Common Pool Resources:

Strategic Behavior, Inefficiencies, and Incomplete Information

Chapter 2: Common Pool Resources in a Static Setting

Outline

- Modeling the Common Pool Resources CPR
- Finding equilibrium appropriation
- Common pool resources – Socially optimal appropriation
- Facing our first inefficiency
- Inefficient exploitation with more general functions
- Policy instruments

Modeling the Common Pool Resources CPR

Assumption

- Assume that N firms (or individuals) have free access to the resource.
- Perfect competition (Every unit of appropriation is sold in the international market)
  - Every fisherman's appropriation represents a small share of industry catches, thus not affecting market prices for this variety of fish
- Every firm takes the market price p as given (normalize to $p = \$1$)

2.2 Modeling the Common Pool Resources CPR

- Every firm faces the following cost function;

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

➢ Where;

- $S > 0$ denotes the stock of the resource, which reduces fisherman i's cost when the resource becomes more abundant.

- q_i represents fisherman i's appropriation.

- $Q_{-i} = \sum_{i \neq j} q_j$ reflects aggregate appropriations by individuals other than i.

➤ ***Case 1:*** Having only two fishermen exploit the resource.

- The total cost function simplifies to

$$C(q_1, q_2) = \frac{q_1(q_1 + q_2)}{S} \qquad \textit{for fisherman 1}$$

$$C(q_2, q_1) = \frac{q_2(q_2 + q_1)}{S} \qquad \textit{for fisherman 2}$$

➢ ***Propositions on Cost Functions:***

- The cost function is increasing in fisherman *i*'s own appropriation, q_i, and in his rival's appropriations, Q_{-i}

- Intuitively, the fishing ground becomes more depleted as other firms appropriate fish, making fisherman *i* more difficult to catch fish.

➢ ***Case 2:*** What if we have three fishermen exploiting the resource?

- The same principle applies, as seen from the following derivatives.

$$\frac{\partial C(q_i, Q_{-i})}{\partial q_i} = \frac{2q_i + Q_{-i}}{S} > 0$$

$$\frac{\partial C(q_i, Q_{-i})}{\partial Q_{-i}} = \frac{q_i}{S} > 0$$

- Agent *i*'s profit-maximization problem
  - Every fisherman chooses its appropriation level q_i to maximize its profits as follows;

$$\max_{q_i \geq 0} \pi_i = q_i - \frac{q_i(q_i + Q_{-i})}{S}$$

  - The first term represents the fisherman's revenue from additional units of appropriation (recalling that $p_i = \$1$).
  - The second term indicates the total cost that the fisherman incurs when appropriating q_i units of fish while his rivals appropriate Q_i units.

Finding equilibrium appropriation

2.3.1 Comparative statics

2.3.2 Extension - What if fishermen have some market power?

2.3 Finding equilibrium appropriation

Goal: Find the appropriation that each fisherman chooses in equilibrium.

- Every agent chooses its appropriation level simultaneously.
- The information about the stock and agents' cost functions is common knowledge (complete information).

Cournot game of simultaneous quantity competition

How to solve this game?

Step 1: Solve each player's profit maximization problem which provides us with the players best response function

Step 2: Use the best response function of all players (step 1) to identify the Nash equilibrium of the game.

➢ ***Step 1:*** Find fisherman *i*'s best response function.

- Differentiating with respect to q_i in the above maximization problem for fisherman i we obtain;

$$\underbrace{1}_{\text{MR}} - \underbrace{\frac{2q_i + Q_{-i}}{S}}_{\text{MC}} = 0$$

- The first term captures the marginal revenue from catching additional units of fish.

- The second term indicates the marginal cost that the firm experiences from these additional catches.

- That is, the fisherman increases appropriation until the marginal revenue and marginal cost exactly offset each other.
  - Rearranging the expression yields

$$S = 2q_i + Q_{-i}$$

- Fisherman i's best response function is

$$q_i(Q_{-i}) = \frac{S}{2} - \frac{1}{2}Q_{-i} \qquad (BRF_i)$$

- It describes how many units to appropriate, q_i, as a response to how many units his rivals appropriate, Q_{-i}.

- She appropriates half of the available stock, $\frac{S}{2}$, when his rivals do not appropriate any units, $Q_{-i} = 0$
- But his appropriation decreases as his rivals appropriate positive amounts, $Q_{-i} > 0$

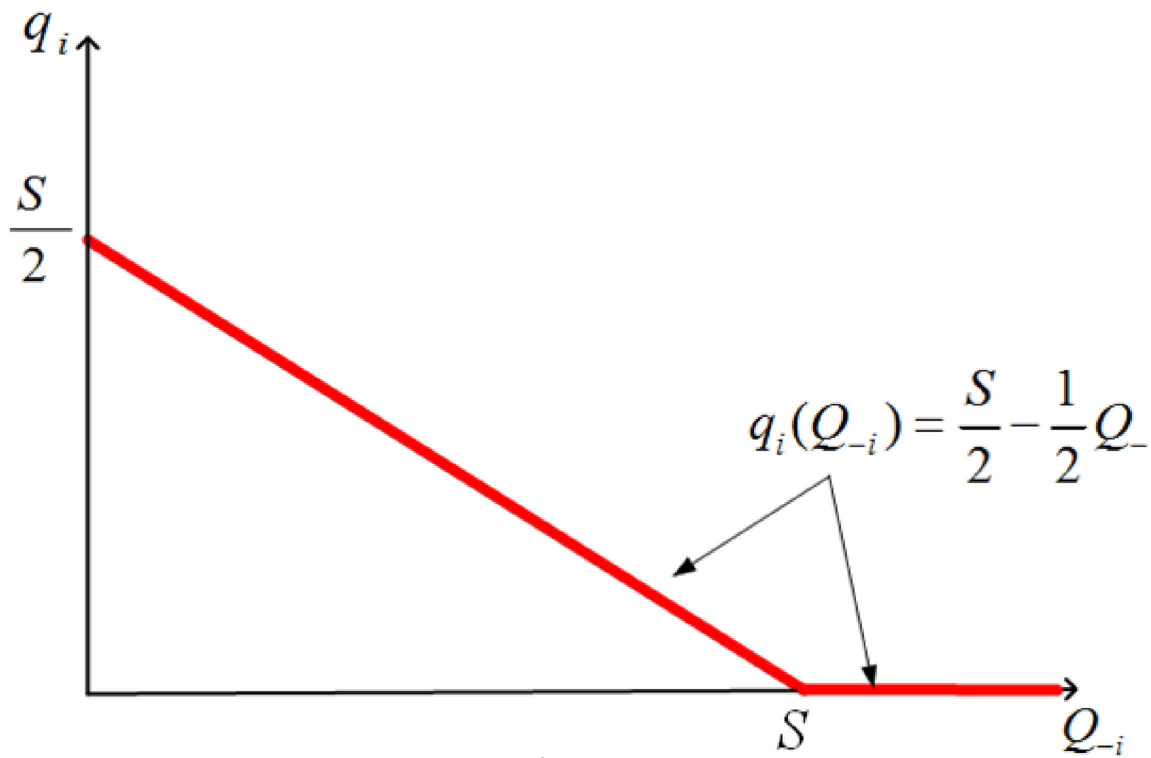

Figure 2.1.

