EconS 424 - Strategy and Game Theory
Homework #2 - Due date: Monday, February 14th, in class.

1. **Exercises from Harrington's textbook:**
   (a) Chapter 4: exercises 1 and 9; and
   (b) Chapter 5: exercises 5 and 11.

EXERCISES

1. One of the critical moments early on in the *The Lord of the Ring* is the meeting in Rivendell to decide who should take the One Mordor. Gimli the Dwarf won't hear of an Elf doing it, whereas (who is an Elf) feels similarly about Gimli. Boromir (who is a opposed to either of them taking charge of the Ring. And then Frodo the Hobbit, who has the weakest desire to take the Ring bu that someone must throw it into the fires of Mordor. In model scenario as a game, assume there are four players: Boromir, Frodc and Legolas. (There were more, of course, including Aragorn and but let's keep it simple.) Each of them has a preference ordering in the following table, as to who should take on the task of carr One Ring.

Preference Rankings for The Lord of the Rings

Person	First	Second	Third	Fourth	Fift
Boromir	Boromir	Frodo	No one	Legolas	Gir
Gimli	Gimli	Frodo	No one	Boromir	Leg
Legolas	Legolas	Frodo	No one	Gimli	Bor
Frodo	Legolas	Gimli	Boromir	Frodo	No

Of the three non-Hobbits, each prefers to take on the task himsel would prefer that other than themselves and Frodo, no one should t Ring. As for Frodo, he doesn't really want to do it and prefers to do if no one else will. The game is one in which all players simultan make a choice among the four people. Only if they all agree—a una voting rule is put in place—is someone selected; otherwise, no one ta this epic task. Find all symmetric Nash equilibria.

2. Consider a modification of driving conventions, shown in the figure in which each player has a third strategy: to zigzag on the road. Su that if a player chooses *zigzag*, the chances of an accident are the whether the other player drives on the left, drives on the right, or z as well. Let that payoff be 0, so that it lies between −1, the payoff w collision occurs for sure, and 1, the payoff when a collision does not Find all Nash equilibria.

Modified Driving Conventions Game

	Drive left	*Drive right*	*Zigzag*
Drive left	1,1	−1,−1	0,0
Drive right	−1,−1	1,1	0,0
Zigzag	0,0	0,0	0,0

7. Return to the Kidnapping game, whose strategic form is shown
all of the Nash equilibria.

Kidnapping

		Vivica (kin of vict	
		Pay ransom	*Do not pa*
Guy (kidnapper)	*Do not kidnap/Kill*	3,5	3,5
	Do not kidnap/Release	3,5	3,5
	Kidnap/Kill	4,1	2,2
	Kidnap/Release	5,3	1,4

8. Queen Elizabeth has decided to auction off the crown jewels,
are two bidders: Sultan Hassanal Bolkiah of Brunei and She
Bin Sultan Al Nahyan of Abu Dhabi. The auction format is a
The Sultan and the Sheikh simultaneously submit a wr
Exhibiting her well-known quirkiness, the Queen specifies
Sultan's bid must be an odd number (in hundreds of millions
pounds) between 1 and 9 (that is, it must be 1, 3, 5, 7, or 9) an
Sultan's bid must be an even number between 2 and 10. The bi
submits the highest bid wins the jewels and pays a price eq
bid. (If you recall from Chapter 3, this is a first-price auction.)
ning bidder's payoff equals his valuation of the item less the
pays, whereas the losing bidder's payoff is 0. Assume that th
has a valuation of 8 (hundred million pounds) and that the S
a valuation of 7.

a. In matrix form, write down the strategic form of this game.
b. Derive all Nash equilibria.

9. Find all of the Nash equilibria for the three-player game here.

Player 3: *A*

		Player 2		
		x	*y*	*z*
Player 1	*a*	1,1,0	2,0,0	2,0,0
	b	3,2,1	1,2,3	0,1,2
	c	2,0,0	0,2,3	3,1,1

Player 3:

		Player	
		x	*y*
Player 1	*a*	2,0,0	0,0,1
	b	1,2,0	1,2,1
	c	0,1,2	2,2,1

Player 3: *C*

		Player 2		
		x	*y*	*z*
Player 1	*a*	2,0,0	0,1,2	0,1,2
	b	0,1,1	1,2,1	0,1,2
	c	3,1,2	0,1,2	1,1,2

3. It is the morning commute in Congestington, DC. Of 100 drivers, each driver is deciding whether to take the *toll road* or take the *back roads*. The toll for the toll road is \$10, while the back roads are free. In deciding on a route, each driver cares only about income, denoted y, and his travel time, denoted t. If a driver's final income is y and his travel time is t, then his payoff is assumed to be $y - t$ (where we have made the dollar value of one unit of travel time equal to 1). A driver's income at the start of the day is \$1,000. If m drivers are on the toll road, the travel time for a driver on the toll road is assumed to be m (in dollars). In contrast, if m drivers take the back roads, the travel time for those on the back roads is $2m$ (again, in dollars). Drivers make simultaneous decisions as to whether to take the toll road or the back roads.
   a. Derive each player's payoff function (i.e., the expression that gives us a player's payoff as a function of her strategy profile.)
   b. Find a Nash equilibrium.

