1. Introduction

Many industries nowadays include firms with different pollution intensities. The literature on environmental regulation, and horizontal mergers, however, mostly considers markets where firms are either all polluting or all green rather than a combination of both. In the energy sector, for instance, mergers are common between energy and utility service providers in which one firm is less polluting than its rivals (Creti and Sanin, 2017).[1]

Allowing for firm-level heterogeneities can help us better understand firms' behavior and predict the welfare effects from the mergers. If the antitrust authorities (AA) incorrectly assume that all firms are symmetric, mergers may be blocked on the basis that they can lead to an overall output reduction. When differences in pollution intensities are taken into account, however, mergers could induce output shifts that mitigate environmental damages, yielding an overall welfare gain in settings where mergers would have been blocked otherwise.

Our model considers an industry with several green and polluting brown firms, where we allow for cost asymmetries and product differentiation between different types of firms. In the first stage, firms choose whether to file a merger request to the AA for approval. In the second stage, the AA approves or blocks the merger. In the third stage, the environmental regulator (such as the Environmental Protection Agency, EPA) sets emission fees, observing whether the merger had been approved or not. In the fourth stage, firms respond choosing their output levels either cooperatively as part of the merger agreement or competing à la Cournot. This timing indicates that merger plans and the AA's response, approving or blocking the merger, are often taken into consideration in a long-run horizon, and staying in place for years or decades before any revisions. In contrast, emission fees and firms' output decisions can be more easily revised after observing a change in the merger decisions in the early stages of the game.[2]

[1] For example, Enbridge, a Canadian oil-pipeline operator, acquired the 50-megawatt Silver State North photovoltaic project from First Solar Inc., a Nevada-based solar power plant in 2012 to honor its neutral footprint commitment (Martin, 2012). Creti and Sanin (2017, Table 1) provide a list of mergers between renewable and non-renewable energy firms under the cap-and-trade program of the Regional Greenhouse Gas Initiative (RGGI). In addition, Hennessy and Roosen (1999) suggest that firms may have incentives to merge and aggregate pollution permits when emission is pooled within a certain attainment area under the "bubble" policy of the US Environmental Protection Agency (EPA).

[2] In addition, we consider the AA taking precedence over the EPA in approving mergers when firms have incentives

We first turn our attention to the case of no environmental regulation. In this setting, mergers induce an output shift from the less to the more cost-efficient firm irrespective of pollution differentials. Such mergers can, however, yield a welfare loss if firms produce relatively homogeneous goods, as in standard merger models without environmental regulation (Kao and Menezes, 2010). Nevertheless, welfare-enhancing mergers can arise when output shifts to the less polluting firms, or output of both green and brown firms is reduced, mitigating more severe environmental damages.[3]

We next consider environmental regulation in our analysis, examining under which conditions mergers entail a welfare loss, as in standard models of market power, and in which cases they yield a welfare gain. In this setting, emission fees create incentives to shift output after the merger from the brown to the green firms–an incentive that did not exist in the absence of environmental regulation. Most importantly, the increase in the green firms' output exceeds the reduction in the brown firms' output, leading to an overall decrease in pollution. Nonetheless, when firms produce relatively homogenous goods and pollution intensities are low, output reduction and price increase due to mergers still reduce social welfare, as in standard merger models (Salant et al., 1983).

We then identify the welfare effects of mergers, finding that they can be welfare-reducing if the polluting firms generate significantly higher environmental damages than their green rivals. Therefore, it is possible that the AA, which ignores environmental damages, can approve welfare-reducing mergers, and we identify situations where those mergers should be blocked; specifically, when firms producing homogeneous goods are relatively symmetric in costs and pollution intensities.

Overall, we may observe the approval of welfare-reducing mergers or the blocking of welfare-enhancing mergers in equilibrium. When the AA ignores environmental damages, mergers can be approved (blocked) but they would have been blocked (approved) by the AA that considers

to merge in anticipation of more stringent environmental standards, such as in the energy sector (Creti and Sanin, 2017).

[3]Monsanto, which manufactures Roundup™ weedkiller, is faced with multiple litigations for allegations of causing non-Hodgkin lymphoma (NHL) and multiple myeloma by its active ingredient glyphosate (Bayer AG, 2019, p.250). Bayer, which acquired Monsanto in 2018, manufactures Natria™ herbicide, an organic substitute for weed and broadleaf control that mitigates the environmental and health impacts associated with the use of chemical herbicides. Our model applies to a broad class of mergers between environmentally differentiated firms, such as the one above where an output shift from conventional to organic herbicides is predicted. Pelaez and Mizukawa (2017) identify a trend of agrochemical industry leaders, which face increasing regulatory costs, acquiring biopesticide companies, which produce more environmental-friendly pest control products.

environmental damages. However, the EPA has the ability, in the subsequent stage of the game, to correct any inefficiencies arisen from the AA's incorrect merger approval decisions. This gives rise to another benefit of environmental policy, often overlooked in the literature, as it can correct output inefficiencies that would not have existed if the AA considered environmental damages when evaluating merger requests. Our results then call for more concerted efforts between the AA and the EPA in evaluating mergers, especially in industries where firms have different pollution intensities.

Our findings contribute to the debate about the factors that competition policy should consider when evaluating mergers. Antitrust authorities often focus on consumer surplus and firms' profits, without considering the potential output shifts that, after the merger, decrease (increase) the production of the brown (green) good. European competition law explicitly considers environmental reasons (see articles 6 and 174 of the Treaty), but they have rarely been used to approve or block mergers.[4] In the case of the United States, the Horizontal Merger Guidelines (US Department of Justice and Federal Trade Commission, 2010) do not explicitly consider the broader impact of mergers on the environment, while the EPA does not actively monitor mergers.[5]

For generality, we consider a setting with several green and brown firms, allowing for mergers of a subset of green firms with a subset of each firm type. While some of the results may be intractable, our findings indicate that, in the absence of environmental policy, mergers can still be welfare-improving under similar conditions as in an industry with one green and one brown firm but are profitable only when merging firms hold a sufficiently high market share, as in Salant et al. (1983) and Gelves (2014). This result is emphasized when firms are subject to environmental regulation, increasing the minimal market share that the more polluting firms must hold to make the merger profitable, which goes in line with Benchekroun et al. (2019) in the context of natural resource extraction, but is ameliorated when firms' output becomes more polluting.

[4]See, for instance, Martinez-Lopez (2000) for a discussion about the agreement between European appliances manufacturers to limit their production of energy-inefficient machines. While this agreement entailed an increase in prices, as the discontinued appliances were the most inexpensive, the European Commission considered that it would help address the environmental externalities that consumers did not take into account in their purchasing decisions.