- Fisherman *j*'s best response function

  - Since firms face the same price for each unit of fish ($1)

  - And they face the same cost function (symmetric)

  - The best response function of any other firm *j* (where $j \neq i$) is symmetric to the best response function of firm *i*;

$$q_j(Q_{-j}) = \frac{S}{2} - \frac{1}{2} Q_{-j} \qquad (BRF_j)$$

➢ ***Step 2:*** Using best response functions to find the Nash equilibrium.

- In a symmetric equilibrium; each fisherman appropriates the same amount of fish

implying that $q_1^* = q_2^* = \cdots = q_N^* = q^*$ *All firms' catches coincide*

Q_{-i} becomes;

$$Q_{-i}^* = \sum_{j \neq i} q^* = (N-1)q^*$$

2.3 Finding equilibrium appropriation

- Inserting this result in the best response function yields

$$q^* = \frac{S}{2} - \frac{1}{2}(N-1)q^*$$

- S is the stock
- N is the number of fishermen
- Rearranging the above expression yields;

$$\frac{S}{2} = \frac{2q^* + (N-1)q^*}{2}$$ or $$S = (N+1)q^*$$

- The equilibrium appropriation becomes ;

$$q^* = \frac{S}{(N+1)}$$

- Numerical example

  - Assume that the stock is $S = 100$ tons of fish

  - The number of fishermen is $N \ = \ 9$

✓ The equilibrium and the aggregate appropriations become

$$q^* = 10\ tons \qquad Q^* = Nq^* = \frac{NS}{N+1} = 90\ tons$$

- ***Case***: Having two firms ($N = 2$) , i and j.

The aggregate appropriation by i's rivals simplifies to $Q_{-i} = q_j$, implying that the best response function of firm i, and j is;

$$q_i(q_j) = \frac{S}{2} - \frac{1}{2}q_j \qquad (BRF_i)$$

$$q_j(q_i) = \frac{S}{2} - \frac{1}{2}q_i \qquad (BRF_j)$$

- Figure 2.2 depicted the Nash equilibrium where both firms' best response functions cross each other….. *(Next slide)*

2.3 Finding equilibrium appropriation

- Since we have two firms $N = 2$;

  - The equilibrium appropriation becomes;

$$q^* = \frac{S}{N+1} = \frac{S}{2+1} = \frac{S}{3}$$

  - The aggregate appropriation becomes

$$Q^* = \frac{2S}{3}$$

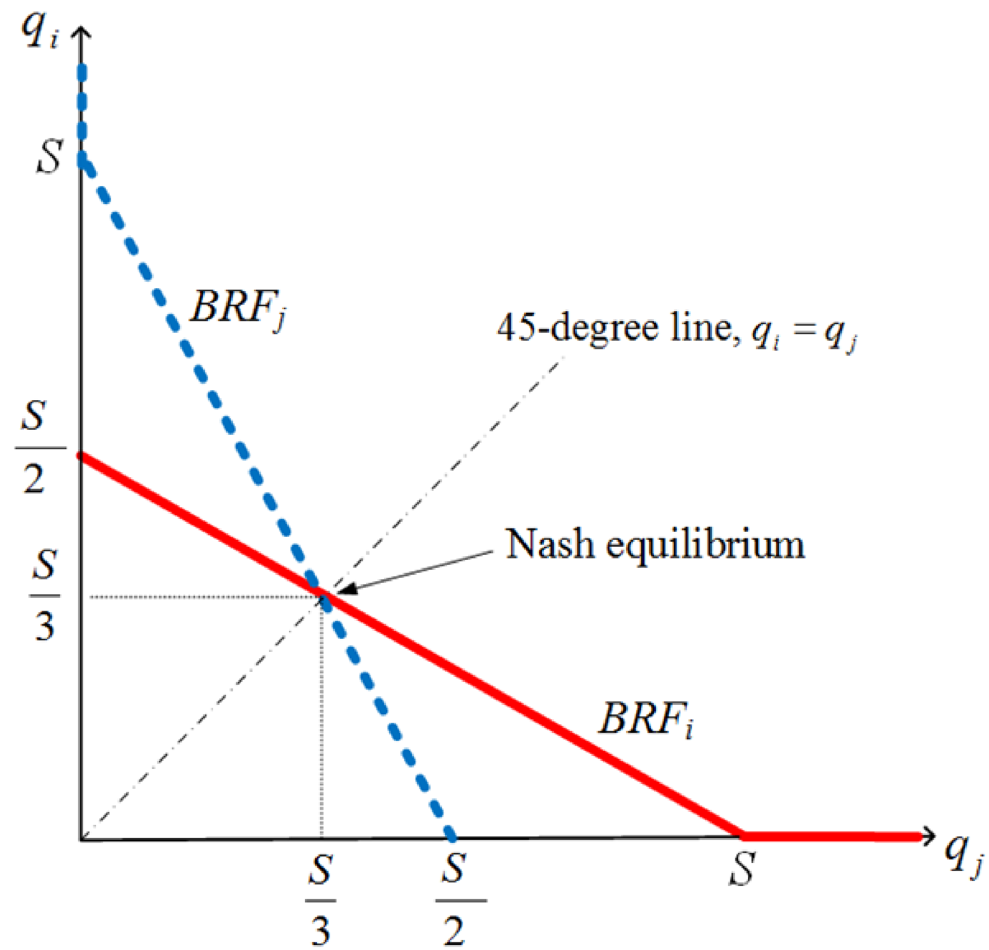

Figure 2.2

Comparative statics

- Discuss how the result is affected by changes in one of the parameters.

Example 1

- The equilibrium appropriation;

$$q^* = \frac{S}{(N+1)}$$

- It only depends on the stock of the resource, S, and the number of firms competing for it, N.

$$\frac{dq^*}{dS} = \frac{1}{(N+1)}$$

- ✓ We can observe the equilibrium appropriation q^* **increases** in S

$$\frac{dq^*}{dN} = -\frac{S}{(N+1)^2}$$

- ✓ We can observe the equilibrium appropriation q^* **decreases** in N

  - ➢ *Intuitively*

    Every fisherman increase his catches as the resource becomes more abundant (higher S) but decreases them as competition becomes fiercer (higher N).

➢ Example 2

- The aggregate appropriation is

$$Q^* = \frac{NS}{N+1}$$

$$\frac{dQ^*}{dN} = \frac{(N+1-N)S}{(N+1)^2} = \frac{S}{(N+1)^2} > 0$$

✓ We can observe the aggregate appropriation Q^* **increases** in N

➢ Figure 2.3a.

