EconS 503 - Microeconomic Theory II
Homework #3 - Answer key

1. **Cost-reducing investment followed by Cournot competition-I.** Consider a duopoly market with two firms selling a homogeneous product, facing inverse demand curve $p(Q) = 1 - Q$, where $Q = q_1 + q_2$ denotes aggregate output, and facing marginal cost c, where $1 > c \geq 0$. In the first stage of the game, every firm i chooses its investment in cost-reducing technology, z_i; and, in the second stage, observing the profile of investment levels (z_i, z_j), firms compete a la Cournot.

   Investment z_i reduces firm i's marginal cost, from c to $c - \frac{1}{4}z_i$; and the cost of investing z_i in the first stage is $\frac{1}{2}z_i^2$. For simplicity, assume no discounting of future payoffs.

   (a) *Second stage.* Operating by backward induction, find firm i's best response function $q_i(q_j)$ in the second stage. How is it affected by a marginal increase in z_i? And by a marginal increase in z_j?

   - Firm i's profit maximization problem in second stage is:

     $$\max_{q_i \geq 0} \pi_{i2} = (1 - q_i - q_j)q_i - \left(c - \frac{1}{4}z_i\right) q_i$$

     Differentiating with respect to q_i, we obtain

     $$1 - 2q_i - q_j - c + \frac{1}{4}z_i = 0$$

     Solving for q_i gives firm i's best response function:

     $$q_i(q_j) = \frac{1 - \left(c - \frac{1}{4}z_i\right)}{2} - \frac{1}{2}q_j$$

     which originates at $\frac{1-\left(c-\frac{1}{4}z_i\right)}{2}$ and decreases in q_j at a rate of $\frac{1}{2}$. Note that when $z_i = 0$, the vertical intercept simplifies to $\frac{1-c}{2}$, as in standard Cournot models of quantity competition.
   - The slope of the best response function is unaffected by z_i, but the vertical intercept increases in z_i since

     $$\frac{\partial}{\partial z_i}\left(\frac{1 - \left(c - \frac{1}{4}z_i\right)}{2}\right) = \frac{1}{8} > 0$$

     Therefore, as firm i's investment in the cost-reducing technology increases, its net cost decreases, and the best response function shifts upward. However, firm i's best response function is unaffected by its rival's investment in the cost-reducing technology, z_j.

(b) Find equilibrium output and profits in the second stage, as a function of investment levels (z_i, z_j). Are they increasing in z_i? Are they increasing in z_j? Interpret.

- Inserting $q_j(q_i)$ into firm i's best response $q_i(q_j)$, we obtain

$$q_i = \frac{1-(c-\frac{1}{4}z_i)}{2} - \frac{1}{2}\left(\underbrace{\frac{1-(c-\frac{1}{4}z_j)}{2} - \frac{1}{2}q_i}_{q_j(q_i)}\right)$$

which simplifies to

$$q_i(z_i, z_j) = \frac{1-c}{3} + \frac{2z_i - z_j}{12}.$$

When $z_i = z_j = 0$, firm i produces the same output as under standard Cournot competition, $q_i = \frac{1-c}{3}$; when $z_i < \frac{z_j}{2}$, firm i produces fewer units; and when $z_i > \frac{z_j}{2}$, firm i produces more units.

- Inserting $q_i(z_i, z_j)$ into firm j's best response function, we find a symmetric output for firm j, as follows,

$$q_j(z_i, z_j) = \frac{1-c}{3} + \frac{2z_j - z_i}{12}$$

Differentiating with respect to z_i and z_j, we obtain

$$\frac{\partial q_i(z_i, z_j)}{\partial z_i} = \frac{\partial q_j(z_i, z_j)}{\partial z_j} = \frac{1}{6} > 0 \text{ and}$$

$$\frac{\partial q_i(z_i, z_j)}{\partial z_j} = \frac{\partial q_j(z_i, z_j)}{\partial z_i} = -\frac{1}{12} < 0$$

Therefore, as firm i's (j's) investment in cost-reducing technology z_i (z_j) increases, firm i produces more (fewer, respectively) units.

