Mergers in Cournot markets with environmental externality and product differentiation

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Motivation & Contribution

- There are not many studies examining how differences in pollution parameters between post and pre-merger markets affect the attractiveness of merger deals.

- The contribution of this study is to examine some conditions under which mergers are more profitable in industries that produce differentiated goods. We extend the discussion to firms producing goods with a negative environmental externality and facing an emission tax.

Research questions

The specific research questions addressed are:

- What is the effect of mergers on the optimally determined emission tax rate, if any?
- How does the attractiveness of a merger deal change when the industry's product differentiation changes?
- How does the effect of product differentiation on the attractiveness of a deal depend on whether the merged entity has modified its technology or product?
- What is the role of the abatement induced by the emission tax on the attractiveness of a merger deal?

The Model

- Consider a Cournot oligopoly model with n firms competing in quantities.
- Every firm i faces the inverse demand curve

$$p_i = \alpha - (\beta - \gamma)q_i - \gamma \sum_{i=1}^{n} q_i \,,$$

$\beta \geq \gamma \geq 0, \;\; \alpha > 0 \; \forall i$, where γ represents the degree of product differentiation, i.e., $\beta = \gamma$ products are completely homogeneous goods.

- Cost function for each firm i is $C(q_i, e_i) = \; cq_i + \frac{(\delta q_i - e_i)^2}{2}$, where q_i denotes output level, $\delta > 0$ is pollution intensity, $c > 0$ and e_i is net emissions.

- Each firm pays a per unit emission tax, t.
- The government choose emission tax to maximize the social welfare:

$$\max_t W = CS\left(\sum_{i=1}^{n} q_i\right) + \left(\sum_{i=1}^{n} \pi_i\right) + t\left(\sum_{i=1}^{n} e_i\right) - \overbrace{\varphi\left(\sum_{i=1}^{n} e_i\right)}^{damages}$$

$\frac{d\varphi(\cdot)}{de} = \; \varphi' > 0$ denotes the marginal damage from emissions

Cont. The Model

- The govt. & firms play a Two-Stage game
  - Where the govt. sets policy (i.e., τ) via social welfare maximization.
  - Firms then take the policy as given and maximize profits by simultaneously choosing the level of output and emissions in a Cournot–Nash fashion.

- We assume interior solutions and symmetric equilibrium throughout the analysis.
  - Under symmetry we have $q_1 = q_2 = \cdots = q$ and thus the market demand function can be expressed as $p_i = \alpha - (\beta - \gamma(n-1))q$.

- We solve the model by backwards induction,
  - In the second stage we solve for the firm's problem.
  - In the first stage we solve for welfare maximization.

Pre-merger Equilibrium

- In the Second Stage:

Solving for the firm's problem:

$$\max_{q_i,\, e_i} \pi_i = (p_i - c)q_i - \frac{1}{2}(\delta_i q_i - e_i)^2 - te_i$$

Taking the first order conditions then, imposing symmetry :

$$q = \frac{\alpha - c - \delta t}{2\,\beta + \gamma(n-1)}, \qquad e = \frac{\delta(\alpha - c - \delta t)}{2\,\beta + \gamma(n-1)} - t, \qquad \pi = \beta(q)^2 + \frac{1}{2}t^2$$

$$\frac{\partial q}{\partial \gamma} < 0, \qquad and \quad \frac{\partial \pi}{\partial \gamma} < 0$$

- In the First Stage:

Solving for welfare maximization:

$$\max_{t} W = CS(nq) + n\pi + net - \varphi(ne)$$

Totally differentiating and re-arranging terms yields:

$$\frac{dW}{dt} = n\beta q\frac{dq}{dt} + (t - \varphi')n\frac{de}{dt}$$

$$t = \varphi' - \frac{\beta q\left[\frac{dq}{dt}\right]}{\frac{de}{dt}}$$

Cont. Pre-merger Equilibrium

$$t = \frac{\varphi'\left(\frac{\delta^2}{2\beta+\gamma(n-1)}+1\right) - \frac{\delta\beta(\alpha-c)}{(2\beta+\gamma(n-1))^2}}{\frac{\delta^2(\beta+\gamma(n-1))}{(2\beta+\gamma(n-1))^2}+1}$$

Assumption 2.2. Optimal emission tax in the pre-merger market is strictly positive.