- Depicts the equilibrium appropriation q^* as a function of the number of firms exploiting the commons.

$$q^* = \frac{S}{(N+1)} = \frac{100}{N+1}$$

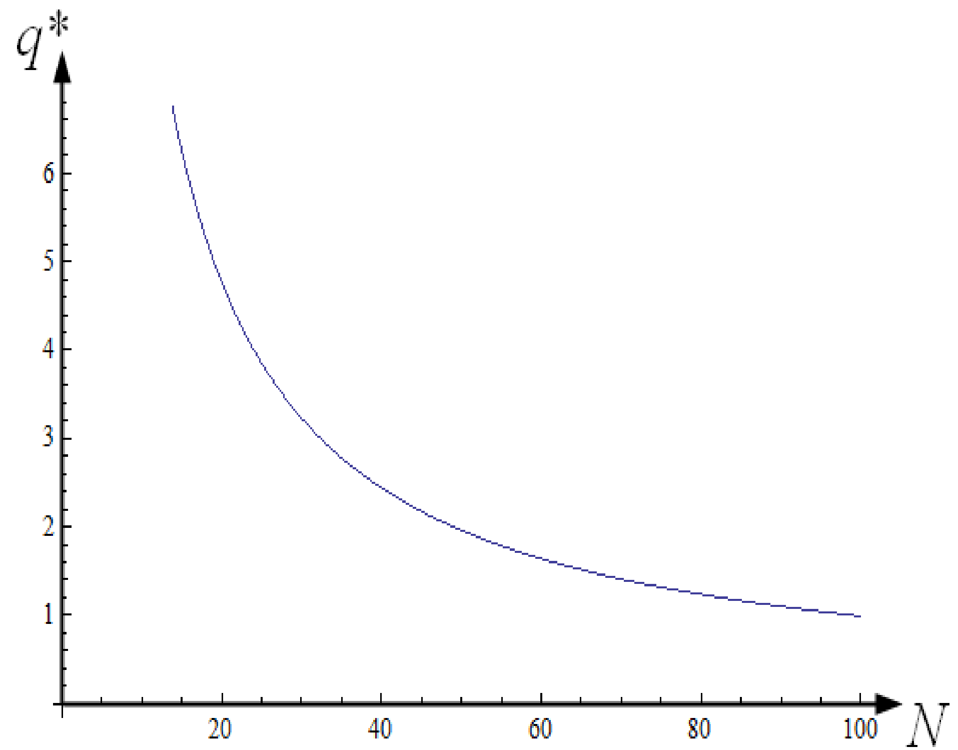

Figure 2.3a. Equilibrium appropriation q^* as a function of N.

- Figure 2.3b.

  - Illustrates the aggregate equilibrium appropriation.

  - $S = 100$

$$Q^* = \frac{NS}{N+1} = \frac{100N}{N+1}$$

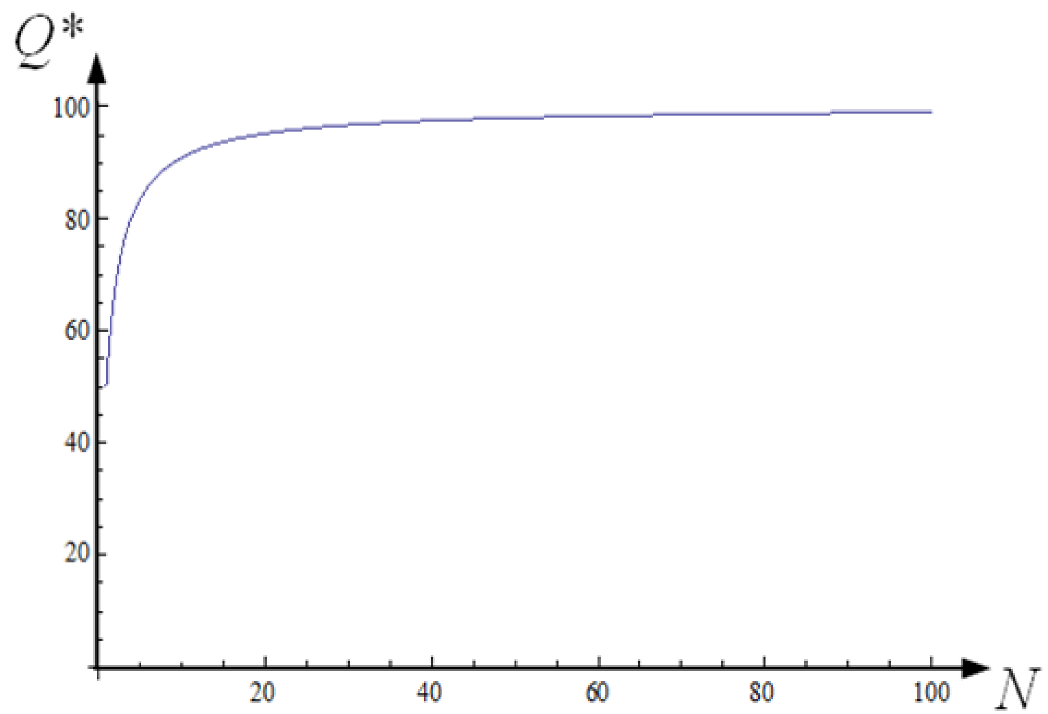

Figure 2.3b. Aggregate equilibrium appropriation Q^* as a function of N.

Figure 2.4

- Depicts q^* as a function of the available stock, S

- $N = 2$

$$q^* = \frac{S}{(N+1)} = \frac{S}{3}$$

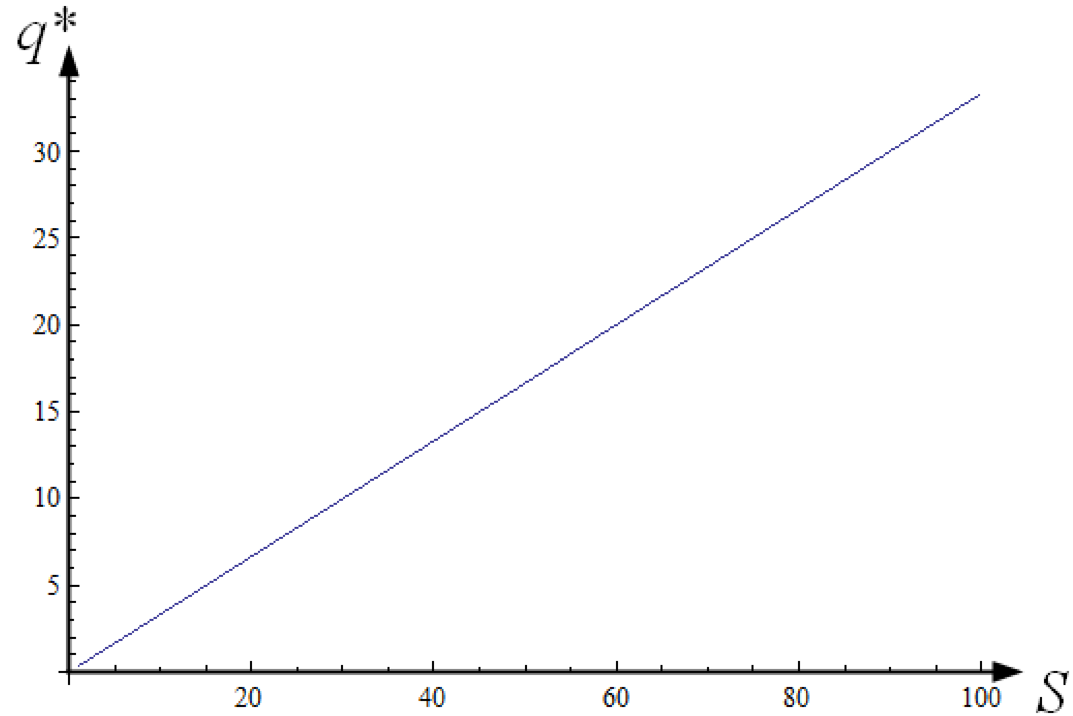

Figure 2.4. Equilibrium appropriation q^* as a function of S.

2.3.2 Extension - What if fishermen have some market power?

- In this stetting, we assume that;
  - Finite number of firms selling homogeneous products.
  - Other CPRs can be characterized by a few firms.
  - Each selling a relatively large share of total appropriations
    (Ex. North Sea, the Bering Sea, and the Western Pacific)
- In this setting, **we can no longer** assume that fishermen take prices as given.

