

EconS 424 - Strategy and Game Theory
Homework #5 - Due date: Friday, April 1st, in class.

1. **Exercises from Harrington:**

(a) **Chapter 10:** Exercises 2 and 5.

2. **Bargaining under incomplete information, allowing for general valuations.**

Consider a bargaining game between a seller and a buyer we described in class, but assume that the buyer's valuation for the object is either high, v^H , with probability $\alpha \in [0, 1]$, and low, v^L , with probability $1 - \alpha$; where $v^H > v^L > 0$. The seller does not observe the buyer's valuation, but knows that it's distributed according to the above probabilities. The seller is the first mover in this game, and without observing the realization of the buyer's valuation, he sets a price p for the good. Observing this price, but observing his valuation for the object (v^H or v^L), the buyer responds accepting or rejecting the deal.

(a) *Buyer's response.* Since the buyer is the privately informed player in this game, identify his best response: first, when his valuation is v^H , and, second, when his valuation is v^L .

(b) *Seller's price.* Find the price that the seller sets in the BNE of the game. [*Hint:* You should find two BNEs.]

(c) How are the BNEs you identified in part (b) affected by an increase in v^H ? And by an increase in v^L ? Interpret.

3. **Cournot competition with incomplete information, allowing for a general probability.**

Consider the duopoly market under incomplete information discussed in class, where firms face an inverse demand function $p(Q) = 1 - Q$, and $Q \geq 0$ denotes aggregate output, $Q = q_1 + q_2$. Firm 2 privately observes its marginal cost, $c_H = \frac{1}{2}$ or $c_L = 0$, with probability p and $1 - p$, respectively. Firm 1's marginal costs are low, $c_L = 0$, which is common knowledge among both firms.

(a) *Privately informed firm.* Find firm 2's best response function when its production cost is $c_H = \frac{1}{2}$, and denote it as $q_2^H(q_1)$. Is it increasing or decreasing in probability p ? Interpret.

(b) *Privately informed firm.* Find firm 2's best response function when its production cost is $c_L = 0$, and denote it as $q_2^L(q_1)$. Is it increasing or decreasing in probability p ? Interpret.

(c) *Uninformed firm.* Find firm 1's best response function, and denote it as $q_1(q_2^H, q_2^L)$. Is it increasing or decreasing in probability p ? Interpret.

(d) Find equilibrium output levels, q_1 , q_2^H , and q_2^L . How is each of them affected by a marginal increase in probability p ? Interpret.

(e) Evaluate the equilibrium output levels, found in part (d), at probability $p = 1$. Then, evaluate these output levels at $p = 0$. Interpret your results.