[5]If the emission fees are administratively costly, or if they require a close monitoring of firms' production decisions, then the AA which considers environmental externalities may help the EPA's task by approving (blocking) mergers that (do not) contribute to an overall welfare improvement.

1.1 Related Literature

A large body of the literature analyzes horizontal mergers in oligopolistic markets with environmental externalities.[6] Specifically, Canton et al. (2012) consider the upstream ecoservice industry supplying pollution abatement goods to the downstream polluting market. In this context, horizontal mergers in the ecoservice industry are profitable if a sufficient number of firms merge, as in Salant et al. (1983), or if costs are relatively convex, as in Perry and Porter (1985).[7] In addition, mergers are welfare-improving when emission fees are intermediate; that is, the fees are not too high (low) to yield insufficient output (abatement). Lambertini and Tampieri (2012, 2014) seek to understand mergers of Cournot triopolists under environmental regulation, and show that firms' merger incentives and social objectives are aligned if output reduction, which increases profits and decreases pollution, more than offsets losses in consumer welfare. Recent developments include Davidson and Mukherjee (2007), Erkal and Piccinin (2010), Berchicci et al. (2012), Berchicci et al. (2017), Benchekroun et al. (2019), and Meglio and Park (2019). Our paper differs from the previous literature by considering endogenous emission fees and allowing for asymmetric firms. We show that mergers can be both profitable and welfare-enhancing under different forms of asymmetries, and how the EPA can strategically use emission fees to facilitate or hinder mergers among firms.

A branch of literature in industrial organization analyzes differentiated oligopolies with asymmetric costs. Zanchettin (2006) extends Singh and Vives (1984)'s model of differentiated duopoly and Fauli-Oller (2002)'s model of asymmetric costs to consider pricing strategies and equilibrium profits in a setting of differentiated and asymmetric firms. Unlike our paper, however, these studies do not consider firms' incentives to merge. Kao and Menezes (2010) show that welfare-enhancing mergers can arise in the context of output shifts from the less to the more cost-efficient firm, and the

[6]Our analysis focuses on non-cumulative pollutants, such as carbon dioxide, sulphur dioxide, and suspended particulate; see Benchekroun and Ray Chaudhuri (2008) for an overview of studies on cumulative pollutants.

[7]A recent example in the pollution abatement industry is the acquisition of Auburn FilterSense LLC, a US provider of particulate emissions monitors and intelligent controls for industrial particulate/dust filtration systems, by Nederman, the Swedish environmental technology company (Filtration + Separation, 2018). Another example in the waste management industry is the acquisition of Quantex Environmental Inc., an Ontario based company, by Covanta Environmental Solutions, a New Jersey based environmental services provider (Waste 360, 2018).

same result holds in our model without environmental damages. However, our model also demonstrates that mergers can be welfare-improving if output shifts mitigate environmental damages. Gelves (2014) considers two-firm mergers in an oligopolistic setting with N firms similar to that in Salant et al. (1983), but allowing for cost asymmetries and product differentiation, suggesting that welfare-enhancing mergers are more likely when products become more differentiated.

The closest articles to our paper are Fikru and Gautier (2016, 2017), who examine mergers in Cournot markets with product differentiation and emission fees but assuming symmetric costs and no pollution differentials between firms. We show that mergers can be welfare-improving when costs are asymmetric even in the absence of environmental externalities, since output shifts from the less efficient to the more cost-efficient firm help save production costs. When externalities are present, however, we propose another channel of welfare gains. Specifically, emission fees on the polluting firm induce this firm to reduce its output even when firms do not merge. Our research then complements Fikru and Gautier (2017, 2020), since we also find that regulation should be more stringent in markets with homogeneous goods and high pollution intensities. However, we demonstrate that stringent regulation is required when firms are relatively asymmetric in production costs. In addition, we identify under which conditions welfare-enhancing (welfare-reducing) mergers are blocked (approved) by the AA, leading to an overall welfare loss.

Fikru and Gautier (2020) consider two countries (home and foreign), with n and m firms in each country, respectively, and k^h and k^f firms merging in each country, denoted as "insiders", which leaves $n - k^h$ and $m - k^f$ unmerged firms as "outsiders". In the first stage, each country's government simultaneously and independently chooses the emission fee. In the second stage, every firm chooses its output and emissions, either to maximize joint profits if the firm is an insider or independently if the firm is an outsider. However, the number of merging firms in each country is exogenous, so every firm does not choose whether to merge or not at any point in the game, and the socially optimal emission fee is not defined (the authors study its effect on aggregate output and welfare).[8] While we only consider firms in one country, we allow for firms' merging decisions and the AA's merger approvals to be endogenous, and identify socially optimal emission fees in different

[8] As in our paper, their setting allows for firms to be asymmetric in their production costs or pollution intensities (although all firms located in a country are symmetric) and for product differentiation between firms located in different countries (all firms in a given country sell homogeneous products).

contexts, analyzing how the absence of environmental regulation gives rise to socially excessive or insufficient mergers. Our setting thus helps us examine the interaction between the EPA and the AA, especially when their objective functions differ, and how emission fees can help correct the AA's regulatory mistakes to promote welfare-enhancing and avoid welfare-reducing mergers.

The remainder of this paper is organized as follows. Section 2 develops the model. Section 3 examines merger incentives and welfare effects without environmental regulation, whether the AA considers pollution (in section 3.2) or not (in section 3.2.1). Section 4 evaluates these effects when emission fees are present, whether the AA considers pollution (in section 4.3) or not (in section 4.5). Section 5 then compares welfare level across different regulatory regimes, and finally, section 6 concludes. The following table summarizes the different regulatory settings in our paper.

Insert Table 1 here.

2. Model

Consider a market with n_G green and n_B brown firms, where $k_G \leq n_G$ green and $k_B \leq n_B$ brown firms consider merging into one firm. Every firm i faces an inverse demand function

$$p_i(q_i) = 1 - Q_i - \beta Q_j,$$

where $i, j = \{B, G\}$ and $j \neq i$, and Q_i (Q_j) represents the aggregate output of all firms with type i (j, respectively). For instance, if firm i is green, the inverse demand function becomes $p_G(q_G) = 1 - Q_G - \beta Q_B$, where $Q_G = q_i + \sum q_k$ and $Q_B = \sum q_B$, for which q_k indicates the output of other n_G green firms while q_B reflects the output of n_B brown firms. Parameter $\beta \in [0, 1]$ represents the degree of product differentiation. When $\beta = 0$, goods are completely differentiated but when $\beta = 1$, goods are perfect substitutes; see Singh and Vives (1984). In addition, c_i denotes firm i's marginal cost of production, where $0 \leq c_i < 1$.