For Assumption 2.2 to hold the marginal damage from industry pollution should be higher than a given threshold φ_0', that is $\varphi' > \varphi_0'$, where $\varphi_0' = \frac{(\alpha-c)\beta\delta}{(2\beta+(n-1)\gamma)(2\beta+(n-1)\gamma+\delta^2)}$.

The optimal tax is related to the industry's product differentiation,γ, as follows:

$$\frac{dt}{d\gamma} = \frac{\beta(n-1)\delta(\alpha-c-\delta\varphi')(4\beta+2n\gamma-2\gamma+\delta^2)}{[4\beta^2+\gamma^2(n-1)^2+\gamma(n-1)\delta^2+\beta(4\gamma(n-1)+\delta^2]^2}$$

where $\frac{dt}{d\gamma} < 0$ if $\varphi' > \varphi_1'$ where $\varphi_1' = \frac{(\alpha-c)}{\delta} > \varphi_0'$. This condition implies that higher product differentiation increases the optimal tax rate. Intuitively, this is because higher product differentiation leads to higher output $\left(\frac{\partial q}{\partial \gamma} < 0\right)$ and so higher taxes are required to reduce the resulting environmental externality.

In the remainder of the paper, we assume $\varphi' > \varphi_1'$ so that $\frac{dt}{d\gamma} < 0$ holds.

Cont. Pre-merger Equilibrium

Two merge □rms, and $n-2$ are the outsiders.

- ## In the Second Stage:

Solving for the merge firm and the outsider problem:

$$\max_{q_1,e_1,q_2,e_2} \pi_1 + \pi_2 = \left[(p_1 - c)q_1 - \frac{(\delta_1 q_1 - e_1)^2}{2} - e_1 t\right] + \left[(p_2 - c)q_2 - \frac{(\delta_2 q_2 - e_2)^2}{2} - e_2 t\right]$$

$$\max_{\tilde{q}_i,\tilde{e}_i} \tilde{\pi} = \left[(\tilde{p}_i - c)\tilde{q}_i - \frac{(\delta_i \tilde{q}_i - \tilde{e}_i)^2}{2} - \tilde{e}_i t\right], \quad i = 3, \dots, n$$

Taking F.O.C, using the zero condition and imply symmetry $\delta_1 = \delta_2 = \delta$

$$q_1 = q_2 = \frac{(2\beta-\gamma)(\alpha-c-\delta t)}{2(2\beta^2+\beta\gamma(n-1)-\gamma^2)}, \qquad \tilde{q} = \frac{\beta(\alpha-c-\delta t)}{2\beta^2+\beta\gamma(n-1)-\gamma^2}, \qquad \pi_1 + \pi_2 = 2(\beta+\gamma)(q_1)^2 + t^2$$

The merger occurs as long as $\pi_1 + \pi_2 - 2\pi > 0$.
$\downarrow$ in γ $\uparrow$ the merged entity's profit $\pi_1 + \pi_2$ (for given tax).

Cont. Pre-merger Equilibrium

- In the First Stage:

Solving for the government optimal policy

$$\max_t W = CS(q_1 + q_2 + (n-2)\tilde{q}) + \pi_1 + \pi_2 + (n-2)\tilde{\pi} + e_1 t + e_2 t + (n-2)\tilde{e}t - \varphi(e_1 + e_2 + (n-2)\tilde{e})$$

Totally differentiating, re-arranging terms yields, and using the zero condition see Appendix B

$$t_m = \varphi' - \frac{2q_1(\beta+\gamma)\left(\frac{\partial q_1}{\partial t_m}\right)+(n-2)\beta\tilde{q}\left(\frac{\partial \tilde{q}}{\partial t_m}\right)}{2\delta\left(\frac{\partial q_1}{\partial t_m}\right)+(n-2)\delta\left(\frac{\partial \tilde{q}}{\partial t_m}\right)-n}$$

$$t_m = \frac{\eta\varphi' - (\alpha - c)\delta(2\beta^3 n - 3\beta\gamma^2 + \gamma^3)}{\eta - \delta^2(2\beta^3 n - 3\beta\gamma^2 + \gamma^3)}, \qquad \eta > 0$$

Assumption: The post-merger optimal policy is positive if

For any given $n > 2$, $t_m > 0$ if $\varphi' > \varphi_2'$ *where* $\varphi_2' = \frac{\delta(\alpha-c)(2n\beta^3-3\beta\gamma^2+\gamma^3)}{\gamma}$. We find that $\varphi_1' > \varphi_2'$ which implies that as long as $\varphi' > \varphi_1'$ the post-merger optimal policy is positive.

