EconS 503 - Microeconomic Theory II
Homework #4 - Due date: Friday, March 4th.

1. **Cournot competition with both firms uninformed.** Consider the duopoly market where firms face an inverse demand function $p(Q) = 1 - Q$, and $Q = q_1 + q_2 \geq 0$ denotes aggregate output. Let us now assume that both firms are uninformed about each other's costs, that is, every firm i privately observes its cost, c_i, which is $c_H = \frac{1}{2}$ or $c_L = 0$, with probability p and $1-p$, respectively. Firm i, however, does not observe its rival's cost, c_j, which is also $c_H = \frac{1}{2}$ or $c_L = 0$, which occurs with probability p and $1 - p$, respectively.

   (a) Find firm i's best response function when its production cost is $c_H = \frac{1}{2}$, and denote it as $q_i^H(q_j^H, q_j^L)$. Is it increasing or decreasing in probability p? Interpret.

   (b) Find firm i's best response function when its production cost is $c_L = 0$, and denote it as $q_i^L(q_j^H, q_j^L)$. Is it increasing or decreasing in probability p? Interpret.

   (c) Find equilibrium output levels, q_i^H and q_i^L.

   (d) How is equilibrium output q_i^H affected by a marginal increase in probability p? Interpret.

   (e) How is equilibrium output q_i^L affected by a marginal increase in probability p? Interpret.

2. **Cournot competition under incomplete information-A twist.** Consider a duopoly market where firms face an inverse demand function $p(Q) = 1-Q$, and $Q = q_1+q_2 \geq 0$ denotes aggregate output. Assume that the marginal production cost of firm 1 (2) is high, $c_H = \frac{1}{2} > 0$, with probability p (q, respectively), where $p, q \in [0, 1]$. Similarly, the marginal cost of firm 1 (2) is low, $c_L = 0$, with probability $1 - p$ ($1 - q$, respectively).

   (a) Find firm 1's best response function when its costs marginal costs are low, $q_1^L(q_2^H, q_2^L)$. Find firm 2's best response function when its marginal costs are low, $q_2^L(q_1^H, q_1^L)$.

   (b) Find firm 1's best response function when its costs marginal costs are high, $q_1^H(q_2^H, q_2^L)$. Find firm 2's best response function when its marginal costs are high, $q_2^H(q_1^H, q_1^L)$.

   (c) Use your results from parts (a) and (b) to find the BNE of the game. [*Hint*: You cannot invoke symmetry since best response functions are not symmetric in this case.]

   (d) How are the equilibrium output levels $\left(q_1^H, q_1^L, q_2^H, q_2^L\right)$ affected by a marginal increase in p? And by a marginal increase in q? Interpret.

   (e) *Symmetric probabilities.* Evaluate your equilibrium results in the special case where both firms' costs occur with the same probability, $p = q$. What if, in addition, these probabilities are both 1/2?

(f) *Special cases.* Evaluate your equilibrium results in the special case where both firms' types are certain, as under complete information: (1) $p = q = 1$, (2) $p = 1$ and $q = 0$, (3) $p = 0$ and $q = 1$, and (4) $p = q = 0$. Interpret.

3. **FPAs with budget constrained bidders.** Consider a FPA with $N \geq 2$ bidders, but assume that every bidder privately observes his valuation for the object, v_i, and his budget, w_i, both being uniformly and independently drawn from the $[0, 1]$ interval. For simplicity, assume that if a bidder wins the auction and the winning price is above his budget, w_i, he cannot afford to pay this price, and the seller imposes a fine on the buyer, $F > 0$, for having to renege from his bid.

   (a) Show that bidding above his budget, $b_i > w_i$, is a strictly dominated strategy for every bidder i.

   (b) If bidder i's valuation, v_i, satisfies $\frac{N-1}{N} v_i \leq w_i$, show that bidding according to $b_i(v_i) = \frac{N-1}{N} v_i$ (as found in class) is still a weakly dominant strategy.

   (c) If bidder i's valuation, v_i, satisfies $\frac{N-1}{N} v_i > w_i$, show that submitting a bid equal to his budget, $b_i = w_i$, is a weakly dominant strategy.

   (d) Combine your results from parts (b) and (c) to describe the equilibrium bidding function in the first-price auction with budget constraints, $b_i(v_i, w_i)$. Depict it as a function of v_i.

4. **Equilibrium bidding function in a lottery auction.** Consider a lottery auction with $N \geq 2$ bidders, all of them assigning the same value to the object, v. Every bidder i's utility from submitting a bid b_i is

$$EU_i \left[b_i | v \right] = \frac{b_i}{b_i + B_{-i}} v - b_i$$

   where the ratio represents bidder i's probability of winning, which compares his bid relative to the aggregate bids submitted by all players, where $B_{-i} \equiv \sum_{j \neq i} b_j$. The second term indicates that bidder i must pay his bid both when winning and losing, as in APAs. For simplicity, assume that bidders use a symmetric bidding strategy, $b(v)$.

   (a) Find bidder i's equilibrium bidding function.

   (b) *Comparative statics.* How does bider i's equilibrium bid change with v and N?

   (c) *Bidding coordination.* Find equilibrium bids if bidders could coordinate their bidding decisions. Compare your results with those of part (b).