- Substituting the equilibrium outputs into firm's profit function, we obtain that

$$\begin{aligned}
\pi_{i2} &= \left[1 - \left(\frac{1-c}{3} + \frac{2z_i - z_j}{12}\right) - \left(\frac{1-c}{3} + \frac{2z_j - z_i}{12}\right)\right]\left(\frac{1-c}{3} + \frac{2z_i - z_j}{12}\right) \\
&\quad - \left(c - \frac{1}{4}z_i\right)\left(\frac{1-c}{3} + \frac{2z_i - z_j}{12}\right) \\
&= \left(\frac{1-c}{3} + \frac{1}{4}z_i - \frac{z_i + z_j}{12}\right)\left(\frac{1-c}{3} + \frac{2z_i - z_j}{12}\right) \\
&= \frac{[4(1-c) + 2z_i - z_j]^2}{144}
\end{aligned}$$

Differentiating profits with respect to z_i and z_j, we obtain

$$\frac{\partial \pi_{i2}(z_i, z_j)}{\partial z_i} = \frac{2z_i - z_j}{36} + \frac{1-c}{9}, \text{ and}$$

$$\frac{\partial \pi_{i2}(z_i, z_j)}{\partial z_j} = -\frac{2z_i - z_j}{72} - \frac{1-c}{18}$$

In a symmetric equilibrium, where $z_i = z_j = z$, we obtain that the first (second) derivative is positive (negative), thus indicating that firm i's profits increase (decrease) in its own investment (its rival's investment) in cost-reducing technology, z_i (z_j, respectively).

(c) *First stage.* Find the equilibrium investment levels that firms choose in the first stage, z_i^* and z_j^*.

- Firm i's profit maximization problem in first stage is:

$$\max_{z_i \geq 0} \pi_{i1} = \underbrace{\frac{[4(1-c) + 2z_i - z_j]^2}{144}}_{\pi_{i2}\text{, found in part (b)}} - \frac{1}{2}z_i^2$$

Differentiating with respect to z_i, we obtain

$$\frac{2z_i - z_j}{36} + \frac{1-c}{9} - z_i = 0$$

Solving for z_i, we have

$$z_i(z_j) = \frac{2(1-c)}{17} - \frac{1}{34}z_j$$

which originates at $\frac{2(1-c)}{17}$ and decreases in z_j at a rate of $\frac{1}{34}$. Intuitively, an increase in firm j's investment, z_j, induces firm i to respond decreasing its own investment, z_i.

- In a symmetric equilibrium, both firms invest the same amount in cost-reducing technologies, $z_i = z_j = z$. Inserting this property into the above first-order condition, yields

$$\frac{z}{36} + \frac{1-c}{9} - z = 0$$

Solving for z, we obtain

$$z^* = \frac{4(1-c)}{35}.$$

(d) *Joint venture.* If, in the first stage, firms could coordinate their investment levels (z_i, z_j) to maximizes their joint profits, what would their investment levels be? This investment decision resembles a "joint venture," where firms coordinate their R&D activities, or any other decision, and then compete in a subsequent stage (in this case, a la Cournot). Compare your results with those in part (c).

- Firms' joint profit maximization problem in first stage is:

$$\begin{aligned}\max_{z_i, z_j} \pi_1 &= \pi_{i1} + \pi_{j1} \\ &= \left[\frac{[4(1-c) + 2z_i - z_j]^2}{144} - \frac{1}{2}z_i^2\right] + \left[\frac{[4(1-c) + 2z_j - z_i]^2}{144} - \frac{1}{2}z_j^2\right]\end{aligned}$$

Differentiating with respect to z_i and z_j, we obtain

$$\frac{1-c}{18} - \frac{67z_i + 4z_j}{72} = 0$$

$$\frac{1-c}{18} - \frac{67z_j + 4z_i}{72} = 0$$

In a symmetric equilibrium, both firms invest the same amount in cost-reducing technologies, $z_i = z_j = z$. Inserting this property into the above first-order conditions, yields

$$\frac{1-c}{18} - \frac{71z}{72} = 0$$

Solving for z, we get

$$z^{JV} = \frac{4(1-c)}{71}$$

where the superscript JV denotes "joint venture."