➢ Modelling the CPRs

- The market demand $p(Q) = a - bQ$

- Q denotes the aggregate appropriation

- *where* $Q = q_i + Q_{-i}$ is the sum of fisherman *i*'s and those of all his rivals

- $a,\ b > 0$ are both positive parameters

- $b > 0$ indicates a larger appropriation decreases the market price at which all fishermen sell their product.

➢ Continue modelling the CPRs

- Every firm faces the following cost function;

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

- The market demand can be expressed as

$$p(Q) = a - bq_i - bQ_{-i}$$

- Fisherman i's profit maximization problem

$$\max_{q_i \geq 0} \ \pi_i = \underbrace{(a - bq_i - bQ_{-i})q_i}_{\text{Total revenue}} - \underbrace{\frac{q_i(q_i + Q_{-i})}{S}}_{\text{Total costs}}$$

- ***How to solve this game?***

Step 1: Finding fisherman i's best response function.

Step 2: Using best response functions to find the Nash equilibrium.

- ***Step 1:*** Finding fisherman *i*'s best response function.

  - Differentiating the profit function with respect to q_i yields

$$a - 2bq_i - bQ_{-i} = \frac{2q_i + Q_{-i}}{2}$$

  - Solving for q_i;

$$q_i(Q_{-i}) = \frac{aS}{2(1 + bS)} - \frac{1}{2}Q_{-i} \qquad (BRF_i\,)$$

➢ Numerical example

- When $a = 1$ and $b = 0$

I. The market price collapses to $p(Q) = \$1$,

II. The best response function simplifies to

$$q_i(Q_{-i}) = \frac{S}{2} - \frac{1}{2}Q_{-i}$$

- The market prices are ***insensitive to sales*** (due to $b = 0$)

Numerical example

- When $a = 1$ and $b > 0$

I. The best response function simplifies to

$$q_i(Q_{-i}) = \frac{aS}{2(1+bS)} - \frac{1}{2}Q_{-i}$$

II. When b increases, the vertical intercept of the best response function ***decreases.***

- producing a downward shift without affecting its slope, $-\frac{1}{2}$

- Intuitively,

For a given appropriation by *i*'s rivals, and by treating Q_{-i} as given ;

> *the appropriation by fisherman i* ***decreases*** *when the market price becomes more sensitive to aggregate appropriation (when parameter b increases)*

- **The opposite effect** arises when demand increases (as captured by an increase in a), as the vertical intercept of the best response function, $\frac{aS}{2(1+bS)}$ now increases, shifting the function upwards.

- ***Step 2:*** Using best response functions to find the Nash equilibrium.

- In a symmetric equilibrium; each fisherman appropriates the same amount of fish

implying that $q_1^* = q_2^* = \cdots = q_N^* = q^*$ All firms' catches coincide

so Q_{-i} becomes;

$$Q_{-i}^* = \sum_{j \neq i} q^* = (N-1)q^*$$

- Inserting this result in the best response function yields

$$q^* = \frac{aS}{2(1+bS)} - \frac{1}{2}(N-1)q^*$$

- The equilibrium appropriation becomes

$$q^* = \frac{aS}{(N+1)(1+bS)}$$

➢ Numerical example

- $When\ a = 1 \quad and \quad b = 0$

I. The equilibrium appropriation simplifies to

$$q^* = \frac{S}{N+1}$$

➢ Numerical example

- $When\ N > 1 \quad and \quad b > 0$

I. The equilibrium appropriation simplifies to

$$q^* = \frac{aS}{(N+1)(1+bS)}$$

✓ II. When b increases every firm decreases its equilibrium appropriation q^*.

✓ III. Its sales create now a negative effect on the market price which did not exist when such a price was given.

- Intuitively, the firm anticipates that selling more units will reduce market prices, so that it does not appropriate as much fish as when prices are insensitive to its catches.

- The aggregate equilibrium appropriation is

$$Q^* = Nq^* = \frac{aNS}{(N+1)(1+bS)}$$

which is increasing in N and S but decreasing in b because

$$\frac{\partial Q^*}{\partial b} = -\frac{aNS^2}{(N+1)(1+bS)^2} < 0$$

$$\frac{\partial Q^*}{\partial N} = \frac{aS}{(N+1)^2(1+bS)} > 0$$

$$\frac{\partial Q^*}{\partial S} = \frac{aN}{(N+1)(1+bS)^2} > 0$$

Common pool resources Socially optimal appropriation

Question….!

Is equilibrium appropriation excessive from a social point of view?

- To answer that question, we start by defining the socially optimal appropriation;

➢ *Definition 1:*

The socially optimal appropriation is the one maximizing the fishermen's joint profits

$$W = PS$$

where $PS = \sum_{i=1}^{N} \pi_i$ denotes the sum of all firms' profits

- In the case of only two fishermen (a CPR cartel)

$$W \text{ collapses to } \quad W = \pi_1 + \pi_2$$

- *Definition 2:*

General welfare function is the sum of consumer and producer surplus;

$$W = CS + PS$$

where $CS = \int_0^Q p(Q)dQ$ denotes consumer surplus

Continue definition 2:

Welfare function in definition 2 is more common in CPRs where catches are sold in the domestic market, thus affecting domestic consumers.

Definition 3:

Welfare function can be further generalized to

$$W = (1 - \lambda)\, CS + \lambda PS$$

where;

λ : the weight that the social planner assigns to producer surplus

$(1 - \lambda)$ captures the weight that she assigns to consumer surplus

➢ ***Special cases on λ;***

- When $\lambda = 1$

  ✓ The welfare function collapses to;

$$W = PS$$

- Indicating that the social planner does not care about consumer surplus.

- ***This case happened*** when all appropriation is sold overseas so domestic consumers are not affected by the price of the good as, in short, they do not buy the product

- When $\lambda = \frac{1}{2}$
  - ✓ The welfare function becomes

$$W = \frac{1}{2}(CS + PS)$$

- Since $\frac{1}{2}$ enters as a constant, it can be graphically understood as a vertical shifter of $CS + PS$, and as a result;

$W = \frac{1}{2}(CS + PS)$ coincides with that maximizing $W = CS + PS$

2.4 Common pool resources Socially optimal appropriation

- When $\lambda = 0$

  ✓ The welfare function collapses to

$$W = CS$$

- Indicating that the social planner does not assign any weight to fishermen's profits

- ***This case happened*** if they are all foreign firms operating at a CPR overseas which does not have effects on domestic welfare, other than those channeled through the demand function and CS

- Find the socially optimal appropriation that maximizes welfare

  - In the next slides we will discuses how to find the socially optimal appropriation *under special cases* when;

    - Only profits matter (In section 2.4.1)
    - Consumers and profits matter (In section 2.4.2)

- We focus on the case in which;

  - Fishermen take prices as given $p = \$1$
  - There are two fishermen $N = 2$

2.4.1 Socially optimal appropriation when only profits matter

- When $\lambda = 1$

The social planner considers the welfare function $W = PS$

$$\max_{q_1, q_2 \geq 0} W = PS = \pi_1 + \pi_2$$

which can be rewritten as

$$\max_{q_1, q_2 \geq 0} \pi_1 + \pi_2 = \left(q_1 - \frac{q_1(q_1 + q_2)}{S}\right) + \left(q_2 - \frac{q_2(q_2 + q_1)}{S}\right)$$