4. Return to *Poshing Up the Cockney* in Section 5.2. Of the n women, let k denote the number of women who have Lilly clothes in their closet and assume $k < n/2$. The other $n - k$ women have Goth in their closets. Now suppose it costs a woman p to buy a Lilly dress or buy a Goth outfit. Thus, one of the k women who already own a Lilly dress can wear it at no cost, but if she wants to go Goth, then it'll cost her p. Similarly, one of the $n - k$ women who already own a Goth outfit can wear it at no cost, but if she wants to go Lilly, then it'll cost her p. Assume that $0 < p < 1$. A woman's payoff is as described in Table 5.1, except that you have to subtract p if she buys clothing. Find all Nash equilibria.

Ch. 5

5. Suppose several friends go out to dinner with the understanding that the bill will be divided equally. The problem is that someone might order something expensive, knowing that part of the cost will be paid by others. To analyze such a situation, suppose n diners are present, and, for simplicity, they have the same food preferences. The accompanying table states the price of each of three dishes on the menu and how much each person values it. Value is measured by the maximum amount the person would be willing to pay for the meal.

Dining Dilemma

Dish	*Value*	*Price*	*Surplus*
Pasta Primavera	**\$21.00**	**\$14.00**	**\$7.00**
Salmon	**\$26.00**	**\$21.00**	**\$5.00**
Filet Mignon	**\$29.00**	**\$30.00**	**−\$1.00**

Surplus is just the value assigned to the meal, less the meal's price. The pasta dish costs \$14 and each diner assigns it a value of \$21. Thus, if a diner had to pay for the entire meal, then each diner would buy the pasta dish, since the surplus of \$7 exceeds the surplus from either salmon or steak. In fact, a diner would prefer to skip dinner then to pay the \$30 for the steak, as reflected by a negative surplus. A player's payoff equals the value of the meal she eats, less the amount she has to pay. The latter is assumed to equal the total bill divided by the number of diners. For

example, if three diners are present and each orders a differen
the payoff to the one ordering the pasta dish is

$$21 - \left(\frac{14 + 21 + 30}{3}\right) = 21 - 21.67 = -0.67$$

the payoff for the person ordering the salmon is

$$26 - \left(\frac{14 + 21 + 30}{3}\right) = 4.33$$

and the payoff to whoever is ordering the steak is

$$29 - \left(\frac{14 + 21 + 30}{3}\right) = 7.33$$

Not surprisingly, the people who order the more expensive mea since all pay the same amount.

a. Suppose two diners are present ($n = 2$). What will they orde Nash equilibrium)?

b. Suppose four diners ($n = 4$) are present. What will they orde Nash equilibrium)?

6. Consider again the entry game from Section 5.3, but now suppo potential entrants are identical in that each faces the same en $300. Given the total number of companies in the market, the a ing table reports a company's net profit (or payoff) if it enters. the payoff from staying out of the market is zero, and each co choose either *enter* or *do not enter*. Find all Nash equilibria.

Entry Game with Identical Companies

Number of Firms	*Gross Profit per Firm*	*Net Profit per Firm*
1	1,000	700
2	400	100
3	250	−50
4	150	−150
5	100	−200

7. A rough neighborhood has $n \geq 2$ residents. Each resident has whether to engage in the crime of theft. If an individual chooses to and is not caught by the police, he receives a payoff of W. If he is the police, his payoff is Z. If he chooses not to commit theft, he rece payoff. Assume that $W > 0 > Z$. All n residents simultaneously decid or not to commit theft. The probability of a thief being caught e where m is the number of residents who choose to engage in theft. probability of being caught is lower when more crimes are committ police have more crimes to investigate. The payoff from being a t that $m - 1$ other people have also chosen to be thieves, is then

$$\left(\frac{m-1}{m}\right)W + \left(\frac{1}{m}\right)Z$$

Find all Nash equilibria.

8. Consider the following game.

Player 2

Player 1	x	y	z
a	2,2	1,2	0,0
b	2,1	2,2	1,3
c	0,0	3,1	0,0

a. Find all Nash equilibria.
b. Provide an argument for selecting among those equilibria.