- Comparing firms' investment in part (c) and (d) by taking difference between z^* and z^{JP}, we have

$$\begin{aligned} z^* - z^{JV} &= \frac{4(1-c)}{35} - \frac{4(1-c)}{71} \\ &= \frac{144(1-c)}{2485} > 0 \end{aligned}$$

Intuitively, when firms internalize their investment in cost-reducing technologies (in the joint venture), they invest less than in equilibrium. In other words, when every firm i independently chooses its investment in cost-reducing technologies, z_i, it ignores the externality that its investment imposes on its rival (namely, expanding the cost differential between the two firms).

2. **Collusion with imperfect monitoring (Continuous strategies and firms competing a la Cournot).** Consider two firms selling a homogenous product, competing a la Cournot, and facing a linear inverse demand function $p(Q) = 1 - Q$, where Q denotes aggregate output. For simplicity, assume that they are symmetric in their marginal production cost, c, which is normalized to $c = 0$.

   (a) *Stage game.* If the game is not repeated, find the equilibrium output, q^C, and profit for each firm.

   - Firm i's profit maximization problem is:

$$\max_{q_i} \pi_i = (1 - q_i - q_j)q_i$$

with FOC

$$1 - 2q_i - q_j = 0$$

Solving for q_i gives firm i's best response function:

$$q_i(q_j) = \frac{1}{2} - \frac{q_j}{2}$$

By symmetry, i.e.,$q_i = q_j$, we have the equilibrium output

$$q_i^C = q_j^C = q^C = \frac{1}{3}$$

Given the equilibrium output, we can obtain the equilibrium price

$$p^C = 1 - q_i^C - q_j^C = \frac{1}{3}$$

and the equilibrium profit

$$\pi^C = p^C q^C = \frac{1}{9}$$

(b) *GTS with perfect monitoring.* For the remainder of the exercise, assume that firms interact in an infinitely-repeated game. Find the minimal discount factor sustaining cooperation if firms use a standard GTS, namely, in the first period, every firm produces half of the monopoly output, $\frac{q^m}{2}$; in all subsequent periods, every firm produces $\frac{q^m}{2}$ if $\left(\frac{q^m}{2}, \frac{q^m}{2}\right)$ was the outcome in all previous periods. Otherwise, every firm reverts to the NE of the stage game, q^C, thereafter. In this part of the exercise, assume that firms can immediately observe deviations (perfect monitoring).

- The monopolist's profit maximization problem is

$$\max_q \pi = (1 - q)q$$

with FOC

$$1 - 2q = 0$$

Solving for q gives monopolist's equilibrium output:

$$q^m = \frac{1}{2}$$

Given the monopoly output, we can obtain the monopoly price

$$p^m = 1 - q^m = \frac{1}{2}$$

and the monopoly profit

$$\pi^m = p^m q^m = \frac{1}{4}$$

- Of which each firm gets half, or

$$\pi_{it} = \frac{\pi^m}{2} = \frac{1}{8}$$

Summing up each firm's lifetime profits, we have

$$\begin{aligned} \frac{\pi^m}{2} + \delta\frac{\pi^m}{2} + \delta^2\frac{\pi^m}{2} + ... &= \frac{1}{1-\delta}\frac{\pi^m}{2} \\ &= \frac{1}{8(1-\delta)} \end{aligned}$$

