EconS 503 - Microeconomic Theory II
Homework #3 - Due date: Wednesday, February 23rd.

1. **Cost-reducing investment followed by Cournot competition-I.** Consider a duopoly market with two firms selling a homogeneous product, facing inverse demand curve $p(Q) = 1 - Q$, where $Q = q_1 + q_2$ denotes aggregate output, and facing marginal cost c, where $1 > c \geq 0$. In the first stage of the game, every firm i chooses its investment in cost-reducing technology, z_i; and, in the second stage, observing the profile of investment levels (z_i, z_j), firms compete a la Cournot.

   Investment z_i reduces firm i's marginal cost, from c to $c - \frac{1}{4}z_i$; and the cost of investing z_i in the first stage is $\frac{1}{2}z_i^2$. For simplicity, assume no discounting of future payoffs.

   (a) *Second stage.* Operating by backward induction, find firm i's best response function $q_i(q_j)$ in the second stage. How is it affected by a marginal increase in z_i? And by a marginal increase in z_j?

   (b) Find equilibrium output and profits in the second stage, as a function of investment levels (z_i, z_j). Are they increasing in z_i? Are they increasing in z_j? Interpret.

   (c) *First stage.* Find the equilibrium investment levels that firms choose in the first stage, z_i^* and z_j^*.

   (d) *Joint venture.* If, in the first stage, firms could coordinate their investment levels (z_i, z_j) to maximizes their joint profits, what would their investment levels be? This investment decision resembles a "joint venture," where firms coordinate their R&D activities, or any other decision, and then compete in a subsequent stage (in this case, a la Cournot). Compare your results with those in part (c).

2. **Collusion with imperfect monitoring (Continuous strategies and firms competing a la Cournot).** Consider two firms selling a homogenous product, competing a la Cournot, and facing a linear inverse demand function $p(Q) = 1 - Q$, where Q denotes aggregate output. For simplicity, assume that they are symmetric in their marginal production cost, c, which is normalized to $c = 0$.

   (a) *Stage game.* If the game is not repeated, find the equilibrium output, q^C, and profit for each firm.

   (b) *GTS with perfect monitoring.* For the remainder of the exercise, assume that firms interact in an infinitely-repeated game. Find the minimal discount factor sustaining cooperation if firms use a standard GTS, namely, in the first period, every firm produces half of the monopoly output, $\frac{q^m}{2}$; in all subsequent periods, every firm produces $\frac{q^m}{2}$ if $\left(\frac{q^m}{2}, \frac{q^m}{2}\right)$ was the outcome in all previous periods. Otherwise, every firm reverts to the NE of the stage game, q^C, thereafter. In this part of the exercise, assume that firms can immediately observe deviations (perfect monitoring).

(c) *GTS with imperfect monitoring.* Assume now that firms suffer from imperfect monitoring: if firm j deviates from the collusive output $\frac{q^m}{2}$, firm i observes its deviation with probability

$$p_i = \frac{q_j - \frac{q^m}{2}}{1 - \frac{q^m}{2}},$$

where q_j denotes firm j's output. This probability is positive when firm j deviates from the collusive output, $q_j > \frac{q^m}{2}$, and increases monotonically when firm j produces a larger output. Intuitively, firm i may not observe its rival's deviation when firm j produces barely above the collusive output $\frac{q^m}{2}$, but it will likely observe its deviation when firm j produces a large output. In this context, the GTS described in part (b) must be rewritten as follows: In the first period, every firm produces half of the monopoly output, $\frac{q^m}{2}$; in all subsequent periods, every firm produces $\frac{q^m}{2}$ if $\left(\frac{q^m}{2}, \frac{q^m}{2}\right)$ was the outcome *observed* in all previous periods. Otherwise, every firm reverts to the NE of the stage game, q^C, thereafter.

3. **Exercises from Tadelis:**

   (a) Chapter 10: Exercises 10.6, 10.9, and 10.11.