- This problem is equivalent to that of a fishermen cartel where fishermen 1 and 2 coordinate their catches, q_1 and q_2, to maximize their joint profits

2.4.1 Socially optimal appropriation when only profits matter

- Differentiating with respect to q_1, q_2;

$$1 - \frac{2(q_1 + q_2)}{S} = 0$$

➢ Intuitively

- The first term represents the marginal revenue (MR) from additional catches
- The second term captures fisherman *i*'s marginal cost (MC)
- Increasing catches produces twice as much marginal costs. ***Why*?**

✓ Since every fisherman takes into account not only the increase in his own costs but also the increase in his rival's cost

- *In brief;*

Every fisherman internalizes the cost externality that his appropriation generates on other fishermen, as a larger q_i increases the cost of fisherman j.

$$S = 2(q_1 + q_2)$$

- Solving for q_1;

$$q_1(q_2) = \frac{S}{2} - q_2 \qquad \text{for fisherman 1}$$

$$q_2(q_1) = \frac{S}{2} - q_1 \qquad \text{for fisherman 2}$$

❖ ***The discussion in the next slide.***

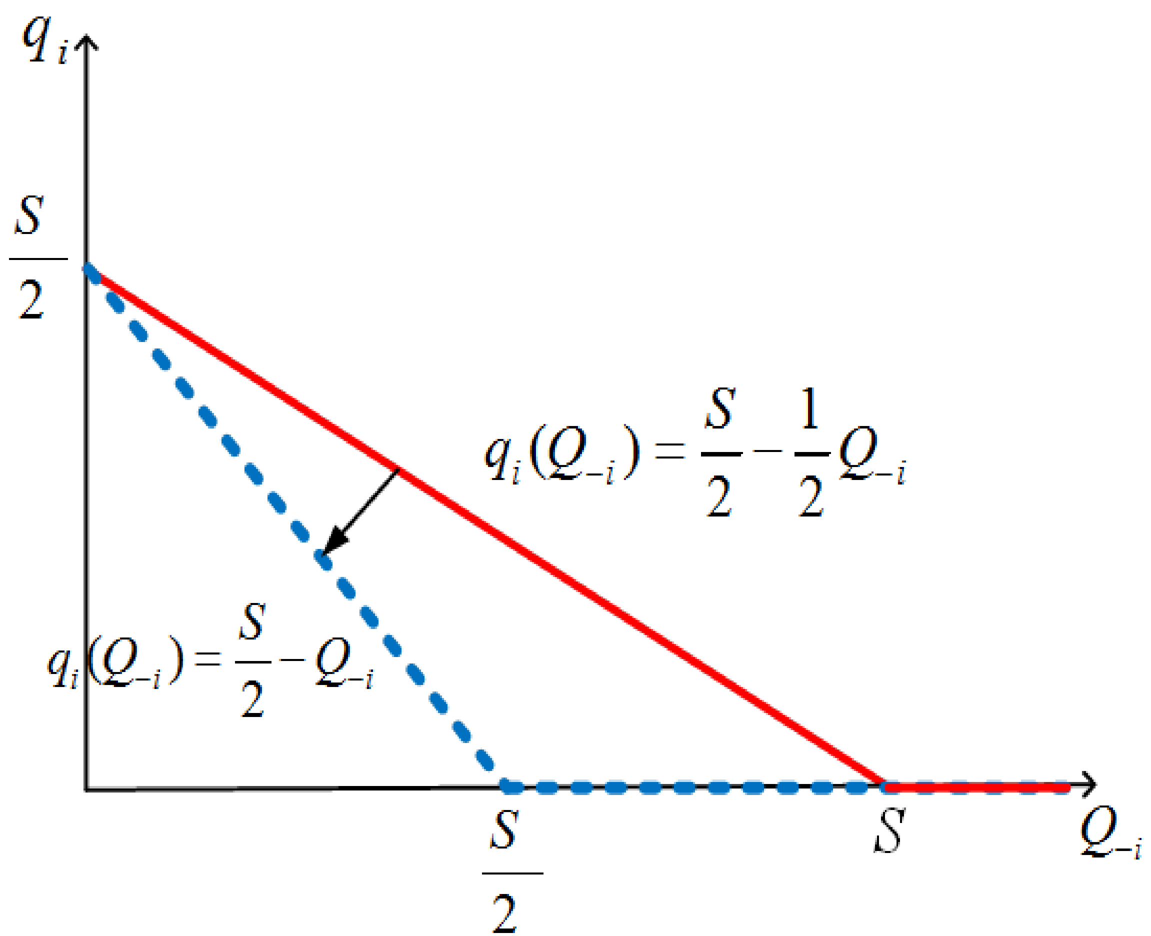

Figure 2.5. Equilibrium vs. joint profit maximization in the commons

- Figure 2.5 indicates the following:

- For a given amount of appropriation from firm 2 (q_2), firm 1 chooses to appropriate fewer units its when firms coordinate their exploitation of the resource (jointly maximizing profits) than when every firm independently selects its own appropriation

- If fisherman 2 appropriates half of the available stock, $q_2 = \frac{S}{2}$, fisherman 1 responds by not appropriating anything, $q_1 = 0$

❖ *Question….!*

How to find the horizontal intercept of expression $q_1(q_2) = \frac{S}{2} - q_2$*?*

- *Confirm the finding;*

I. Let us simultaneously solve for appropriation levels q_1 *and* q_2

$$q_1(q_2) = \frac{S}{2} - q_2 \qquad \text{for fisherman 1}$$

$$q_2(q_1) = \frac{S}{2} - q_1 \qquad \text{for fisherman 2}$$

II. We consider that, among all optimal pairs, a natural equilibrium is that in which both firms appropriate the same amount.

- Since firms are symmetric, the socially optimal output, q^{SO}, becomes

$$q_1^{SO} = q_2^{SO} = q^{SO}$$

III. Inserting $q_1^{SO} = q_2^{SO} = q^{SO}$ in the equation for fisherman 1

$$q^{SO} = \frac{S}{2} - q^{SO}$$

and solving for q^{SO};

$$q^{SO} = \frac{S}{4}$$

IV. When agents are independent,

$$q^* = \frac{S}{N+1}$$

Evaluating at the case of $N = 2$ fishermen;

$$q^* = \frac{S}{2+1} = \frac{S}{3}$$

- Comparing the results

$$q^* > q^{SO}$$

$$\frac{S}{3} > \frac{S}{4}$$

- *In words,*

The agents exploit the resource less intensively when they coordinate their appropriation decisions (and thus internalize the cost externalities their appropriation generates on others) than when they do not coordinate their exploitation.