Let $m_i \equiv \frac{1-c_i}{1-c_j}$ measure firm i's efficiency relative to firm j's, where $j \neq i$. When firm i has a higher efficiency, $c_i < c_j$, we obtain that $m_i > 1$, as empirically shown in a sample of relatively polluting (brown) Swedish exporting firms by Forslid et al. (2018). The opposite holds if firm i is less efficient than j, where $c_i > c_j$, yielding that $m_i < 1$, as found in environmentally-friendly (green) Canadian firms by Najjar and Cherniwchan (2021).

Environmental damages, $Env(Q_B, Q_G) = d(Q_B + \alpha Q_G)^2$, are increasing and convex in the aggregate output of brown (green) firms Q_B (Q_G), where $d \geq 0$ stands for the pollution intensity.[9] In this context, parameter $\alpha \in [0, 1]$ denotes how clean the green firm is relative to its brown rival, being completely clean when $\alpha = 0$, as in Gelves (2014), or equally polluting when $\alpha = 1$.

The timing of the game is as follows:

(1) Every firm i decides whether to merge with firm j or not, where at least one firm must be of the other type, $i \in \{B, G\}$ and $i \neq j$.[10] If one firm opposes, the merger does not ensue and the game proceeds to stage 3. If all participating firms agree to merge, the merger proposal is sent to the AA.

(2) The AA, receiving the merger proposal, decides whether to approve or block the merger.

(3) The EPA sets an emission fee t_i^l to every firm i, which depends on its type $i \in \{B, G\}$ and the industry structure $l \in \{M, NM\}$, where M (NM) denotes merger (no-merger), respectively.

(4) All firms that merge jointly choose output q_i^M and those that do not merge independently choose q_i^{UM}. If the merger does not ensue, every firm i independently chooses output q_i^{NM}. Superscripts M and UM denote the merged and unmerged firm after the merger, respectively, and NM represents the case of no merger.

We first examine the setting without environmental regulation (where stage 3 is absent) and solve for firms' equilibrium output and profits with and without the merger, examining merger incentives and the AA's role in approving mergers. Then, we evaluate how equilibrium outcomes, merger incentives, and administrative decisions of the AA are affected when the EPA introduces emission fees t_i^l in stage 3. For simplicity, we assume $m_i \geq \frac{\beta(n_j - k_j + 1)}{n_j - k_j + 2}$ throughout this paper.[11]

[9] See Lambertini and Tampieri (2012), Lambertini (2013), and Espínola-Arredondo and Muñoz-García (2015).

[10] See Canton et al. (2012) for mergers of homogeneous firms with environmental externalities.

[11] If, instead, $m_i < \frac{\beta(n_j - k_j + 1)}{n_j - k_j + 2}$ $(m_i > \frac{n_i - k_i + 2}{\beta(n_i - k_i + 1)})$, then every firm i (rival firm j) does not produce any output whether it is merged or not, rendering the merger problem uninteresting.

3. Equilibrium analysis without regulation

3.1 Third stage - Output decisions

Operating by backward induction, we begin our analysis with the third stage of the game.[12] Lemma 1 identifies firm i's output before and after the merger, where for compactness, define $r \equiv \beta + 2\alpha d$, $s_B \equiv 1 + 2d$, $s_G \equiv 1 + 2\alpha^2 d$, $v_i \equiv (n_i - k_i + 2)(n_j - k_j + 1) - \beta^2(n_i - k_i + 1)(n_j - k_j)$, $w_i \equiv \beta^2(n_i - k_i) + n_j - k_j + 2$, $x \equiv (n_i - k_i + 2)(n_j - k_j + 2) - \beta^2(n_i - k_i)(n_j - k_j)$, $y \equiv (n_i + 1)(n_j + 1) - \beta^2 n_i n_j$, and $z \equiv n_i - k_i + n_j - k_j + 2$, where $r, s_i, v_i, w_i, x, y, z > 0$. For presentation purposes, all proofs are relegated to Appendix 1.

Lemma 1. *Before the merger, every firm i produces* $q_i^{NM} = \frac{(n_j+1)(1-c_i)-\beta n_j(1-c_j)}{y}$ *units, which is positive if and only if* $m_i \geq \frac{\beta n_j}{n_j+1}$. *After the merger, every merged and unmerged firm i produces:*

(1) $q_i^M = 0$ *and* $q_i^{UM} = \frac{(n_j-k_j+2)(1-c_i)-\beta(n_j-k_j+1)(1-c_j)}{v_j}$ *if* $\underline{m}_i \leq m_i < \frac{\beta z}{w_i}$,

(2) $q_i^M = \frac{w_i(1-c_i)-\beta z(1-c_j)}{(1-\beta^2)x}$ *and* $q_i^{UM} = \frac{(n_j-k_j+2)(1-c_i)-\beta(n_j-k_j)(1-c_j)}{x}$ *if* $\frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z}$, *and*

(3) $q_i^M = q_i^{UM} = \frac{(n_j-k_j+1)(1-c_i)-\beta(n_j-k_j)(1-c_j)}{v_i}$ *if* $\frac{w_j}{\beta z} \leq m_i \leq \overline{m}_i$;

where $\underline{m}_i \equiv \frac{\beta(n_j-k_j+1)}{n_j-k_j+2}$ *and* $\overline{m}_i = \frac{n_i-k_i+2}{\beta(n_i-k_i+1)}$.

Intuitively, when firms are relatively cost-symmetric, where $\frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z}$, both types of firms produce positive units of output before and after the merger. In contrast, when type-i firms are relatively efficient, type-j firms shut down after the merger.

Lemma 2 finds firm i's output change due to the merger, as captured by $\Delta q_i^{NR} \equiv q_i^M - q_i^{NM}$.

Lemma 2. *After the merger, firm i: (1) increases output if* $m_i \geq \widehat{m}_i$*; (2) reduces output if* $\frac{\beta z}{w_i} \leq m_i < \widehat{m}_i$*; and (3) stops production otherwise, where* $\widehat{m}_i \equiv \frac{\beta[yz-(1-\beta^2)n_j x]}{w_i y-(1-\beta^2)(n_j+1)x}$.

When $m_i \geq \widehat{m}_i$, firm i is more efficient than firm j and produces more units after the merger. Otherwise, firm i produces fewer units if its relative efficiency m_i falls within $\frac{\beta z}{w_i} \leq m_i < \widehat{m}_i$, and stops production if its cost disadvantage widens. Comparative statics of cutoff $\widehat{m}_i$ with respect to n_i and n_j yield intractable expressions, so we next provide a numerical simulation of these cutoffs.

[12] Environmental regulation (stage 3) is absent in this section, implying that the game has only three stages.

