Sequential Interaction and Reputation

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- We have seen, players were able to use their ongoing relationship to support cooperative behavior in an infinitely repeated games.
- Examples: infinitely repeated Prisoner's Dilemma or infinitely repeated Cournot game.
- This was not enforcing in the short run for the possibility of temptation to deviate from it.
- Two elements of such cooperative behavior:
  - No predetermined terminal period.
  - Discount factor not too small.

Cooperation as Reputation

- Players in a repeated relationship can create a reputation to cooperate with each other.
- As long as a player maintains his reputation of being cooperative, other player will trust him and respond in kind.
- If a player fails to be cooperative at any stage then he will lose his good will and the players will move to a non-cooperative phase of their engagement (e.g. grim trigger strategies)
- When there is no future or when the future is unimportant, such reputational concerns may be dampened or disappear altogether.

- An example where reputational incentives can facilitate better outcomes than in a one-shot or infinitely repeated setting.

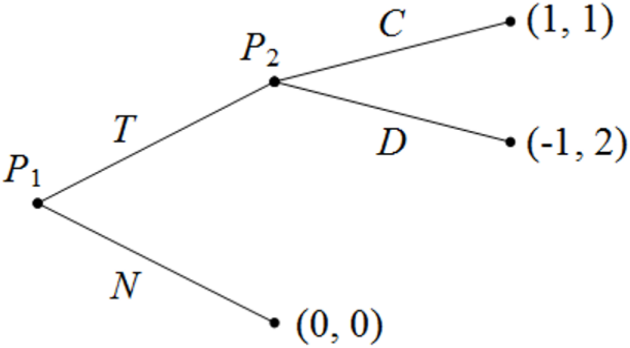

Figure 10.1

- In this "trust game", Player 1 first chooses whether to ask for services of player 2. He can trust player 2 (T) or not trust him (N).
- If player 1 plays T,player 2 to can either cooperate (C) or defect (D).

- In a one-shot game or finitely repeated game, cooperative behavior is not possible as shown in the following figure. Subgame-perfect equilibrium will be (N,D).

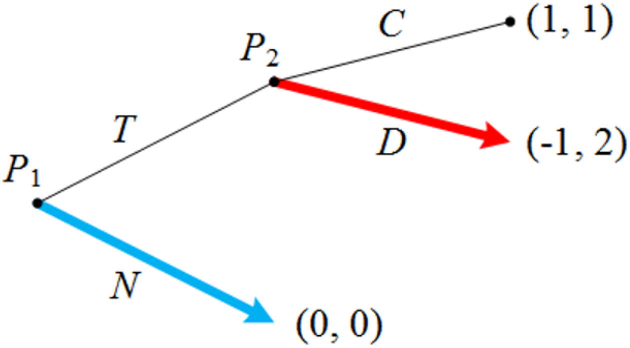

Figure 10.1(a)

- If this one shot game is repeated infinitely with a discount factor of δ, we can create incentives that support cooperative behavior.
- Consider the grim-trigger strategies:
  - **Player 1:** In period 1 I will trust player 2, and so long as there were no deviations from the pair (T,C) in any period then I will continue to trust him. Once such a deviation occurs then I will not trust him forever after.
  - **Player 2:**In period 1 I will cooperate, and as long as there were no deviations from the pair (T,C) in any period then I will continue to do so. Once such a deviation occurs then I will deviate forever after.
- These two strategies will form subgame-perfect equilibrium if no player wants to deviate at any stage.

- Using one-stage deviation principle, no player wants to deviate off the equilibrium path.
- On the equilibrium path:
  - **Player 1** is playing a best response as stream of payoffs of 1 is better than stream of payoffs of 0.
  - **Player 2** will prefer to stick to the equilibrium path if and only if,

$$\frac{1}{1-\delta} \geq 2$$

$$\implies \delta \geq \frac{1}{2}$$

- This commitment is not possible in possible in one shot game.

Third-party Institutions as Reputation Mechanisms

- Possible to provide incentives to player 2 (e.g. trust game) that extend beyond the terminal period of a one-shot interaction?
- One way is to use a third player who acts as a guarantor, or enforcement institution.

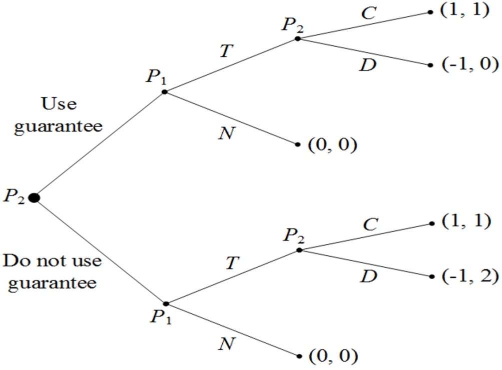

Figure 10.2

- Imagine that player 2 deposits a payment equivalent to 2 in the hands of the guarantor, player 3 with the following contract:

  "If player 1 trusts me and I defect then you keep the payoff of 2 that I deposited in your hands, anything else you return it to me."
- Assume player 3 must follow the contract & has no discretion.