Comparative analysis

- Comparative analysis using the post-merger policy:

$$\frac{dt_m}{d\gamma} = 2\delta(\alpha - c - \delta\varphi')\frac{J}{D^2}$$ see eq. (25)

Where for any given n, $J > 0$. Under the given assumption of positive tax, $\frac{dt_m}{d\gamma} < 0$. Similar to the pre-merger case, a higher product differentiation leads to a higher tax required to reduce the environmental externality resulting from more output.

$$\frac{dt_m}{d\delta} = \frac{(\alpha - c)a_0 + \varphi'\delta a_1}{(a_2)^2}$$ see eq. (26)

Where $a_1 > 0$. Under the given assumption (i.e.,$\varphi' > \varphi_1'$ and $t_m > 0$) we find $\frac{dt_m}{d\delta} > 0$. This is intuitive since a higher pollution intensity triggers a larger tax rate than a lower pollution intensity.

Comparing Pre- & Post-merger Outcomes

- Output level in Pre- & Post-merger

$$q_1 = q_2 = \frac{(2\beta-\gamma)(\alpha-c-\delta_m t_m)}{2(2\beta^2+\beta\gamma(n-1)-\gamma^2)}, \qquad \tilde{q} = \frac{\beta(\alpha-c-\delta_m t_m)}{2\beta^2+\beta\gamma(n-1)-\gamma^2}, \qquad q = \frac{\alpha- c - \delta t}{2\,\beta+\gamma(n-1)}$$

Hence, for $\beta > \gamma > 0$
(i) $q_1 < \tilde{q}$,
(ii) if $t_m \leq t$ and $\delta_m \leq \delta$ then $q < \tilde{q}$ and
(iii) if $t_m = t$ and $\delta_m = \delta$ then $q_1 < q$

The reduction in output post-merger by the merged entity reflects output coordination in order to push the market price upwards.

- Industry output in pre- and post-merger:

$$nq = \frac{n(\alpha- c - \delta t)}{2\,\beta+\gamma(n-1)}, \qquad q_1 + q_2 + (n-2)\tilde{q} = \frac{(\beta n-\gamma)(\alpha-c-\delta_m t_m)}{2\beta^2+\beta\gamma(n-1)-\gamma\text{^}2}$$

If $t_m = t$ and $\delta_m = \delta$, we find that the industry output pre-merger is higher than the industry output post-merger. Thus, a merger reduces industry output and increases price which is the usual market power motive for merger.

Cont. Comparing Pre- & Post-merger Outcomes

- Optimal Policies in Pre- & Post-merger

$$\frac{dW}{dt} = n\beta q \frac{dq}{dt} + (t - \varphi')n \frac{de}{dt}$$

Evaluated at t_m

$$\frac{\partial W}{\partial t}\Big|_{t_m} = n\beta q \frac{\partial q}{\partial t} - \left(\frac{2q_1(\beta+\gamma)\left(\frac{\partial q_1}{\partial t_m}\right)+(n-2)\beta\tilde{q}\left(\frac{\partial \tilde{q}}{\partial t_m}\right)}{\underbrace{2\delta_m\left(\frac{\partial q_1}{\partial t_m}\right)+(n-2)\delta_m\left(\frac{\partial \tilde{q}}{\partial t_m}\right)-n}_{(-)}} \right) n \frac{\overbrace{\partial e}^{(-)}}{\partial t}$$

Multiplying both sides by $-\left[2\delta_m\left(\frac{\partial q_1}{\partial t_m}\right) + (n-2)\delta_m\left(\frac{\partial \tilde{q}}{\partial t_m}\right) - n\right]$ and collecting terms gives:

$$-\overbrace{\psi}^{(-)} \frac{dW}{dt}\Big|_{t_m} = \overbrace{2n \frac{\partial q}{\partial t}\frac{\partial q_1}{\partial t_m}[\delta q_1(\beta+\gamma) - \delta_m \beta q] + n(n-2)\delta\beta \frac{\partial q}{\partial t}\frac{\partial \tilde{q}}{\partial t_m}[\tilde{q}\delta - q\delta_m]}^{capture\ the\ effect\ of\ damages\ from\ pollution}$$

$$-n\overbrace{\left[2(\beta+\gamma)q_1\frac{\partial q_1}{\partial t_m} + (n-2)\beta\tilde{q}\frac{\partial \tilde{q}}{\partial t_m} - n\beta q\frac{\partial q}{\partial t}\right]}^{the\ abatement\ induced\ by\ the\ tax}$$