Figure 1a depicts the case of an equal number of brown and green firms ($n_B = n_G = 10$), and 9 firms of each type merge together. Above cutoff $\widehat{m}_i$, firm i's relative efficiency allows this firm to increase output after the merger, but decrease it otherwise. When firm i is less efficient than firm j, it stops production after the merger in the region below cutoff $\frac{\beta z}{w_i}$ and above cutoff $\frac{2\beta}{3}$. When firm i's cost disadvantage to firm j widens, it remains dormant before and after the merger below cutoff $\frac{2\beta}{3}$. When firms produce more homogeneous goods (higher β), however, they engage in more intensive competition with the unmerged firms, shrinking (expanding) the range of m_i under which firm i can increase (must reduce) output after the merger, which occurs if m_i lies above (below) cutoff $\widehat{m}_i$.

Figure 1b (1c), in comparison, considers the case of 11 (10) type-i and 10 (11) type-j firms, still with the same number of firms merging together. Given one more homogeneous (heterogeneous) outsider, competition now becomes more (less) intense, so that cutoff $\widehat{m}_i$ shifts up (down), and it becomes more (less) restrictive for firm i to increase output after the merger.

In addition, when $n_i = n_j = k_i = k_j = 1$, Figure 1d illustrates that the merging firms become a single firm (a monopolist). Still with the merger between one brown and one green firm ($k_i = k_j = 1$), when the market is comprised of 4 firms of each type ($n_i = n_j = 4$), Figure 1e shows that competition becomes more intense, and cutoff $\widehat{m}_i$ shifts up, indicating that it becomes more restrictive for every merged firm i to increase output after the merger. Finally, when 2 out of 4 firms of each type merge ($n_i = n_j = 4$ and $k_i = k_j = 2$), Figure 1f illustrates that cutoff $\widehat{m}_i$ shifts down, entailing that as every merged firm i gains more market power, it can produce more units of output after the merger over a wider range of parameters.

Insert Figures 1a, 1b, 1c, 1d, 1e, and 1f here.

Figure 1. Output change under no regulation

Next, Corollary 1 compares the output change of each firm type as a result of the merger.

Corollary 1. *Firm i increases output more than firm j's, that is,* $\Delta q_i^{NR} \geq \Delta q_j^{NR} \geq 0$, *if and only if* $m_i \geq \widetilde{m}_i$, *where* $\widetilde{m}_i \equiv \frac{(w_j+\beta z)y-\left(1-\beta^2\right)(n_i+\beta n_j+1)x}{(w_i+\beta z)y-(1-\beta^2)(\beta n_i+n_j+1)x}$.

In this context, firm i increases output more than firm j after the merger when this firm is relatively more efficient than its rivals, that is, $m_i \geq \widetilde{m}_i$. This result embodies the following three cases: (1) firm i and firm j decrease their output after the merger, but firm i's output reduction is smaller than its rival; (2) firm i increases its output while firm j decreases its own; and (3) firm i increases its output while firm j shuts down its operation.[13]

Corollary 2 studies how type-i firms' output changes before and after the merger, as captured by $\Delta q_i^M \equiv q_i^M - k_i q_i^{NM}$ for the merged firm and $\Delta q_i^{UM} \equiv q_i^{UM} - q_i^{NM}$ for every unmerged firm.

Corollary 2. *Mergers reduce (increase) the output of the merged (every unmerged) firm, that is, $\Delta q_i^M < 0$ ($\Delta q_i^{UM} > 0$), under all parameter conditions.*

When firms merge, the merged firm obtains a larger market share and reduces output to capture more profits. Every unmerged firm, however, takes advantage of the merger to produce more units.

The next section shows, however, that the output loss can be offset by the increase in profits stemming from efficiency gains due to output shifts, and by the reduction in environmental damages.

3.2 Second stage - Merger approval

In the second stage, the AA anticipates aggregate output by each type of firm, Q_B and Q_G, in the third stage, and evaluates the social welfare as follows. (For generality, section 3.2.1 considers alternative welfare functions.)

$$SW(Q_B, Q_G) = CS + PS - Env(Q_B, Q_G) \tag{1}$$

where $CS = \frac{1}{2}\left(Q_B^2 + 2\beta Q_B Q_G + Q_G^2\right)$ denotes consumer surplus,[14] $PS = (p_B - c_B) Q_B + (p_G - c_G) Q_G$ stands for the firms' aggregate profits (including both merged and unmerged firms), and $Env(Q_B, Q_G) = d(Q_B + \alpha Q_G)^2$ represents environmental damages, as defined in section 2.[15]

The AA approves the merger if and only if it improves social welfare, that is, $\Delta SW \equiv SW\left(Q_B^M, Q_G^M\right) - SW\left(Q_B^{NM}, Q_G^{NM}\right) \geq 0$, where $Q_i^{NM} = n_i q_i^{NM}$ and $Q_i^M = q_i^M + (n_i - k_i) q_i^{UM}$, for all

[13] For example, when $n_i = k_i = k_i = k_j = 1$, this cutoff becomes $\widetilde{m}_i = 1$, and firm i increases its output to a larger extent than firm j if and only if firm i is more efficient than firm j. When $1 \leq m_i \leq \widehat{m}_i$, however, firm i is more efficient than firm j, and reduces output less significantly than its rival after the merger, or increases it otherwise.

[14] Our results do not suffer from path dependence problems since our quasilinear-quadratic utility produces no wealth effects, see Johansson (1999, p.748) and Hsu (2005).

[15] While parameter $d > 0$ measures the pollution intensity of both firms, it could alternatively capture the difference between the EPA and AA's welfare function (for example, their preference divergence). When $d = 0$, both agencies have the same welfare function. When $d > 0$, the EPA assigns a weight to environmental damages but the AA does not.

$i \in \{B, G\}$, as stated in Lemma 3. For compactness, define $\Delta CS \equiv CS\left(Q_B^M, Q_G^M\right) - CS\left(Q_B^{NM}, Q_G^{NM}\right)$ and $\Delta PS \equiv PS\left(Q_B^M, Q_G^M\right) - PS\left(Q_B^{NM}, Q_G^{NM}\right)$, where cutoffs $\underline{m}_i^{SW}$ and $\overline{m}_i^{SW}$ are identified in the proof.

Lemma 3. *The AA approves the merger if and only if $\underline{m}_i^{SW} \leq m_i \leq \overline{m}_i^{SW}$, and blocks it otherwise.*

For presentation purposes, we focus on the brown firm's relative efficiency, as represented by m_B, in the vertical axis of our figures. As an illustration, we consider the case of $n_B = n_G = 10$ and $k_B = k_G = 9$, as depicted in Figures 2a and 2b. When firms do not generate environmental damages, $d = \alpha = 0$, mergers reduce welfare for all parameter conditions and are blocked by the AA. This occurs because the outsiders' output increase does not offset the insiders' output decrease, yielding a reduction in aggregate output after the merger, with an associated loss of consumer surplus that exceeds the gain in producer surplus.