- We use backward induction in the modified game in the figure 10.2(a)

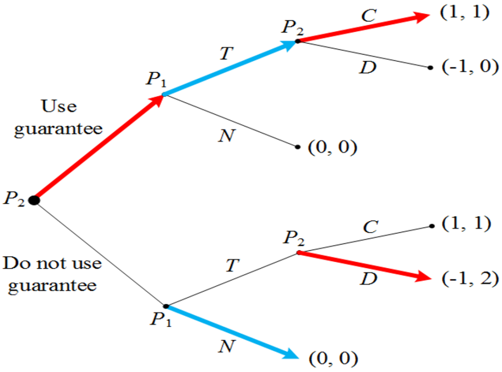

Figure 10.2(a)

- In the unique subgame-perfect equilibrium:
  - Player 2 chooses to use the guarantor and player 1 then rationally anticipates that player 2 will cooperate, which indeed happens.

- Two important question arises:
  - Why would such a committed guarantor exist in the first place?
  - If the guarantor is a 3rd player, shouldn't he be given the option: return/not return the money to player 2 after player 2 cooperates.
- If we add the guarantor as player as player 3, player 3 has a dominant strategy: don't return the money to player 2.
- Consider this:
  - Player 3 is some local official (judge/arbitrator), provides service in $t = 1, 2, ...$ with a discount factor $\delta < 1$ to evaluate future payoffs.
  - In every period t, a new pair of players P_1^t & P_2^t arrive, each player playing a one-shot trust game with no future interaction.
  - Player P_2^t can choose to post a bond equal to 2 with player 3 (figure 10.2), but must pay him a fee of $w = 0.1$ for his service. Alternatively, he can choose not to use player 3's service.

- **Equilibrium:** every pair pf players P_1^t & P_2^t chooses to use player 3's services and successfully engages in cooperative trade in every period, despite not having their own mutual future trades on which to rely?

- **Yes!** Consider the following strategies:

  - **Player** P_1^t: If P_2^t posted a bond with player 3 then trust him, else don't.
  - **Player** P_2^t:**Bond posting:** In $t = 1$ post a bond with player 3. In $t > 1$, if player 3 followed the bond contract in all periods before period t then post a bond with player 3, else don't.
  - **Player 3**: If player P_2^t posts a bond in period T & pays the wage then return the bond to player P_2^t if he cooperated, else don't return the bond.

- To confirm subgame-perfect equilibrium, check if players are playing best responses on & off the equilibrium path, regardless of the history.
- In equilibrium path, players P_1^t &P_2^t are playing a best response in every period (see figure 10.2).
- Player 3: If he follows his proposed strategy, he gets infinite stream of $w = 0.1$ per period. If he defects in any period t, then he will get 2 in that period, and 0 forever after.
- Player 3 will follow his proposed strategy if and only if,

$$\frac{0.1}{1-\delta} \geq 2$$

$$\implies \delta \geq 0.95$$

- Hence, for large enough discount factor, player 3 will happily play the role of bondholder and follow the contract as described.

Reputation Transfers without Third Parties

- Here we only need an abstract entity that can be passed on from one player to the next.
- The entity will have will have have a value in its own right.
- Imagine:
  - In every period t, a new pair of players P_1^t &P_2^t are matched to play a one-shot game (figure 10.1), with no future interaction.
  - Before 1st period starts, player P_2^1 can create a unique or brand name-call it Trusted Associates.
  - Once he creates the brand name player P_2^1 has the sole right to sell that name in the future.

- Now consider the following strategies:
  - **Player** P_1^1: If P_2^1 created a unique brand name then trust him, else don't.
  - **Player** P_2^1: Choose a unique name, and cooperate with player P_1^1. Afterwards offer the brand name for sale to player P_2^2 at a price $p* > 1$.
  - **Player** $P_1^t(t > 1)$:If (1) P_2^t bought the brand name from P_2^{t-1} and (2) no abuse of trust (defection) ever occurred under that brand name then trust him, else don't.
  - **Player** $P_2^t(t > 1)$:If (1) P_2^{t-1} bought the brand name from P_2^{t-2} at price $p*$ and (2) no abuse of trust (defection) ever occurred under that brand name then buy the brand name from P_2^{t-1}, and then cooperate with p_1^t.Afterwards offer the brand name for sale to player P_2^{t+1} at a price $p*$.

- The above strategies will constitute a subgame-perfect equilibrium (you are asked to do this in exercise 10.11).

- The idea behind the construction of these strategies is simple and appealing:

    - The so-called brand name acts as a bearer of firm reputation that is passed on from one player 2 to the next.

    - The investment made in the name before play begins in period t acts like a bond, and it will only be recovered if that period's player 2 will indeed cooperate.