- If a firm chooses to deviate, it will produce

$$q^{Dev} = \frac{1}{2} - \frac{\frac{q^m}{2}}{2} = \frac{3}{8}$$

and receive deviation profit for the period it deviates to q^{Dev}:

$$\pi^{Dev} = (1 - \frac{3}{8} - \frac{1}{4})\frac{3}{8} = \frac{9}{64}$$

Once the deviation is detected, both firms will charge the NE of the stage game q^C for all future periods and receive π^C thereafter. The profit if firm deviates is

$$\begin{aligned} \pi^{Dev} + \delta\pi^C + \delta^2\pi^C + \ldots &= \pi^{Dev} + \frac{\delta}{1-\delta}\pi^C \\ &= \frac{9}{64} + \frac{\delta}{9(1-\delta)} \end{aligned}$$

- Therefore, to sustain cooperation, we must have the lifetime discounted profits be weakly higher than seizing the market for one period and receiving π^C afterwards, i.e.,

$$\frac{1}{8(1-\delta)} \geq \frac{9}{64} + \frac{\delta}{9(1-\delta)} \Longrightarrow \delta \geq \frac{9}{17}$$

The GTS is a SPE for $\delta \geq \frac{9}{17}$. Collusion can be sustained if the firms are patient enough.

(c) *GTS with imperfect monitoring.* Assume now that firms suffer from imperfect monitoring: if firm j deviates from the collusive output $\frac{q^m}{2}$, firm i observes its deviation with probability

$$p_i = \frac{q_j - \frac{q^m}{2}}{1 - \frac{q^m}{2}},$$

where q_j denotes firm j's output. This probability is positive when firm j deviates from the collusive output, $q_j > \frac{q^m}{2}$, and increases monotonically when firm j produces a larger output. Intuitively, firm i may not observe its rival's deviation when firm j produces barely above the collusive output $\frac{q^m}{2}$, but it will likely observe its deviation when firm j produces a large output. In this context, the GTS described in part (b) must be rewritten as follows: In the first period, every firm produces half of the monopoly output, $\frac{q^m}{2}$; in all subsequent periods, every firm produces $\frac{q^m}{2}$ if $\left(\frac{q^m}{2}, \frac{q^m}{2}\right)$ was the outcome *observed* in all previous periods. Otherwise, every firm reverts to the NE of the stage game, q^C, thereafter.

- If no firm has observed deviation, the firm receives:

$$\begin{aligned} \frac{\pi^m}{2} + \delta\frac{\pi^m}{2} + \delta^2\frac{\pi^m}{2} + \ldots &= \frac{1}{1-\delta}\frac{\pi^m}{2} \\ &= \frac{1}{8(1-\delta)} \end{aligned}$$

- However, if firm j deviates, firm i observes j's deviation with probability $p_i = \frac{q_j - \frac{q^m}{2}}{1-\frac{q^m}{2}}$, where $\frac{q^m}{2} = \frac{1}{4}$ from part (b). Therefore, the probability of detection is

$$
\begin{aligned}
p_i &= \frac{q_j - \frac{1}{4}}{1 - \frac{1}{4}} \\
&= \frac{q_j - \frac{1}{4}}{\frac{3}{4}} \\
&= \frac{4q_j - 1}{3}
\end{aligned}
$$

where probability p_i is well behaved since $p_i \geq 0$ for all $q_j \geq \frac{1}{4}$, which holds as long as firm j produces at least the collusive output $\frac{q^m}{2} = \frac{1}{4}$; and $p_i \leq 1$ for all $q_j \leq \frac{1}{2}$, which holds if firm j produces less than the aggregate output under monopoly.
Hence, the payoff of deviation is

$$
\begin{aligned}
&\pi_j + \delta \left[p_i \frac{\overbrace{\pi^C}^{\text{Detected}}}{1-\delta} + (1-p_i) \frac{\overbrace{\frac{\pi^m}{2}}^{\text{Undetected}}}{1-\delta} \right] \\
= &\underbrace{\left(1 - \frac{1}{4} - q_j\right) q_j}_{\pi_j} + \delta \left[\overbrace{\frac{4q_j - 1}{3}}^{p} \overbrace{\frac{1}{9}\frac{1}{1-\delta}}^{\text{Detected}} + \overbrace{\left(1 - \frac{4q_j - 1}{3}\right)}^{1-p} \overbrace{\frac{1}{8}\frac{1}{1-\delta}}^{\text{Undetected}} \right] \\
= &\left(\frac{3}{4} - q_j\right) q_j + \frac{\delta}{1-\delta}\left(\frac{7 - q_j}{54}\right)
\end{aligned}
$$