9. Consider the following game.

Player 2

Player 1	x	y	z
a	1,0	1,2	0,1
b	0,0	2,1	3,3
c	1,2	1,1	0,1

a. Find all Nash equilibria.
b. Provide an argument for selecting among those equilibria.

10. Consider a country with n citizens, and let v_i be the value that citizen i attaches to protesting. Enumerate the citizens so that citizen 1 attaches more value to protesting than citizen 2, who attaches more value than citizen 3, and so forth: $v_1 > v_2 > \ldots > v_n (= 0)$, where citizen n attaches no value to protesting. Assume that the cost of protesting is the same for all citizens and is c/m where $c > 0$ and m is the number of protestors. Then the payoff to citizen i from protesting is $v_i - (c/m)$, while the payoff from not protesting is zero. Assume that $v_1 - c < 0$. Find all Nash equilibria.

11. n pre-med students are planning to take the MCAT. Each student must decide whether to take a preparatory course prior to taking the test. Let x_i denote the choice of student i, where $x_i = 0$ indicates that she will not take the course and $x_i = 1$ indicates that she will take the course. A student cares about her ranking in terms of her MCAT score and whether or not she took the prep course. Let s_i denote student i's MCAT score and r_i denote the ranking of student i among the n students who took the test. Specifically, r_i equals 1 plus the number of students who scored strictly higher than student i. To clarify this specification, here are three examples: If $s_i \geq s_j$ for all $j \neq i$, then $r_i = 1$. (In other words, if nobody's score is higher than that of student i, then her rank is 1.) If $s_i < s_j$ for all $j \neq i$, then $r_i = n$. (In other words, if student i has the lowest score, then her rank is n.) Finally, if $s_1 > s_2 > s_3 = s_4 > s_5$, then $r_1 = 1, r_2 = 2, r_3 = r_4 = 3, r_5 = 5$. Now, assume that student i's payoff equals $b(n - r_i) - x_i c$, where $b > c > 0$. Note that taking the prep course entails a cost to a student equal to c. Note also that a student adds to her payoff by an amount b if her rank increases by 1. Student

i's score is assumed to be determined from the formula $s_i = a_i + x_i$
$a_i > 0$ and $z > 0$. a_i is related to the innate ability of the student and
she would score if she did not take the prep course. If she takes
course, she adds to her score by an amount z. Assume that

$$a_1 > a_2 > \cdots > a_{n-1} = a_n.$$

This means that student 1 is, in a sense, smarter than student 2, s
is smarter than student 3, . . . , student $n - 2$ is smarter than studen
and students $n - 1$ and n are equally smart. The final assumption

$$a_{i+1} + z > a_i \text{ for all } i = 1, 2, \ldots, n - 1$$

In this game, n students simultaneously deciding whether or not to
MCAT preparatory course. Derive a Nash equilibrium.

12. For the operating systems game, let us now assume the intrinsic sup
of Mac is not as great and that network effects are stronger for W
These modifications are reflected in different payoffs. Now, the pay
adopting Windows is $50 \times w$ and from adopting Mac is 15 +
n consumers are simultaneously deciding between Windows and M

a. Find all Nash equilibria.

b. With these new payoffs, let us now suppose that a third option
which is to not buy either operating system; it has a payoff of 1,
Consumers simultaneously decide among Windows, Mac, and n
operating system. Find all Nash equilibria.

13. A billionaire enters your classroom and plunks down \$1 million on
He says that he's here to give away money to the least greedy perso
class. The procedure is that all 30 of the students are to write d
integer between 1 and 1,000,000. Whoever writes down the lowest
walks away with an amount of money equal to his or her number. I
more students write down the same number, and it was the lowest
all students, then those students will equally share the amount th
wrote down. Assume each student's payoff equals how much mone
she wins.

a. Assuming students play according to Nash equilibrium, how mu
that \$1 million does the billionaire give away?

Now suppose the student who wrote down the lowest lone number
receives an amount equal to that number. If there is no lone number—
every number submitted by a student was also submitted by at leas
other student—then no money is distributed.

b. Find a Nash equilibrium in which the amount paid out is \$1.

c. Find a Nash equilibrium in which the amount paid out is \$15.

14. Muhammad and his followers are preparing to battle the Qurays
from Mecca:

> [Muhammad] places his archers on top of a mountain near
> flank, ordering them to "hold firm to your position, so that we
> not be attacked from your direction." To the rest of his men
> shouts his final instructions: "Let no one fight until I comma
> him to fight" Almost immediately, the Quraysh are put to flig
> Muhammad's archers release a steady hail of arrows onto the b
> tlefield, protecting his meager troops and forcing the Meccan ar