Cont. Comparing Pre- & Post-merger Outcomes

- What is the effect of mergers on the optimally determined emission tax rate? "first question"

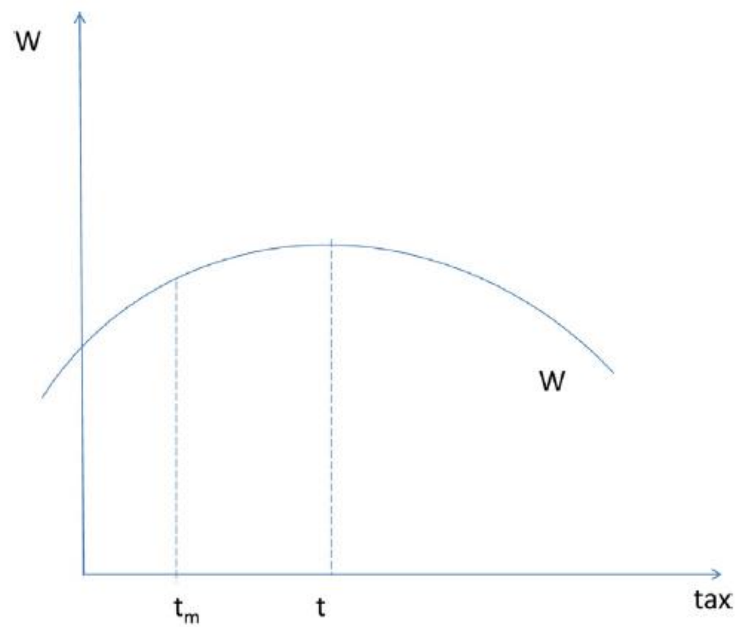

Fig. 1. Pre- and post-merger emission tax.

Proposition 2.3

Mergers result in lower optimal emission tax post-merger (i.e., $t_m < t$) as long as the post-merger pollution intensity, δ_m, is sufficiently small and/or abatement induced by the post-merger tax is sufficiently large and positive.

Profitability of Merger

- Optimal Policies in Pre- & Post-merger

The profitability of the merger is

$$\Delta = \pi_1 + \pi_2 - 2\pi$$

Merger occurs if $\Delta > 0$.

Δ : Incentive to merge in a differentiated market.

$$\Delta = 2(\beta + \gamma)(q_1)^2 + (t_m)^2 - 2\beta q^2 - t^2$$

$$\Delta = 2(\beta + \gamma)\left(\frac{(2\beta - \gamma)(\alpha - c - \delta_m t_m)}{2(2\beta^2 + \beta\gamma(n-1) - \gamma^2)}\right)^2 + (t_m)^2 - 2\beta\left(\frac{\alpha - c - \delta t}{2\beta + \gamma(n-1)}\right)^2 - t^2$$

Δ is ambiguous; depends upon parameter values $\beta \geq \gamma \geq 0$, differences in tax payments, δt and $\delta_m t_m$, and differences in abatement effects between the pre- and post-merger markets, t^2 and t_m^2.

Profitability of Merger

To answer the second and third questions, we consider two cases both of which imply an exogenous change in pollution intensity in the post-merger market:

- A change in δ_m is caused by modification of production technology or process.

$$\delta_m = \delta, \quad \delta_m > \delta, \quad and\ \delta_m < \delta$$

- A change in δ_m is caused by modification of products to be more environmentally friendly.