Therefore, the most profitable deviation, q_j, is that solving

$$
\max_{q_j \geq 0} \left(\frac{3}{4} - q_j\right) q_j + \frac{\delta}{1-\delta}\left(\frac{7 - q_j}{54}\right)
$$

Differentiating with respect to q_j, yields

$$
\frac{3}{4} - 2q_j - \frac{\delta}{54(1-\delta)} = 0
$$

and solving for q_j, we obtain

$$
q_j^* = \frac{3}{8} - \frac{\delta}{108(1-\delta)} \geq 0 \Longrightarrow \delta \leq \frac{81}{83}
$$

Substituting the q_j^* into the profit function, we have

$$\begin{aligned} & \left(\frac{3}{4} - q_j\right) q_j + \frac{\delta}{1-\delta}\left(\frac{7-q_j}{54}\right) \\ = & \left(\frac{3}{4} - \frac{3}{8} + \frac{\delta}{108(1-\delta)}\right)\left(\frac{3}{8} - \frac{\delta}{108(1-\delta)}\right) + \frac{\delta}{1-\delta}\left(\frac{7-\left(\frac{3}{8} - \frac{\delta}{108(1-\delta)}\right)}{54}\right) \\ = & \frac{1421\delta^2 - 7938\delta + 6561}{46656(1-\delta)^2} \end{aligned}$$

- Therefore, to sustain cooperation, we must have the lifetime discounted profits be weakly higher than those from deviation, that is,

$$\frac{1}{8(1-\delta)} \geq \frac{1421\delta^2 - 7938\delta + 6561}{46656(1-\delta)^2}$$

which yields two roots for δ, as follows

$$\frac{27}{49} \leq \delta \leq \frac{27}{29}$$

3. **Exercise 10.6 from Tadelis.**

(a) Let player 1 be the firm who can choose G (good) or B (bad), and player 2 is the consumer who can choose P (purchase) or N (not purchase), as shown in the following matrix. To understand the payoff pairs, note that when players choose (G, P), the firm gets $6-4=2$ and the consumer gets $9-6=3$. If players choose (G, N), then the firm gets $0-4=-4$ and the consumer gets 0. If players choose (B, P), the firm gets $6-0=6$ and the consumer gets $4-6=-2$. Finally, if players choose (B, N), then both firm and consumers earn 0.

Player 1 \ *Player 2*	P	N
G	$2, 3$	$-4, 0$
B	$6, -2$	$0, 0$

(b) It is easy to see that player 1 finds G to be strictly dominated by G. Deleting the row corresponding to G from the above 2x2 matrix, we obtain

Player 1 \ *Player 2*	P	N
B	$6, -2$	$0, 0$

At this point, player 2's best response is to choose N. Therefore, the unique psNE in the second stage if (B, N). By backward induction, (B, N) must also be played in the first stage. Hence, (B, N) played in both stages is the unique subgame perfect equilibrium of the game.

(c) Consider the following Grim Trigger Strategy (GTS):

- At $t = 1$, player 1 chooses G, and continues doing so if he chose G in the past and as long as player 2 purchased. Otherwise, he plays B forever after (his Nash equilibrium strategy in the unrepeated version of the game).
- At $t = 1$, player 2 chooses P and continues doing so if he chose P in the past and as long as player 1 chose G. Otherwise, he plays N forever after (his Nash equilibrium strategy in the unrepeated version of the game).

Player 2 has no incentive to deviate at any stage, but player 1 can earn 6 from switching to B in any period (earning 6 instead of 2). He will not have incentive to deviate if

$$\frac{2}{1-\delta} \geq 6 + \frac{0}{1-\delta}$$

which simplifies to $\delta \geq \frac{2}{3}$.