"The tragedy of the commons"

2.4.2 Socially optimal appropriation with consumers and profits matter

➢ When the social planner considers welfare function

$$W = (1 - \lambda)CS + \lambda PS$$

- She chooses the level of catches q_1 and q_2 to solve

$$\max_{q_1, q_2 \geq 0} \quad W = (1 - \lambda)CS + \lambda PS$$

where

$$CS = \int_0^Q p(Q)dQ$$

2.4.2 Socially optimal appropriation with consumers and profits matter

- The inverse demand function;

$p(Q) = 1 - Q$ is linear in the aggregate appropriation

- Consumer surplus can be expressed as the area of the triangle below the demand function;

$$CS = \frac{1}{2}[1 - (1 - Q)](Q - 0) = \frac{1}{2}Q^2$$

- The aggregate appropriation can be expanded as;

$$Q = q_1 + q_2$$

- The social welfare can be rewritten as;

$$\max_{q_1,q_2 \geq 0} W = (1-\lambda)\frac{1}{2}(q_1+q_2)^2 + \lambda(\pi_1+\pi_2)$$

- Differentiating with respect to q_1 and q_2

$$\frac{\partial W}{\partial q_1} = (1-\lambda)(q_1+q_2) + \lambda\left(1 - \frac{2(q_1+q_2)}{S}\right) = 0$$

$$\frac{\partial W}{\partial q_2} = (1-\lambda)(q_1+q_2) + \lambda\left(1 - \frac{2(q_1+q_2)}{S}\right) = 0$$

2.4.2 Socially optimal appropriation with consumers and profits matter

- In a symmetric social optimum, firms exploit the CPR at the same rate;

$$q_1^{SO} = q_2^{SO} = q^{SO}$$

$$(1-\lambda)(q^{SO} + q^{SO}) + \lambda\left(1 - \frac{2(q^{SO} + q^{SO})}{S}\right) = 0$$

$$2(1-\lambda)q^{SO} + \lambda\left(1 - \frac{4q^{SO}}{S}\right) = 0$$

- Solving for q^{SO}, we obtain the socially optimal appropriation,

$$q^{SO} = \frac{S\lambda}{2[2\lambda - S(1-\lambda)]}$$

2.4.2 Socially optimal appropriation with consumers and profits matter

- Case: when $\lambda = 1$

  ✓ The socially optimal appropriation simplifies to

$$q^{so} = \frac{S}{4}$$

  ✓ The social planner only considered producer surplus $(\lambda = 1)$

Question..!

What is the impact of change in the weight on producer surplus on socially optimal?

2.4.2 Socially optimal appropriation with consumers and profits matter

➢ General case

$$q^{SO} = \frac{S\lambda}{2[2\lambda - S(1-\lambda)]}$$

- Differentiating with respect to λ

$$\frac{\partial q^{SO}(\lambda)}{\partial \lambda} = -\frac{S^2}{2[2\lambda - S(1-\lambda)]^2}$$ (which is negative)

➢ Intuitively,

The regulator decreases the socially optimal appropriation when she assigns a larger weight to producer surplus.

Facing our first inefficiency

- *From previous section,*

  - Our results help us to identify the first inefficiency in the exploitation of the commons by individual firms.

- Firms' equilibrium appropriation is larger than that a social planner would select. This happens regardless of the welfare function that she considers, that is, both when;

  I. she only seeks to maximize firms' joint profits

$$W = \pi_1 + \pi_2$$

II. her objective is to maximize a weighted sum of consumer and producer surplus

$$W = (1 - \lambda)CS + \lambda PS$$

- Intuitively;

- Every individual fisherman ignores the negative cost externality that his appropriation produces on the other fishermen, and thus exploits the resource above the socially optimal level.

  - ***Ex.*** The Chilean jack mackerel in the Southeast Pacific, and the Peruvian anchovy in the Southeast Pacific.

- Our result is analogous to that in the standard Cournot model of quantity competition, where firms tend to produce too much, relative to the output that would maximize their joint profits in a cartel,

  - Since they ignore the negative effect that their sales generate on their rivals' revenues

    (as these sales decrease the market price which, in turn, reduce the total revenue of all firms in the industry)

- This negative effect is, however, internalized when firms coordinate their production decisions to maximize their joint profits or, more generally, when a social planner determines individual output levels

Inefficient exploitation with more general functions

2.6 Inefficient exploitation with more general functions

- In this section, we want to show that;
  - The appropriation is excessive relative to the social optimum,
  - Or more compactly, the equilibrium appropriation is socially excessive

$$q^* > q^{SO}$$

- We show this result **without** assuming a specific cost function
  - Our previous analysis considered a specific cost function for every firm i

$$C_i(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

2.6 Inefficient exploitation with more general functions

- We only assume that firm *i*'s marginal cost $MC_i = \frac{\partial C_i(q_i, Q_{-i})}{\partial q_i}$ satisfies the following properties:

  - ***Assumption 1*:**
    Positive, $MC_i > 0$, and increasing in firm *i*'s own appropriation, $\frac{\partial MC_i}{\partial q_i} > 0$;

  - ***Assumption 2*:**
    Decreasing in the available stock, $\frac{\partial MC_i}{\partial S} < 0$;

  - ***Assumption 3*:**
    Increasing in the appropriation of any rival firm *j*, $\frac{\partial MC_i}{\partial q_j} > 0$, where $j \neq i$.

➢ Intuitively;

- Assumption 1 says that every fisherman i faces a positive and increasing cost for every additional unit the firm appropriates.

- Assumption 2 suggests that fisherman i can capture q_i tons of fish more easily when the stock becomes more abundant.

- Assumption 3 indicates that, when other fishermen increase their appropriation Q_i, the resource becomes more scarce, increasing the time and effort that fisherman i needs to spend to appropriate a given amount.

2.6 Inefficient exploitation with more general functions

Given that $$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

we have $$\frac{\partial C(q_i, Q_{-i})}{\partial q_i} = \frac{2q_i + Q_{-i}}{S} = MC_i$$

which is

✓ Positive and increasing in q_i (as required by Assumption 1)

✓ Decreasing in the stock S (as required by Assumption 2)

✓ Increasing in the appropriation by firm *i*'s rivals, Q_{-i} (as required by Assumption 3)

➢ Equilibrium appropriation

$$\max_{q_i \geq 0} \pi_i = q_i - C_i(q_i, Q_{-i})$$

$$\frac{\partial \pi_i}{\partial q_i} = 1 - \frac{\partial C_i(q_i, Q_{-i})}{\partial q_i} = 0$$

- We can express the above result more compactly as

$$MC_i = 1$$

- In words,

every fisherman *i* increases his individual appropriation until the point where his marginal revenue from additional sales coincides with the marginal cost of this additional appropriation.