A function of γ.

$$\frac{d\delta(\gamma)}{d\gamma} = 0, \quad and\ \frac{d\delta_m(\gamma)}{d\gamma} = \delta_m' \geq 0$$

Keeping this two cases in mind, we will study the effect of an exogenous change in γ on the Δ:

$$\frac{d\Delta}{d\gamma} = 2(q_1)^2 + 4(\beta+\gamma)q_1\frac{dq_1}{d\gamma} - 4\beta q\frac{dq}{d\gamma} + 2t_m\frac{dt_m}{d\gamma} - 2t\frac{dt}{d\gamma}$$

$$\frac{dq_1}{d\gamma} = \overbrace{\frac{\partial q_1}{\partial t_m}}^{(-)}\frac{dt_m}{d\gamma} + \overbrace{\frac{\partial q_1}{\partial \gamma}}^{(-)} + \overbrace{\frac{\partial q_1}{\partial \delta_m}}^{(-)}\delta_m', \qquad \frac{dq}{d\gamma} = \underbrace{\overbrace{\frac{\partial q}{\partial t}}^{(-)}\overbrace{\frac{dt}{d\gamma}}^{(-)}}_{(+)} + \overbrace{\frac{\partial q}{\partial \gamma}}^{(-)}, \text{ and} \qquad \frac{dt_m}{d\gamma} = \overbrace{\frac{\partial t_m}{\partial \gamma}}^{(-)} + \overbrace{\frac{\partial t_m}{\partial \delta_m}}^{(+)}\delta_m'$$

Assuming β, n, α and c remain fixed

Merged Entity & Cleaner Technology

We examine how differences between the Pre- & Post-merger pollution intensity potentially affect merger profitability: where $t_m < t$

Assume $\frac{dt}{d\gamma} = 0, \frac{dt_m}{d\gamma} = 0$, and $\delta_m'(\gamma) = 0$

$\gamma > 0,\ \frac{dq_1}{d\gamma} < 0,\ and \frac{dq}{d\gamma} < 0$

$$\Delta\Big|_{\gamma=0} = \delta q t - t^2 - \delta_m q_1 t_m + t_m^2 + q_1 \delta t - q \delta_m t_m$$

$$\Delta|_{\gamma=\beta=1} = \frac{(n+1)^2(\alpha-c-\delta_m t_m)^2 - 2n^2(\alpha-c-\delta t)^2}{n^2(n+1)^2} + t_m^2 - t^2; \quad 2n^2 > (n+1)^2$$

$$\frac{d\Delta}{d\gamma} = [-2(q_1)^2\gamma] + \frac{4\beta q^2(n-1)}{2\beta+\gamma(n-1)}$$

$$\frac{d\Delta}{d\gamma}|_{\gamma=0} = \frac{(n-1)(q+q_1)(-\delta t+\delta_m t_m)}{\beta}$$

$$\frac{d\Delta}{d\gamma}|_{\gamma=\beta=1} = \frac{4n^3(n-1)(\alpha-c-\delta_m t_m)^2-(n+1)(n-2)(\alpha-c-\delta t)^2}{\beta} + t_m^2 - t^2;$$

Remark 3.1

For any degree of product differentiation $\gamma \epsilon [0, \beta]$ profitability is positive for sufficiently small post-merger pollution intensity, δ_m, and sufficiently large abatement effect post-merger.

Cont. Merged Entity & Cleaner Technology

$$\Delta = 2(\beta+\gamma)\left(\frac{(2\beta-\gamma)(\alpha-c-\delta_m t_m)}{2(2\beta^2+\beta\gamma(n-1)-\gamma^2)}\right)^2 + (t_m)^2 - 2\beta\left(\frac{\alpha-c-\delta t}{2\,\beta+\gamma(n-1)}\right)^2 - t^2$$

Case 1: Pollution intensity is not higher post-merger, $\delta_m \leq \delta$

- Consider $\delta_m = \delta$ and (i) $\Delta|_{\gamma=0} > 0$, if $t_m^2 - t^2$ small; (ii) $\Delta|_{\gamma=\beta=1} < 0$, if $t_m < t$ small; and (iii) evaluated at $\gamma = 0, \frac{d\Delta}{d\gamma} < 0$. Then, there is a $0 < \tilde{\gamma} < \beta$ such that $\Delta = 0$. A range of γ for which $\Delta \lesseqgtr 0$

Why?

