EconS 503 - Microeconomic Theory II
Homework #6 - Due date: April 1st, in class.

1. **Cheap talk with a continuum of types.** Consider the Crawford-Sobel cheap talk game with a continuum of types. In class we discussed the maximal number of partitions n that can be sustained as a PBE of the game.

   (a) Find the ex-ante expected utility that the sender obtains in equilibrium. (By "ex-ante" we mean before observing the realization of parameter θ.)

   (b) Find the ex-ante expected utility that the receiver obtains in equilibrium.

   (c) How are the above ex-ante expected utilities affected by an increase in the preference divergence parameter δ?

2. **Signaling when the expert receives imprecise signals.** Consider the following signaling model between an expert (E), such a special interest group, and a decision maker (DM), such as a politician. For simplicity, assume that the state of the world is discrete, either $\theta = 1$ or $\theta = 0$ with prior probability $p \in (0, 1)$ and $1 - p$, respectively. The expert privately observes an informative but noisy signal s, which also takes two discrete values $s \in \{0, 1\}$. The precision of the signal is given by the conditional probability

$$\text{prob}(s = k|\theta = k) = q,$$

where $k = \{0, 1\}$, and $q > \frac{1}{2}$. In words, the probability that the signal s coincides with the true state of the world θ is q (precise signal), while the probability of an imprecise signal where $s \neq \theta$ is $1 - q$. The time structure of the game is as follows:

   1) Nature chooses θ according to the prior p.
   2) Expert observes signal s and reports a message $m \in \{0, 1\}$
   3) Decision maker observes m and responds with $x \in \{0, 1\}$
   4) θ is observed and payoffs are realized

The payoff function for the decision maker is

$$u(x, \theta) = \left(\theta - \frac{1}{2}\right) x$$

while that of the expert is

$$v(m, \theta) = \begin{cases} 1, & \theta = m \\ 0, & \theta \neq m \end{cases}$$

which, in words, indicates that the expert's payoff is 1 when the message she sends coincides with the true realization of the state of the world, but becomes zero otherwise. Importantly, her payoff is unaffected by the signal, which she only uses to infer the actual realization of parameter θ. Intuitively, $v(m, \theta)$ is often understood as a "reputation function" since it provides the expert with a payoff of 1 only when his message was an accurate representation of the true state of the world (which in this model he does not precisely observe).

(a) Is there a Perfect Bayesian equilibrium (PBE) in which the expert reports his signal truthfully?

3. **Exercise from MWG.** Chapter 13, exercise 13.C.2.

4. **Too-Cool-for-School, or Countersignaling, based on Feltovich et al. (2002).**[1] Consider the following labor signaling game with three types. Bob privately observes his productivity, θ_L, θ_M, or θ_H (denoting low, medium, or high productivity), where $0 < \theta_L < \theta_M < \theta_H$. The prior probability that Bob's type is being θ_L is λ_L, the prior probability that his type is θ_M is λ_M, and the probability that his type is θ_H is $1 - \lambda_L - \lambda_M$. Bob's acquires an education level $e \geq 0$, with cost function $c(e, \theta) = \frac{e}{\theta}$. A manager (Alice) observes Bob's education, e, but doesn't observe his type, and responds offering a salary $w \geq 0$ to Bob. Alice's utility function is $(\theta - w)^2$, which depends on Bob's true type, θ, and the salary she pays, w. Bob's utility function is $w - c(e, \theta)$. Each agent is an expected utility maximizer.

   We modify the above three-type labor market signaling game as follows. Carol, Bob's former employer, has learned something about his type. In particular, if Bob's type is θ_L , Carol believes that Bob is "sloppy." If Bob's types is θ_H, however, Carol believes that Bob is a "pro." If Bob's type is θ_M, Carol believes that he is sloppy with probability $p \in (0, 1)$ and a pro with probability $1 - p$. Here is the time structure of the game:

   1) At time $t = 0$, Alice can privately meet Carol and learn whether, in Carol's opinion, Bob is sloppy or a pro.
   2) At time $t = 1$, Bob acquires an education level $e \geq 0$ at the cost $c(e, \theta)$.
   3) At time $t = 2$, Alice meets Bob and observes his education e, and pays him a salary $w \geq 0$.
   4) At time $t = 3$, Alice's and Bob's utilities are realized.

   Answer the following questions:

   (a) Assuming that Alice does not meet Carol at time $t = 0$ and that Bob knows that Alice does not meet Carol, solve for the least-costly separating equilibrium. Interpret.

   (b) Assume that Alice meets Carol at time $t = 0$ and learns whether Bob is sloppy or a pro, and that Bob knows that Alice meets Carol but does not know what Alice learns. Identify the parameter conditions for which there exists a so-called countersignaling (or too-cool-for school) equilibrium, in which Bob's chooses education levels $e_L = 0$, $e_M > 0$, and $e_H = 0$. Why do you think that this equilibrium is called "countersignaling"?

   (c) Does the equilibrium found in part (b) survive the Cho and Kreps' Intuitive Criterion?

[1] Feltovich, N., R. Harbaugh, and T. To (2002) "Too Cool for School? Signaling and Countersignaling", RAND Journal of Economics, 33(4), pp. 630-49. Downloadable in Rick Harbaugh's webpage: https://kelley.iu.edu/riharbau/.

(d) Would Alice be better off if she could publicly commit not to discuss Bob's past job performance with Carol? Would Bob be better off?

(e) In 1970s, Gola Meir, then the Israeli prime minister, reproached a U.S. diplomat, who had just given a speech in Jerusalem: "You shouldn't be so humble; you are not so great." Discuss Golda Meir's remark in the context of the countersignaling model.