❖ *The discussion in the next slide….!*

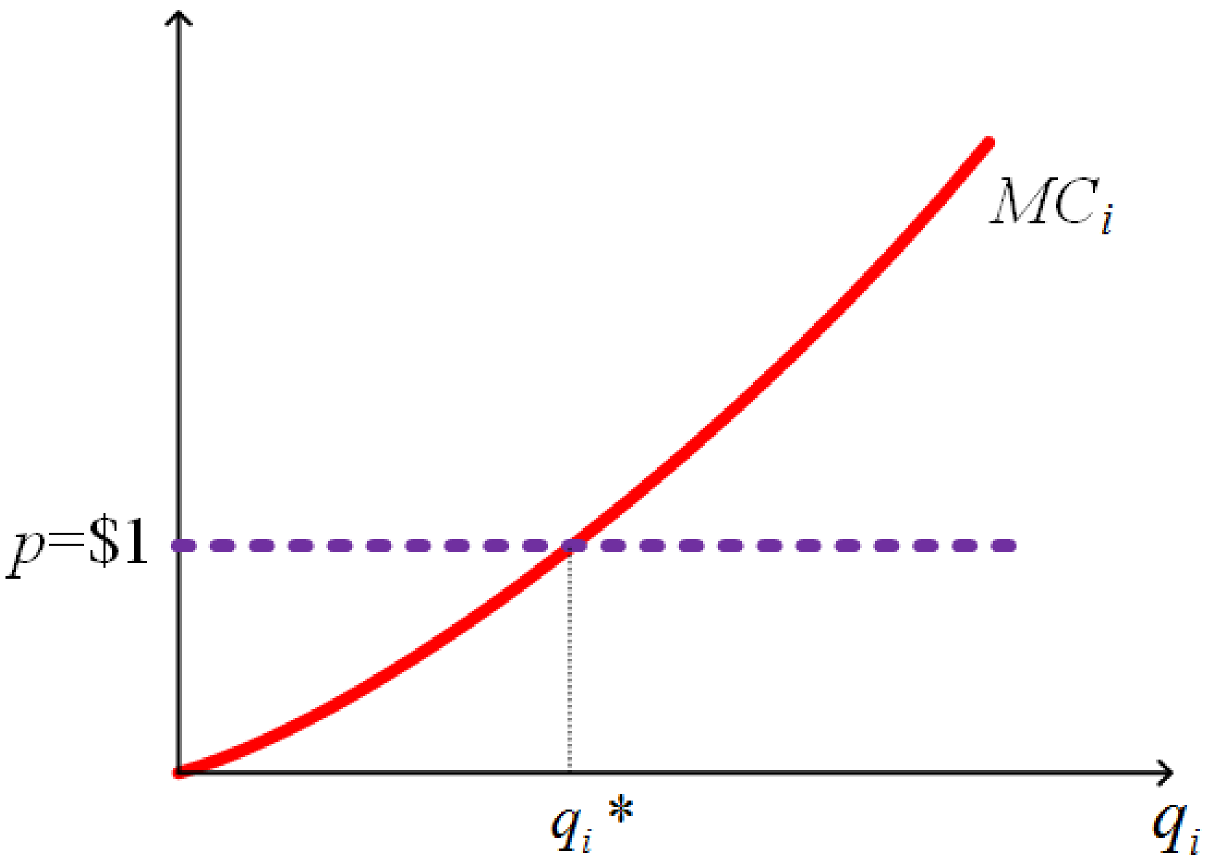

Figure 2.6. Equilibrium appropriation q_i^*

2.6 Inefficient exploitation with more general functions

- Figure 2.6 depicts condition $MC_i = 1$, by separately plotting the price $p = \$1$ and the marginal cost MC_i. This marginal cost is increasing in firm *i*'s appropriation q_i since, ***by Assumption 1***, $\frac{\partial MC_i}{\partial q_i} > 0$.

- When firm j increases its individual appropriation q_j, firm *i*'s marginal cost MC_i increases, since $\frac{\partial MC_i}{\partial q_j} > 0$ by ***Assumption 3***; whereas the marginal revenue in the right-hand side of $MC_i = 1$ is unaffected.

- In Figure 2.6, curve MC_i shifts upward, entailing that the crossing point between MC_i and \$1 moves to the left, reducing firm *i*'s equilibrium appropriation q_i.

➢ The socially optimal appropriation

- Assuming the welfare function considers only joint profits, the social planner solves a problem that is;

$$\max_{q_1,\ldots,q_N} W = PS = \sum_{i=1}^{N} \pi_i = \sum_{i=1}^{N} [q_i - C_i(q_i, Q_{-i})]$$

2.6 Inefficient exploitation with more general functions

- Which can be expanded as the sum of firm *i*'s profits plus the profits of all its rivals $\pi_i + \sum_{j\neq i}\pi_j$, as follows

$$\max_{q_i,\ldots,q_N\geq 0} W = [q_i - C_i(q_i, Q_{-i})] + \sum_{j\neq i}[q_j - C_j(q_j, Q_{-j})]$$

Differentiating with respect to every q_i, we find

$$1 - \frac{\partial C_i(q_i, Q_{-i})}{\partial q_i} - \sum_{j\neq i}\frac{\partial C_j(q_j, Q_{-j})}{\partial q_i} = 0$$

2.6 Inefficient exploitation with more general functions

➢ Since Q_{-j} includes q_i as one of its components,

- we can rearrange the expression as;

$$MC_i + \sum_{j \neq i} \frac{\partial C_j\left(q_j, Q_{-j}\right)}{\partial q_i} = 1$$

✓ Our result then coincides with equilibrium condition $MC_i = 1$, except for the new term $\sum_{j \neq i} \frac{\partial C_j\left(q_j, Q_{-j}\right)}{\partial q_i}$.

- Intuitively,

Every firm i increases its individual appropriation until the point where its marginal revenue from appropriating one more unit ($p = \$1$) coincides with the sum of its own additional cost, MC_i, and the additional cost that its appropriation generates on all other firms, $\sum_{j \neq i} \frac{\partial C_j(q_j, Q_{-j})}{\partial q_i}$.

- Relative to the equilibrium condition $MC_i = 1$, every firm now internalizes the negative cost externality that its individual appropriation q_i produces on its rivals.

  - As a result of this additional cost, firm i chooses a lower exploitation in the social optimum than in equilibrium, $q_i^{SO} < q_i^*$

❖ Figure 2.7 illustrates this result and compares it against that emerging from equilibrium condition, $MC_i = 1$.