- Difference in abatement effect too small. (affect profitability)
- Product differentiation ($\gamma \simeq 0$) vs ($\gamma \simeq \beta$). (affect profitability)

If (i.e., $t_m^2 - t^2$ is large) $\rightarrow \Delta < 0$

- Consider $\delta_m < \delta$: ↑ prob. of positive in very diff. market (i.e., $t_m^2 - t^2$ is small)

If (i.e., $t_m^2 - t^2$ is large) $\rightarrow \Delta < 0$

* Important role for a smaller δ_m to raise profitability of merger.

Cont. Merged Entity & Cleaner Technology

Case 2: Pollution intensity is higher post-merger, $\delta_m > \delta$

If δ_m, is sufficiently large, then $\Delta < 0$ for very Diff. products.
Why?

- Tax payments of the merged entity rise relatively more
- Pre-merger profits are larger due to a relatively higher abatement effect, i.e., $t_m^2 < t^2$. Thus, with a sufficiently large δ_m, profitability declines via higher tax payments post-merger and higher abatement effect pre-merger as γ becomes smaller.

Proposition 3.2

The attractiveness of a merger deal increases with the increase in product differentiation when the merged entity is relatively less pollution-intensive post-merger than pre-merger.

Merged Entity & Modified Product

We examine the potential role an environmentally conscious firm in the post-merger equilibrium may have on profitability.

Case 1:

$$\frac{\partial t_m}{\partial \gamma} = 0\,, \frac{\partial t_m}{\partial \delta_m} = 0, \frac{\partial t}{\partial \gamma} = 0, and \;\; \delta_m'(\gamma) > 0$$

Case 2:

$$\frac{\partial t_m}{\partial \gamma} = 0\,, \frac{\partial t_m}{\partial \delta_m} > 0, \frac{\partial t}{\partial \gamma} = 0, and \;\; \delta_m'(\gamma) > 0$$

Case 3:

$$\frac{\partial t_m}{\partial \gamma} < 0\,, \frac{\partial t_m}{\partial \delta_m} = 0, \frac{\partial t}{\partial \gamma} < 0, and \;\; \delta_m'(\gamma) = 0$$

Case 4:

$$\frac{\partial t_m}{\partial \gamma} < 0\,, \frac{\partial t_m}{\partial \delta_m} > 0, \frac{\partial t}{\partial \gamma} < 0, and \;\; \delta_m'(\gamma) > 0$$

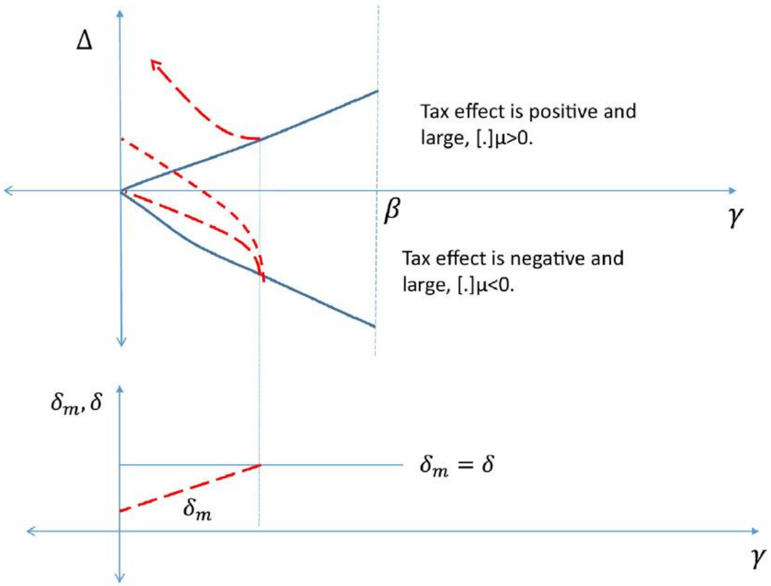

Fig. 2. $dt/d\gamma < 0$ and $dt_m/d\gamma < 0$; profitability and changes in profitability.

Proposition 4.1

In markets where firms are differentiating products to be environmentally conscious, the attractiveness of a merger deal increases with product differentiation if the pollution intensity of the merged entity is smaller post-merger than pre-merger.

Conclusion

Findings Suggest:

- Merger deals could result in lower t_m.
- As products become more diff., the attractiveness of a deal increases.
- The attractiveness of a merger a deal increases as δ_m is not large.
- When merged entities modify products to be greener, they are more likely to benefit more from the deal if δ_m is not large.