When emissions only originate from the brown firms, $d = \frac{1}{4}$ and $\alpha = 0$, as depicted in Figure 2a, mergers above cutoff $\overline{m}_B^{SW}$ reduce aggregate output of brown firms and decrease their environmental damages, so that the AA approves welfare-enhancing mergers in the shaded region of Figure 2a.

To further understand this result, consider the case of cost-symmetric firms with a dotted horizontal line at $m_B = 1$. This line lies entirely below cutoff $\underline{m}_B^{SW}$, indicating that mergers are welfare-reducing for all values of parameter β. This setting embodies, as a special case, that firms sell homogeneous goods ($\beta = 1$), as in standard oligopoly models where mergers are unambiguously welfare-reducing. Intuitively, mergers produce a loss in consumer surplus, $\Delta CS < 0$, and an increase in producer surplus, $\Delta PS > 0$, as firms maximize joint profits after the merger. However, when all firms are completely cost-symmetric and sell homogeneous goods, $m_B = \beta = 1$ as depicted in the bullet point of this figure, the former effect offsets the latter, yielding an overall welfare reduction.

This welfare decrease from the merger is emphasized when goods become more differentiated; graphically, we move leftward along the dotted horizontal line, $m_B = 1$, to lower values of β. In this setting, every firm does not significantly affect its rival's price before the merger and, as a result, firms do not need to substantially change their output levels after the merger. Consequently, the increase in profits that they enjoy from the merger is smaller than when goods are homogeneous, yielding a small gain in producer surplus, ΔPS. Overall, we obtain that the increase in producer surplus is so small that the above overall welfare decrease holds for larger parameter conditions.

When firms become cost-asymmetric, $m_B \neq 1$, however, the merging firms now benefit from a new effect, as they can start shifting their output towards the most efficient firm. This new effect increases ΔPS, allowing for welfare increases to hold if firms are sufficiently asymmetric and goods are relatively homogeneous; as depicted in shaded region above cutoff $\underline{m}_B^{SW}$ Figure 2a.[16] When the dotted vertical line shifts leftward, we find that the merger becomes welfare-reducing under larger parameter conditions (graphically, a longer segment of the vertical line between cutoffs $\underline{m}_B$ and $\underline{m}_B^{SW}$). Intuitively, when products become more differentiated, the merger induces a less significant output shift from the least to the most efficient firm, thus yielding smaller welfare gains.[17]

When both firms are polluting, $d = \frac{1}{4}$ and $\alpha = \frac{1}{2}$, as depicted in Figure 2b, mergers above cutoff $\underline{m}_B^{SW}$ and below cutoff $\overline{m}_B^{SW}$ help reduce aggregate output of both brown and green firms, but more significantly from the brown firms, which mitigate relatively substantial environmental damages. Compared to Figure 2a, cutoff $\underline{m}_B^{SW}$ shifts down as α increases, implying that the shaded region of welfare-improving mergers expands. This allows the merger of cost-symmetric firms to enhance welfare as now the dotted horizontal line representing $m_B = 1$ lies entirely within the shaded region.

In summary, if firms are polluting, $d > 0$, but symmetric in their pollution intensities, $\alpha = 1$, a new positive effect from mergers arises, since aggregate output reduction improves social welfare, thus expanding the region of parameter values for which mergers can be approved by the AA. When firms are polluting but asymmetric in their pollution intensities, $\alpha < 1$, this new positive effect from mergers is attenuated, shrinking the region of parameter values for which mergers improve welfare (graphically, a decrease in parameter α is illustrated when one moves from Figure 2b to 2a).

Insert Figures 2a and 2b here.

Figure 2. Social welfare under no regulation

3.2.1 Antitrust overlooks environmental effects

Suppose the AA does not consider environmental damages, its welfare function becomes

$$\widehat{SW}(Q_B, Q_G) = CS + PS \tag{2}$$

[16]This result is similar to those in Davidson and Mukherjee (2007). Considering symmetric firms but allowing for free entry and exit, the authors show that firms have incentives to merge even if they benefit from a small cost synergy due to the merger and, as in our model, the merger becomes welfare-enhancing under larger parameter conditions.

[17]Out result indicates that cutoff $\overline{m}_B^{SW}$ does not fall within $\underline{m}_B = \frac{2\beta}{3}$ and $\overline{m}_B = \frac{3}{2\beta}$, so it is absent from Figure 2a.

which welfare gain of approving a merger is defined as $\Delta\widehat{SW} \equiv \widehat{SW}\left(Q_B^M, Q_G^M\right) - \widehat{SW}\left(Q_B^{NM}, Q_G^{NM}\right)$.

The following corollary identifies under which conditions the AA's decision leads to regulatory inefficiencies: blocking mergers that would have been welfare-improving according to ΔSW, which we refer to as "socially insufficient mergers" (SIM), or approving mergers that would have been welfare-reducing according to ΔSW, which we denote as "socially excessive mergers" (SEM).

Corollary 3. *The AA's decision leads to socially insufficient mergers if and only if $\Delta SW \geq 0 > \Delta\widehat{SW}$, and to socially excessive mergers if and only if $\Delta\widehat{SW} \geq 0 > \Delta SW$.*

Following our discussions in Figures 2a (2b), mergers improve welfare in the regions above cutoff $\underline{m}_B^{SW}$ and below cutoff $\overline{m}_B$ ($\overline{m}_B^{SW}$). However, they are blocked by the AA which only considers consumer and producer surpluses (ignoring environmental damages, resembling the case where $d = 0$). Mergers in this region should be approved according to expression (1) since $\Delta SW \geq 0$, but are blocked according to the welfare criterion in expression (2) where $\Delta\widehat{SW} < 0$. This gives rise to SIM, as indicated in the shaded regions of Figures 2a and 2b. In contrast, mergers blocked under expression (1) in the unshaded regions above cutoff $\underline{m}_B$ and below cutoff $\underline{m}_B^{SW}$ continue to be blocked under expression (2), resulting in no SEM that would otherwise be approved by the AA.[18]

The following corollary reports the AA's behavior which considers consumer surplus alone.

$$\widetilde{SW}\left(Q_B, Q_G\right) = CS \tag{3}$$

which welfare gain of approving a merger is defined as $\Delta\widetilde{SW} \equiv \widetilde{SW}\left(Q_B^M, Q_G^M\right) - \widetilde{SW}\left(Q_B^{NM}, Q_G^{NM}\right)$.

