set y equal to the expected value of x, given her belief.)

(b) In one or two sentences, explain why the result of part (a) is interesting from an economic standpoint.

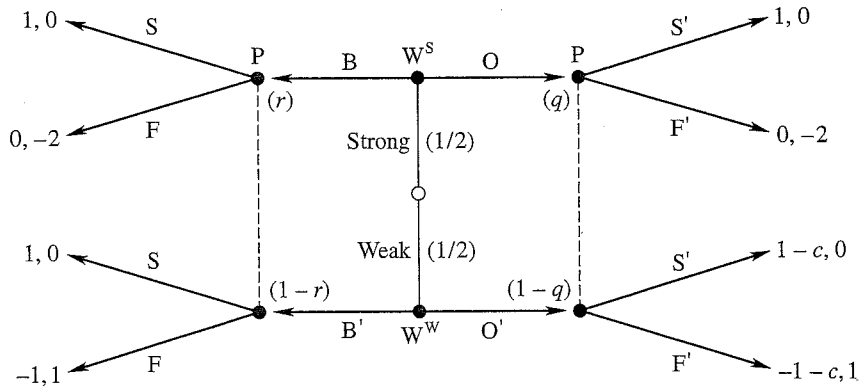
(c) Consider a version of the game in which x is an integer between 0 and K, inclusive, with each of these values equally likely. Compute the perfect Bayesian equilibrium of this game. (Hint: Use your intuition from part (a).)

6. In the classic Rob Reiner movie *The Princess Bride,* there is a scene at the end where Wesley (the protagonist) confronts the evil prince Humperdinck. The interaction can be modeled as the following game: Wesley is one of two types: weak or strong. Wesley knows whether he is weak or strong, but the prince only knows that he is weak with probability 1/2 and strong with probability 1/2. Wesley is lying in a bed in the prince's castle when the prince enters the room. Wesley decides whether to get out of bed (O) or stay in bed (B). The prince observes Wesley's action but does not observe Wesley's type. The prince then decides whether to fight (F) or surrender (S) to Wesley. The payoffs are such that the prince prefers to fight only with the weak Wesley, because otherwise the prince is an inferior swordsman. Also, the weak Wesley must pay a cost c to get out of bed. The extensive-form representation of the game is shown at the top of the next page.

(a) What conditions on c guarantee the existence of a separating PBE? Fully describe such an equilibrium.

(b) For what values of c is there a pooling equilibrium in which both strong and weak Wesleys get out of bed? Fully describe such an equilibrium.

→ FIGURE



7. Consider the "worker status" model of Exercise 3 in Chapter 25. Suppose there are two possible types of worker, H and L. The types differ in the parameters y and q. Specifically, $y = 75$ and $q = 3/5$ for the H type, whereas $y = 65$ and $q = 2/5$ for the L type.

(a) Using your answer from Exercise 3 of Chapter 25, what contract would the firm offer if it knew that the worker's type is L? What if it knew that the worker's type is H?

(b) Consider the game of incomplete information in which the worker knows his own values of y and q, but the firm only knows that the worker is L with probability p and H with probability $1 - p$. Suppose the firm can offer two contracts to the worker, which we can write as $(w^0, 0)$ and $(w^1, 1)$. The interpretation is that the firm is willing to pay w^0 for the safe job and w^1 for the risky job. After observing the firm's offer, the worker decides whether to accept a job and, if so, which contract to take. Suppose that, because of market pressure, the firm is constrained to set $w^0 = 35$. (Other firms are offering job $z = 0$ at wage 35.) Compute the firm's optimal choice of w^1. Explain the steps you take to solve this problem.

(c) Try to find the optimal contract offers for the firm when w^0 is not constrained to equal 35.

8. Compute the PBE of the three-card poker game described in Exercise 4 of Chapter 24. (Hint: Start by determining whether there are any information sets at which a player has an optimal action that is independent of his belief about his opponent's strategy.)