(d) If the price of the drug is lowered to 5, the payoff matrix in part (a) becomes:

Player 2

Player 1		P	N
	G	$1, 4$	$-4, 0$
	B	$5, -1$	$0, 0$

Following the same argument as in part (c), player 2 has no incentive to deviate at any stage. Player 1, however, can earn 5 from switching to B in any period (earning 5 instead of 1). He will not have incentive to deviate if

$$\frac{1}{1-\delta} \geq 5 + \frac{0}{1-\delta}$$

which simplifies to $\delta \geq \frac{4}{5}$.
Hence, if $\frac{4}{5} > \delta \geq \frac{2}{3}$, then, increasing the price from 4 to 5 will cause the good equilibrium, (G, P), to collapse and no trade will occur. In contrast, if $\delta \geq \frac{4}{5}$, the argument in favor of raising the price can be made because, intuitively, consumers benefit at the expense of the firm but there is enough surplus to support the good outcome.

4. **Exercises from Tadelis:**

(a) Chapter 10: Exercises 10.6, 10.9, and 10.11.

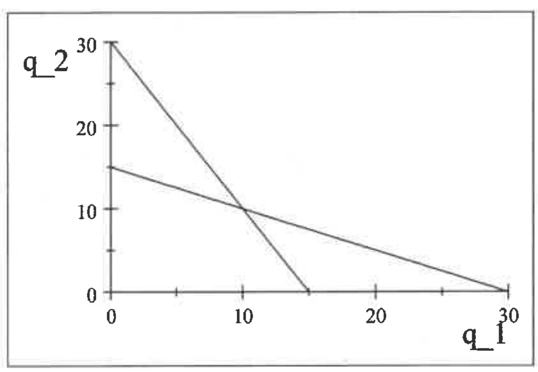
- See scanned pages at the end of this handout.

or $\delta^2 + \delta - 1 \geq 0$, which results in $\delta \geq \frac{1}{2}\sqrt{5} - \frac{1}{2} \approx 0.618$. The reason we need a larger discount factor is that the punishment is less severe as it lasts for only two periods and not infinitely many. ■

9. **Negative Externalities:** Two firms are located adjacent to one another and each imposes an external cost on the other: the detergent that Firm 1 uses in it's laundry business makes the fish that firm 2 catches in the lake taste funny, and the smoke that firm 2 uses to smoke its caught fish makes the clothes that firm 1 hands out to dry smell funny. As a consequence, each firms profits are increasing it its own production and decreasing in the production of its neighboring firm. In particular, if q_1 and q_2 are the firms' production levels then their per-period (stage game) profits are given by $v_1(q_1, q_2) = (30 - q_2)q_1 - q_1^2$ and $v_2(q_1, q_2) = (30 - q_1)q_2 - q_2^2$.

   (a) Draw the firms' best response functions and find the Nash equilibrium of the stage game. How does this compare to the Pareto optimal stage-game profit levels?

   Answer: Each firm maximizes $v_1(q_i, q_j) = (30 - q_j)q_i - q_i^2$ and the first order condition is $30 - q_j - 2q_i = 0$, resulting in the best response function $q_i = \frac{30 - q_j}{2}$ as drawn in the following figure:



   The unique Nash equilibrium is $q_1 = q_2 = 10$ giving each firm a profit of 100. To solve for the Pareto optimal outcome we can maximize the sum of profits,

$$\max_{q_1, q_2} S(q_1, q_2) = (30 - q_2)q_1 - q_1^2 + (30 - q_1)q_2 - q_2^2$$

and the two first order conditions are

$$\frac{\partial S(q_1,q_2)}{\partial q_1} = 30 - q_2 - 2q_1 - q_2 = 0$$
$$\frac{\partial S(q_1,q_2)}{\partial q_2} = 30 - q_1 - 2q_2 - q_1 = 0$$

and solving them together yields $q_1 = q_2 = 7\frac{1}{2}$ and the profits of each firm are $112\frac{1}{2}$. ■

(b) For which levels of discount factors can the firms support the Pareto optimal level of quantities in an infinitely repeated game?