Corollary 4. *The AA's decision leads to socially insufficient mergers if and only if $\Delta SW \geq 0 > \Delta\widetilde{SW}$, and to socially excessive mergers if and only if $\Delta\widetilde{SW} \geq 0 > \Delta SW$.*

In Figures 2a and 2b, since mergers produce a larger loss of consumer surplus than the gain in producer surplus, that is, $\Delta PS > 0 > \Delta CS$ and $|\Delta CS| > |\Delta PS|$, when environmental damages are not considered (as if $d = 0$), all mergers would be blocked by the AA. In this context, welfare-enhancing mergers will not be approved, resulting in SIM in the same shaded regions of the respective figures. In contrast, the universal rejection of all mergers does not result in SEM in equilibrium.

[18] In section 4.5, we find that in the merger between one brown and one green firm in an industry comprised of 10 firms from each type, both SIM and SEM can arise in contexts where the AA overlooks environmental damages.

3.3 First stage - Merger vote

In the first stage of the game, let us investigate firms' incentives to merge. For compactness, define $\Delta\pi^{NR} \equiv \pi_i^M + \pi_j^M - k_i\pi_i^{NM} - k_j\pi_j^{NM}$ to be the merged firms' profit gain from the merger, and cutoffs $\underline{m}_i^{NR}$ and $\overline{m}_i^{NR}$ are included in the proof of the next lemma.

Lemma 4. *Under no regulation, firm i has incentives to merge if and only if* $\underline{m}_i^{NR} \leq m_i \leq \overline{m}_i^{NR}$.

Figure 3a considers an industry comprised of 10 firms of each type ($n_i = n_j = 10$). In this context, 9 firms have incentives to merge ($k_i = k_j = 9$) in the shaded region above cutoff $\underline{m}_i^{NR}$ and below cutoff $\overline{m}_i^{NR}$, where mergers are profitable when firms are relatively symmetric in costs and produce more differentiated goods (low β), since the outsiders are less able to substitute for the output of the insiders.[19] Figure 3b depicts the case with one more outsider producing homogeneous goods ($n_i = 11$ and $n_j = 10$). Facing more intense competition, firm i still has incentives to merge if it is relatively efficient in producing more differentiated goods, as seen by cutoff $\overline{m}_i^{NR}$ rotating counterclockwisely that shrinks the region of (β, m_i)-pairs denoting profitable mergers. Figure 3c depicts the case with one more outsider producing differentiated goods ($n_i = 10$ and $n_j = 11$), where cutoff $\underline{m}_i^{NR}$ rotates clockwisely shrinking the region of (β, m_i)-pairs denoting profitable mergers, so that a less efficient firm i still has incentives to merge only when goods become more differentiated.

Insert Figures 3a, 3b and 3c here.

Figure 3. Merger incentives under no regulation

Figure 4a identifies the subset of profitable mergers assuming $\beta = \frac{1}{3}$, $m_B = \frac{2}{3}$, and $n_B = n_G = 50$. The shaded region above (to the right of) the cutoff finds for each proportion of brown (green) firms on the horizontal (vertical) axis, k_B/n_B (k_G/n_G), the proportion of green (brown) firms, k_G/n_G (k_B/n_B), required for mergers to be profitable for every participating green (brown) firm.[20] The unshaded region in the bottom-left corner, in contrast, indicates that neither the brown nor the green firm has incentives to merge, because there are too few firms merging together to generate sufficient

[19] When the market is comprised of one brown and one green firm, both firms have incentives to merge under all parameter conditions, as in standard merger models of duopolies into monopoly (Salant et al., 1983; Gelves, 2014).

[20] In the case of a merger between one brown and one green firm, we have $k_G/n_G = k_B/n_B = 1$ at the top-right corner of the figure, which is within the shaded region for every firm to find incentives to participate in the merger.

market power to offset the output substitution effect of the outsiders (Salant et al., 1983; Gelves, 2014). Figure 4b shows that when the number of firms increases ($n_B = n_G = 250$ in dashed and $n_B = n_G = 1,000$ in dotted lines), we need a larger proportion of firms merging to compete with an increased number of outsiders in a more competitive market. Similar results are obtained if we assume that goods are more homogeneous (higher β) or firms become more asymmetric in costs.

Insert Figures 4a and 4b here.

Figure 4. Market concentration under no regulation

4. Equilibrium analysis with environmental regulation

In this section, we introduce the regulator, which uses emission fees to affect firms' behavior.

4.1 Fourth stage - output decisions

In the fourth stage, every firm i makes output decisions in response to the emission fees set in the third stage, where the superscript R denotes variables under environmental regulation.

Lemma 5. *Under regulation, firm i's output is* $q_i^{NM,R}\left(t_i,t_j\right) = \frac{(n_j+1)(1-c_i-t_i)-\beta n_j(1-c_j-t_j)}{(n_i+1)(n_j+1)-\beta^2 n_i n_j}$ *before the merger. After the merger, its output becomes* $q_i^{M,R}\left(t_i,t_j\right) = \frac{w_i(1-c_i-t_i)-\beta z(1-c_j-t_j)}{(1-\beta^2)[(n_i-k_i+2)(n_j-k_j+2)-\beta^2(n_i-k_i)(n_j-k_j)]}$ *if firm i joins the merger and* $q_i^{UM,R}\left(t_i,t_j\right) = \frac{(n_j-k_j+2)(1-c_i-t_i)-\beta(n_j-k_j)(1-c_j-t_j)}{(n_i-k_i+2)(n_j-k_j+2)-\beta^2(n_i-k_i)(n_j-k_j)}$ *if it does not.*

Our results, therefore, suggest that firm i reduces its output in response to a higher emission fee t_i, but increases its output when firm j suffers a more stringent emission fee t_j.[21] Evaluating these output levels in an industry with only one brown and one green firm that merge into a single firm, where $n_i = n_j = k_i = k_j = 1$, firm i's output becomes $q_i^{NM,R}\left(t_i,t_j\right) = \frac{2(1-c_i-t_i)-\beta(1-c_j-t_j)}{4-\beta^2}$ before the merger and changes to $q_i^{M,R}\left(t_i,t_j\right) = \frac{(1-c_i-t_i)-\beta(1-c_j-t_j)}{2(1-\beta^2)}$ after the merger.[22] In the special case

[21] Our model, then, implies that the regulator can set different emission fees on each firm type, given its relative pollution intensity as captured by parameter α, which is a common practice when firms in the same industry exhibit asymmetric pollution intensities. In the freight industry, for instance, heavy-duty trucks and freight trains are subject to different air pollution regulation from the EPA. In particular, the EPA issued nitrogen oxides (NOx) and particulate matter emission standards on heavy-duty trucks, making them more stringent in 2007. In contrast, the major non-road freight modes of transportation (trains and marine vessels) were virtually unregulated until the late 1990s, and today remain much less regulated than freight trucks.

