EconS 424 - Strategy and Game Theory
Midterm Exam #1 - Due date: Friday, March 11th, via email.

Instructions:

- This is a take-home exam.
- You can use your class notes and books when working on the exam.
- All questions must be made via email to me at fmunoz@wsu.edu, on March 9th, before midnight.
- Write your answers to each exercise in a different page.
- Show all your work, and be as clear as possible in your answer. You can work in groups, but each student must submit a copy of his/her exam.
- I strongly recommend you to work 3-4 exercises every day, rather than trying to solve all exercises in one day.
- **Submission:**
  - The due date of this take-home exam is Friday, March 11th, at 5:00pm.
  - The exam must be submitted via email to my email address: fmunoz@wsu.edu. You can scan your answers and send them to me (ideally as a PDF file), or type your answers in a Word file, whichever option works best for you.
  - Because this is a take-home exam, late submission will be subject to significant grade reduction.

1. **IDSDS, psNE and msNE.** Consider the following simultaneous-move game played by player 1 (in rows) and player 2 (in columns).

		Player 2		
		x	y	z
	a	$2, 3$	$1, 4$	$3, 2$
Player 1	b	$5, 1$	$2, 3$	$1, 2$
	c	$3, 7$	$4, 6$	$5, 4$
	d	$4, 2$	$1, 3$	$6, 1$

   (a) Which strategy pairs survive the application of iterative deletion of strictly dominated strategies (IDSDS)? [*Hint*: You should be able to delete one strategy for player 2 and two strategies for player 1, leaving you with a 2 by 2 matrix.]

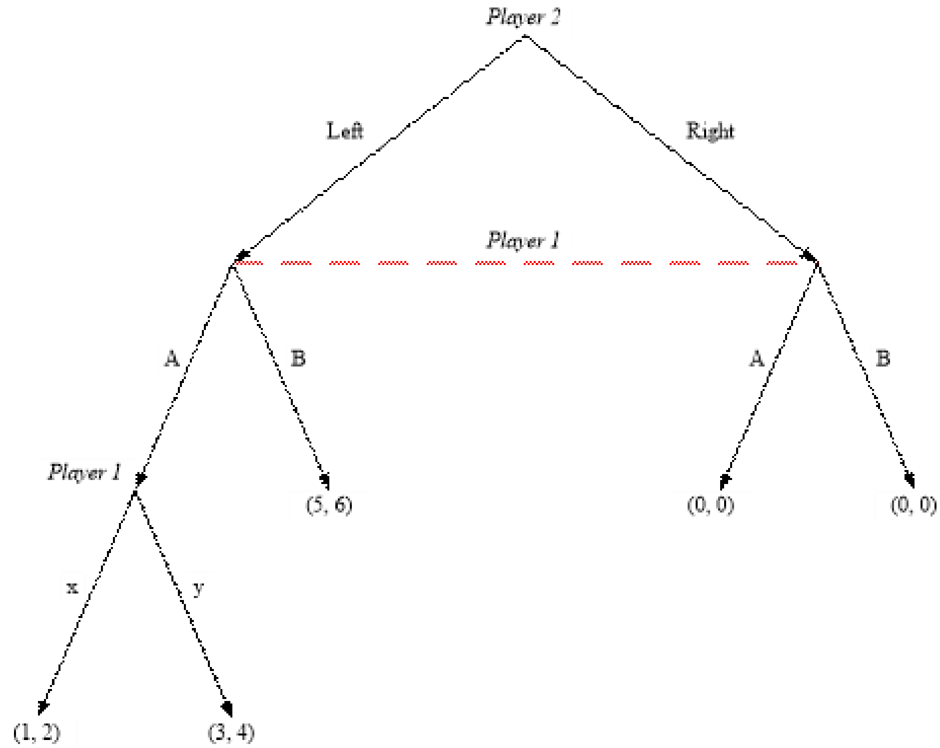
   (b) Using your results from part (a), show that there is no pure strategy Nash equilibrium (psNE) in this game.

   (c) Using your results from part (a), find a mixed strategy Nash equilibrium (msNE) in this game. [*Hint*: You can use p to denote the probability with which player 1 randomizes between b and c, and q to denote the probability with which player 2 randomizes between x and y.]

   (d) In a figure with p in the vertical axis and q in the horizontal axis, draw player 1's best response function, $p(q)$, and player 2's best response function, $q(p)$. (Use different colors, if possible). [*Hint*: The point where both players' best response functions cross each other should coincide with the msNE you previously found in part (b).]

2. **Sequential-move game.** Let us consider the following game, where player 2 chooses firstly whether to go Left (L) or Right (R). Then, without observing what player 2 chose, player 1 is called to choose whether he wants to play A or B. Finally, player 1 is again called to move after playing A in the case in which player 2 initially chose Left.

In this event player 1 can choose between action x and y.



(a) How many strategies does player 1 have in this extensive form game?

(b) Represent the game in its normal form payoff matrix.

(c) Find the pure strategy Nash equilibria of the normal form game you represented in b).

(d) How many proper subgames can you identify? Draw them, and explain why these can be considered proper subgames.

(e) Find the subgame perfect Nash equilibrium (SPNE) of this extensive form game.

3. **Exercise from Harrington:**

   - Chapter 13: Exercises 13 and 15.
   - Chapter 14: Exercise 1 and 11.

4. **R&D tournaments.** Several strategic settings can be modeled as a tournament, whereby the probability of winning a certain prize not only depends on how much effort you exert, but also on how much effort other participants in the tournament exert. For instance, wars between countries, or R&D competitions between different firms in order to develop a new product, not only depend on a participant's own effort, but on the effort put by its competitors. Let's analyze equilibrium behavior in these settings. Consider that the benefit that firm 1 obtains from being the first company

to launch a new drug is \$36 million. However, the probability of winning this R&D competition against its rival (i.e., being the first to launch the drug) is

$$\frac{x_1}{x_1 + x_2},$$

which increases with this firm's own expenditure on R&D, x_1, relative to total expenditure, $x_1 + x_2$. Intuitively, this suggests that, while spending more than its rival, i.e., $x_1 > x_2$, increases firm 1's chances of being the winner, the fact that $x_1 > x_2$ does not guarantee that firm 1 will be the winner. That is, there is still some randomness as to which firm will be the first to develop the new drug, e.g., a firm can spend more resources than its rival but be "unlucky" because its laboratory exploits a few weeks before being able to develop the drug. For simplicity, assume that firms' expenditure cannot exceed 25, i.e., $x_i \leq 25$. The cost is simply x_i, so firm 1's profit function is

$$\pi_1(x_1, x_2) = 36\frac{x_1}{x_1 + x_2} - x_1$$

and there is an analogous profit function for firm 2:

$$\pi_2(x_1, x_2) = 36\frac{x_2}{x_1 + x_2} - x_2$$

You can easily check that these profit functions are concave in a firm's own expenditure, i.e., $\frac{\partial \pi_i(x_i, x_j)}{\partial x_i} \leq 0$ for every firm $i = \{1, 2\}$ where $j \neq i$. Intuitively, this indicates that, while profits increase in the firm's R&D, the first million dollar is more profitable than the 10th million dollar, e.g., the innovation process is more exhausted.

(a) Set up every firm i's profit-maximization problem, differentiate with respect to its investment in R&D, x_i. Find its best-response function, $x_i(x_j)$.

(b) Show that $x_i(x_j)$ is concave in x_j, and find its maximum.

(c) Find a symmetric equilibrium in which both firms invest the same amount in R&D, $x_1^* = x_2^* = x^*$.