Answer: We consider grim trigger strategies of the form "I will choose $q_i = 7.5$ and continue to do so as long as both chose this value. If anyone ever deviates I will revert to $q_i = 10$ forever." The best deviation from $q_i = 7.5$ given that $q_j = 7.5$ is to choose the best response to 7.5 which is $\frac{30-7.5}{2} = 11.25$, and the profit from deviating is $(30 - 7\frac{1}{2})11\frac{1}{4} - (11\frac{1}{4})^2 = \frac{2025}{16} = 126\frac{9}{16}$. Thus, each player will not want to deviate if

$$126\frac{9}{16} + \delta\frac{100}{1-\delta} \leq \frac{112\frac{1}{2}}{1-\delta}$$

which holds for $\delta \in [\frac{9}{17}, 1)$. ■

10. **Law Merchants (revisited):** Consider the three person game described in section ??. A subgame perfect equilibrium was constructed with a bond equal to 2, and a wage paid by every player P_2^t to player 3 equal to $w = 0.1$, and it was shown that it is indeed an equilibrium for any discount factor $\delta \geq 0.95$. Show that a similar equilibrium, where players P_1^t trust players P_2^t who post bonds, players P_2^t post bonds and cooperate, and player 3 follows the contract in every period, for any discount factor $0 < \delta < 1$.

Answer: First notice that the bond need not be equal to 2 because player P_2^t only gains 1 from deviating. Hence, any bond of value $1 + \varepsilon > 1$ will deter player P_2^t from choosing to defect instead of cooperate. Second, notice that for any wage to the third party of $1 - \varepsilon < 1$, player P_2^t still get a positive surplus $\varepsilon > 0$ from engaging the services of the third party. Hence, for any value of $\varepsilon \in (0,1)$, posting a bond of $1+\varepsilon$ and paying the third party $1-\varepsilon$ guarantees that player P_2^t will choose to employ the third party and cooperates if trusted, and in turn, P_1^t will choose to trust. We are left to see whether the third party prefers to return the bond as promised or if he would deviate and give up the future stream of all income. By deviating the third party pockets the bon worth $1+\varepsilon$, and gives up the future series of wages $1-\varepsilon$ for all future periods. Hence, he will not deviate if

$$1+\varepsilon \leq \frac{\delta}{1-\delta}(2-\varepsilon),$$

which for $\varepsilon \in (0,1)$ holds for $\delta \in (\frac{1+\varepsilon}{2}, 1)$. Hence, for any $\delta > \frac{1}{2}$ there exists a small enough $\varepsilon > 0$ for which the inequality above holds. ■

11. **Trading Brand Names:** Show that the strategies proposed in Section ?? constitute a subgame perfect equilibrium of the sequence of trust games.

    Answer: Consider any player P_2^t, $t > 1$. Under the proposed strategies, if trust was never abused and the name was bought up till period $t-1$ then (i) by buying the name and cooperating he is guaranteed a payoff of 1, (ii) by buying the name and defecting he receives 2 but cannot sell the name to the next player 2 and hence he gets $2-p^* < 1$, and (iii) by not buying the name he gets 0. Hence, for any t the strategy of P_2^t is a best response. Consider player P_2^1. If he (i) by creating the name and cooperating he is guaranteed a payoff of $1+p^* > 2$, (ii) by not creating the name he gets 0. Hence, the strategy of P_2^1 is a best response. Last, it is easy to see that any player 1 can expect cooperation, and hence trusting is a best response conditional on no one ever defecting and the name being created and transmitted. ■

12. **Folk Theorem (revisited):** Consider the infinitely repeated trust game described in Figure 10.1.

    (a) Draw the convex hull of average payoffs.

    Answer: ■