[22] In this context, $q_i^{UM,R}$ does not apply to the setting of a full merger with no outsiders.

where only type-i firms are active in the market, their output levels become $q_i^{NM,R}(t_i) = \frac{1-c_i-t_i}{n_i+1}$ before and $q_i^{M,R}(t_i) = \frac{1-c_i-t_i}{n_i-k_i+2}$ after the merger.

4.2 Third stage - Emission fees

In the third stage, the EPA chooses aggregate output Q_i^{SO} for every type-i firm that solves

$$\max_{Q_B,Q_G \geq 0} SW(Q_B, Q_G) = CS + PS + Tax - Env(Q_B, Q_G) \tag{4}$$

where $Tax = t_B Q_B + t_G Q_G$ represents the EPA's total tax revenues from all brown and green firms.[23] The following lemma identifies this aggregate socially optimal output, which applies to whether firms choose to merge or not. We discuss our results and then find which emission fees induce firms to respond choosing a socially optimal output in the fourth stage.[24]

Lemma 6. *The socially optimal aggregate output for all type-i firms is*

$$Q_i^{SO} = \begin{cases} 0 & \text{if} \quad m_i < \underline{m}_i^R \\ \frac{s_j(1-c_i)-r(1-c_j)}{s_i s_j - r^2} & \text{if} \quad \underline{m}_i^R \leq m_i \leq \overline{m}_i^R \\ \frac{1-c_i}{s_i} & \text{if} \quad m_i > \overline{m}_i^R \end{cases}$$

where $\underline{m}_i^R = \frac{r}{s_j}$ *and* $\overline{m}_i^R = \frac{s_i}{r}$.

When no firm pollutes ($d = 0$), Figure 5a shows that it is socially optimal for the relatively inefficient brown (green) firms to shut down when m_B lies below (above) cutoff $\underline{m}_B^R$ ($\overline{m}_B^R$), which is more likely when firms produce relatively homogeneous goods. When the brown firms generate pollution while their green rivals are completely clean ($d = \frac{1}{4}$ and $\alpha = 0$, as illustrated in Figure 5b), it is socially optimal to stop the brown firms' production under the same conditions as in Figure 5a, but it becomes more restrictive to shut down the green firms, as cutoff $\overline{m}_B^R$ shifts upwards. Figure 5c depicts the case of brown and less polluting green firms ($d = \frac{1}{4}$ and $\alpha = \frac{1}{2}$), where cutoff $\overline{m}_B^R$ lies above (below) that in Figure 5a (5b), so it is socially optimal to shut down the green firms

[23] This welfare function is consistent with, for instance, those in Fikru and Gautier (2016, 2017, 2020), and the EPA in our model can be deemed as the social planner.

[24] This approach yields the same results as maximizing social welfare, evaluated at the equilibrium output functions of Lemma 5, with respect to the emission fees. However, to facilitate our calculations, we first identify the socially optimal output levels.

under more (less) restrictive conditions than when neither (only the brown) firms pollute. However, cutoff $\underline{m}_B^R$ shifts up because when firms become less asymmetric in their pollution intensities (α increases), it is socially optimal to shut down the brown firms under less restrictive parameter conditions.[25]

Insert Figures 5a, 5b, and 5c here.

Figure 5. Socially optimal aggregate output

Next, we study how socially optimal output levels change in product differentiation and cost.

Corollary 5. *When $\underline{m}_B^R \leq m_B \leq \overline{m}_B^R$, socially optimal aggregate output Q_B^{SO} (Q_G^{SO}) increases in β if and only if $m_B \geq \overline{m}_B^\beta \equiv \frac{s_B s_G + r^2}{2rs_G}$ ($m_B \leq \underline{m}_B^\beta \equiv \frac{2rs_B}{s_B s_G + r^2}$), and unambiguously decreases in c_B (c_G) but increases in c_G (c_B). Otherwise, Q_B^{SO} (Q_G^{SO}) is unaffected by β and only decreases in c_B (c_G).*

In this context, when the brown (green) firms are relatively efficient, specifically, $m_B \geq \overline{m}_B^\beta$ ($m_B \leq \underline{m}_B^\beta$), the regulator induces more units of output from these firms when goods become more homogeneous (β increases), but fewer units of output when they become less efficient.[26]

We are also interested in how environmental damages affect the socially optimal output levels.

Corollary 6. *Socially optimal aggregate output, Q_B^{SO} and Q_G^{SO}, decreases in d for all parameters.*

Intuitively, when goods become more polluting, the EPA induces fewer units from both firms. The next corollary evaluates how the socially optimal aggregate output varies in pollution intensities.

Corollary 7. *When $\underline{m}_B^R \leq m_B \leq \overline{m}_B^R$, socially optimal aggregate output Q_B^{SO} (Q_G^{SO}) increases (decreases) in α if and only if $m_B \geq \overline{m}_B^\alpha \equiv \frac{s_B(1-\alpha\beta)+r(\beta-\alpha)}{2r(1-\alpha\beta)}$ ($m_B \geq \underline{m}_B^\alpha \equiv \frac{2(\beta-\alpha)s_B}{s_B(1-\alpha\beta)+r(\beta-\alpha)}$). When $m_B > \overline{m}_B^R$ $\left(m_B < \underline{m}_B^R\right)$, however, Q_B^{SO} (Q_G^{SO}) is unaffected by (decreases in) α.*

[25]Cutoff $\underline{m}_B^R$ ($\overline{m}_B^R$) is upward- (downward-) sloping, because when goods become more homogeneous, output shifts can achieve more significant efficiency gains. When environmental damages become more severe (d increases), cutoff $\underline{m}_B^R$ monotonically shifts up for all parameter conditions while cutoff $\overline{m}_B^R$ shifts up if and only if $\alpha < \beta$. Intuitively, it becomes more restrictive to shut down the green firms when they are relatively cleaner than their brown rivals. When the green firms become more polluting (higher α), however, cutoff $\underline{m}_B^R$ shifts up if and only if $d < \frac{1-2\alpha\beta}{2\alpha^2}$, entailing that when environmental damages are not too significant, the brown firms can shut down under wider range of efficiency conditions, while cutoff $\overline{m}_B^R$ shifts down for all parameter values.

[26]A direct comparison reveals that $\underline{m}_B^R < \underline{m}_B^\beta < \overline{m}_B^\beta < \overline{m}_B^R$, which makes the first result of Corollary 5 feasible.

Intuitively, when green firms become more polluting (α increases), the EPA induces fewer units of both goods when firms are relatively symmetric in costs ($\underline{m}_B^\alpha \leq m_B \leq \overline{m}_B^\alpha$). Further increases in α, however, shift cutoff $\overline{m}_B^\alpha$ ($\underline{m}_B^\alpha$) down so the EPA induces more (fewer) units of the brown (green) good over a wider range of m_B when firms become less asymmetric in their pollution intensities.