2.6 Inefficient exploitation with more general functions

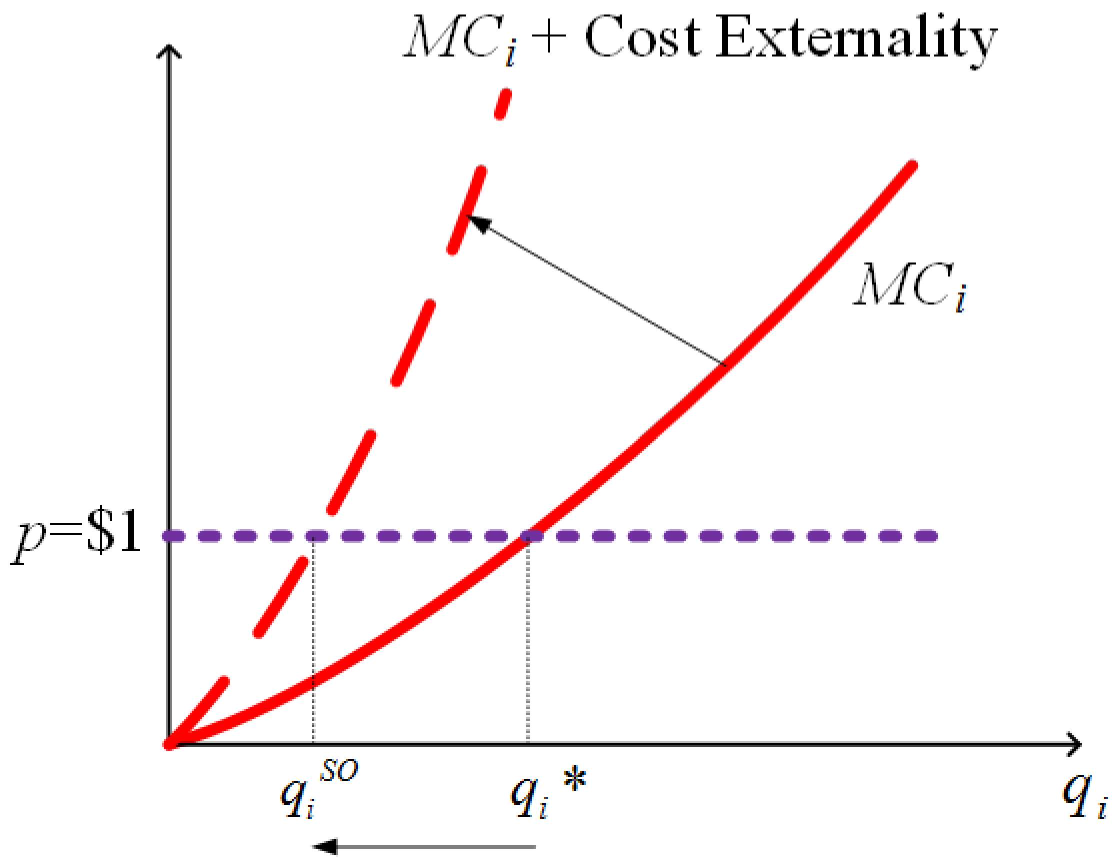

Figure 2.7. Equilibrium and socially optimal appropriation

- In this section,
    - We discuss some policy instruments to correct the socially excessive exploitation that we identified in our previous results.

- Two policy instruments

I. Quotas
II. Appropriation fees

2.7.1. Policy instruments-Quotas-

- The regulator can set a quota that lets fisherman i catch as much fish as the socially optimal level, q_i^{SO}, facing stringent penalties if it exceeds this allowance.

- Quotas are rather common in several CPRs such as;

  - ***Common Fisheries Policy*** in the European Union, which sets quotas on which types of fish each member state can fish.

  - ***Individual transferable quotas*** assigned to each fisherman in the U.S. or New Zealand.

*These quotas are also known as **Catch Share***

2.7.1. Policy instruments-Quotas-

- How these quotas work?

  - The regulator starts by setting a total allowable catch for each species of fish and for a given time period;

  - and then a dedicated portion is assigned to individual fishermen in the form of quotas, which are transferable, and thus can be bought, sold, and leased to other fishermen.

- *Example;*

- In 2008,148 major fisheries and 100 smaller fisheries around the world had adopted some from of individual transferable quota.

➢ *How do these quotas assign?*

- Quotas are often initially assigned according to the recent catch history of the fishermen, implying that those who more intensively appropriate the resource receive larger quotas.

- This assignment rule can, then, induce fishermen to increase their relative appropriation of the resource to receive a larger transferable quota, which they can keep or sell in future periods.

Quota auctions

- Quota auctions have been proposed as an alternative allocation mechanism, which may prevent the previous perverse incentives to increase appropriation before the quota is allocated and, in addition, raises public funds for access to fisheries.

➢ Quotas in aquifers

- Quotas in aquifers are less common, but countries such as Mexico and Spain set limits on private use; otherwise, the farmer can lose his water permit.

➢ Other command-and-control regulations

- Other command-and-control regulations include restrictions on the boat size, fishing gear (such as mesh or net size), limits on the days certain boats can fish, or prohibiting the catch of juvenile fish; among others.

Appropriation fees

- The regulator can set an emission fee to fisherman i, t_i, that induces this fisherman appropriate the socially optimal level q_i^{SO}.

In this setting,

- Every fisherman i solves a problem analogous to

$$\max_{q_i \geq 0} \pi_i = q_i - \frac{q_i(q_i + Q_{-i})}{S}$$

- but with marginal costs increased by t_i.

2.7.2 Policy instruments-Appropriation fees

- Fisherman *i*'s objective function now becomes

$$\max_{q_i \geq 0} \pi_i = q_i - \frac{q_i(q_i + Q_{-i})}{S} - q_i t_i$$

- First-order condition with respect to q_i

$$1 - \frac{2q_i + Q_{-i}}{S} - t_i = 0$$

- Solving for appropriation q_i , we find best response function

$$q_i(Q_{-i}) = \frac{S(1 - t_i)}{S} - \frac{1}{2} Q_{-i}$$

2.7.2 Policy instruments-Appropriation fees

- *When the appropriation fee is absent,* $t_i = 0$

$$q_i(Q_{-i}) = \frac{S}{2} - \frac{1}{2}Q_{-i} \qquad (BRF_i)$$

  - ✓ It coincides with that in section 2.3

- *When the appropriation fee is present* t_i

  - A more stringent fee decreases the vertical intercept of the best response function, $\frac{S(1-t_i)}{S}$, without affecting its slope.

- Graphically;
  - We can imply a parallel downward shift of fisherman *i*'s best response function.

(Try to draw it on Figure 2.1)

- Intuitively,
  - For a given aggregate appropriation from his rivals Q_{-i}, fisherman i decreases his individual appropriation when facing a more stringent fee.
  - This comes at no surprise since this fee increases the fisherman's marginal cost of additional appropriation, reducing his incentives to exploit the resource.

2.7.2 Policy instruments-Appropriation fees

➢ In a symmetric equilibrium,

- $q_i^* = q_j^* = q^*$, which entails that $Q_{-i}^* = (N-1)q_i^*$.

- Inserting this property in the above best response function;

$$q^* = \frac{S(1-t_i)}{2} - \frac{1}{2}(N-1)q^*$$

Rearranging yields $q^*(N+1) = S(1+t_i)$

2.7.2 Policy instruments-Appropriation fees

Solving for q^*;

$$q^*(t_i) = \frac{S(1 - t_i)}{N + 1}$$ *"The equilibrium appropriation"*

- *Case 1: When the appropriation fee is absent,* $t_i = 0$

$$q^*(t_i) = \frac{S}{N + 1}$$

- *Case 2: When the appropriation fee is present,* $t_i > 0$

Nonetheless, equilibrium appropriation is lower

- Questions…!

  - How can the regulator find the appropriation fee t_i that induces fisherman *i* exploit the resource at the socially optimal level q_i^{SO}?

  - What emission fee t_i, inserted in fisherman i's equilibrium appropriation $q^*(t_i)$, induces this fisherman to appropriate q^{SO}?

2.7.2 Policy instruments-Appropriation fees

- The regulator seeks to achieve $q^*(t_i) = q_i^{SO}$;
  - We know that:

$$q^*(t_i) = \frac{S(1-t_i)}{N+1} \qquad \text{and} \qquad q_i^{SO} = \frac{S}{4}$$

$$\frac{S(1-t_i)}{N+1} = \frac{S}{4}$$

Solving for t_i^*;

$$t_i^* = \frac{3-N}{4}$$

✓ which is decreasing in the number of firms

- Intuitively,
  - The regulator seeks to induce the same socially optimal output per firm, $q_i^{SO} = \frac{S}{4}$, regardless of the number of firms.

- What is the impact of the number of firms on the equilibrium appropriation $q^*(t_i)$?

  - ***When few firms*** operate in the common;

  - ✓ The equilibrium exploitation of each firm, $q^*(t_i)$, is substantially larger than q_i^{SO}, requiring a stringent fee to reduce exploitation.

2.7.2 Policy instruments-Appropriation fees

- When ***several firms*** compete:

  - ✓ The equilibrium appropriation of each firm, $q^*(t_i)$, is relatively lower, while $q_i^{SO} = \frac{S}{4}$ is unaffected, leading the regulator to set a lax appropriation fee; which converges to zero when *N* is sufficiently large.

  - ✓ While appropriation fees are less common in fisheries, they are relatively frequent in groundwater agricultural use.