Finally, we identify the emission fees of the EPA inducing socially optimal output, as follows.[27]

Proposition 1. *When it is socially optimal for all firm types to be active, $\underline{m}_i^R \leq m_i \leq \overline{m}_i^R$, the EPA charges every unmerged firm i an emission fee of*

$$t_i^{NM} = \frac{\left[n_i(s_i s_j - r^2) - ((n_i+1)s_j - \beta n_i r)\right](1-c_i) + \left[(n_i+1)r - \beta n_i s_i\right](1-c_j)}{n_i(s_i s_j - r^2)}$$

which, after the merger, changes to

$$t_i^M = \frac{\left[(s_i s_j - r^2)(v_i v_j - \beta^2(x-z)^2) - (1-\beta^2)(s_j v_i - \beta r(x-z))x\right](1-c_i) + (1-\beta^2)(r v_i - \beta s_i(x-z))x(1-c_j)}{(v_i v_j - \beta^2(x-z)^2)(s_i s_j - r^2)}$$

When it is socially optimal that only type-i firms are active, $m_i > \overline{m}_i^R$, the EPA sets a fee $t_i^{NM} = \frac{[n_i(s_i-1)-1](1-c_i)}{n_i s_i}$ before and $t_i^M = \frac{[(n_i-k_i)(s_i-1)+s_i-2](1-c_i)}{(n_i-k_i+1)s_i}$ after the merger for every firm i.

Evaluating firm i's emission fees in an industry with one brown and one green firm merging into a single firm, where $n_i = n_j = k_i = k_j = 1$, we find that $t_i^{NM} = \frac{(2r-\beta s_i)(1-c_j)-[s_j(2-s_i)+r(r-\beta)](1-c_i)}{s_i s_j - r^2}$ before the merger that changes to $t_i^M = \frac{2(r-\beta s_i)(1-c_j)-[s_j(2-s_i)+r(r-2\beta)](1-c_i)}{s_i s_j - r^2}$ after the merger when both firms are active ($\underline{m}_i^R \leq m_i \leq \overline{m}_i^R$). Otherwise, when it is socially optimal that only firm i is active ($m_i > \overline{m}_i^R$), the fee becomes $t_i^{NM} = \frac{(s_i-2)(1-c_i)}{s_i} = t_i^M$ because firm i monopolizes the market.

To better understand our results, Figure 6 plots those emission fees assuming parameter values $\alpha = \frac{1}{2}$, $d = \frac{1}{4}$, $n_B = n_G = 10$, and $k_B = k_G = 1$, where both firms pollute but the green firm is, relatively speaking, less polluting than the brown firm. Figure 6a illustrates that before the merger, the brown (green) firm is subsidized to produce more units above (below) the blue (black) cutoff, which happens when this firm is more efficient and goods are relatively homogeneous (high β). However, when firms are relatively cost-symmetric to each other, as in the shaded region between the two cutoffs, both firms are taxed to reduce output relative to the unregulated levels. When firms

[27] The environmental regulator sets taxes on either or both firms, which we refer to as emission fees, to facilitate our presentation. These taxes, however, seek to correct two market failures: the environmental externality from pollution and the imperfect competition in this industry.

merge, Figure 6b depicts that the shaded region where both firms are taxed shrinks. Intuitively, firms merge to reduce output, thus alleviating the EPA's need to use emission fees to curb emission.

Insert Figures 6a and 6b here.

Figure 6. Optimal emission fees

4.3 Second stage - merger approval

In the second stage, we evaluate the AA's merger approval decisions.

Lemma 7. *Mergers are approved by the AA under all parameter conditions.*

The AA, anticipating that the EPA in the third stage will respond setting emission fees that align firms' aggregate output to the socially optimal levels, both when firms merge and when they do not, can approve all merger proposals under all parameter conditions and industry structures.[28]

4.4 First stage - Merger vote

In the first stage of the game, the following lemma examines every firm i's incentives to merge with firm j. For compactness, cutoffs $\underline{m}_i^{\pi}$ and $\overline{m}_i^{\pi}$ are provided in the proof of Lemma 8.

Lemma 8. *Firm i has incentives to merge if and only if $m_i \leq \underline{m}_i^{\pi}$ or $m_i \geq \overline{m}_i^{\pi}$.*

Evaluating firm i's profit gains in an industry with one brown and one green firm merging into a single firm, where $n_i = n_j = k_i = k_j = 1$, cutoffs in Lemma 8 satisfy $\underline{m}_i^{\pi} = \overline{m}_i^{R}$ and $\overline{m}_i^{\pi} = \underline{m}_i^{R}$, implying that the conditions on m_i in Lemma 8 hold for all values of m_i when both firms produce output (as shown in Lemma 6). As in the case of no environmental regulation, where firms have incentives to merge under the parameter conditions in Lemma 4, they still have incentives to merge under environmental regulation, since the merged firm can conduct output shifts to the less polluting green firm to save emission fees or to the more efficient brown firm to benefit from cost savings.

As an illustration, Figure 7 considers, still with 10 firms of each type, one brown and one green firm merging into a single firm, that is, $n_B = n_G = 10$ and $k_B = k_G = 1$. As a benchmark, Figure

[28]This regulatory setting, where the AA considers pollution and the EPA is present, leads to merger approval decisions that are rarely observed in practice. We nonetheless consider this setting as the first-best against which to compare equilibrium outcomes in other regulatory environments.

7a considers the case of $d = \alpha = 0$, where neither the green nor the brown firm pollutes. In this setting, we find that the brown firm has incentives to merge in the following two shaded regions: (1) above cutoff $\overline{m}_B^{\pi}$ and below cutoff $\overline{m}_B^{R}$, and (2) above cutoff $\underline{m}_B^{R}$ and below cutoff $\underline{m}_B^{\pi}$, where the brown (green) firm is relatively efficient, respectively, and firms produce relatively homogeneous goods, so that when firms are subsidized to produce more units of non-polluting goods, the merged firm receives more generous subsidies on the relatively efficient output that facilities the merger.

Figure 7b depicts the case of $d = \frac{1}{4}$ and $\alpha = 0$, where the merger between a polluting brown and a non-polluting green firm can be supported in the shaded regions. Specifically, the region above cutoff $\underline{m}_B^{R}$ and below cutoff $\underline{m}_B^{\pi}$ expands, relative to the benchmark case in Figure 7a, as shifting output to the relatively efficient green firm can save emission fees and lower production costs. In contrast, the region above cutoff $\overline{m}_B^{\pi}$ and below cutoff $\overline{m}_B^{R}$ shrinks. This occurs because the brown firm faces more stringent emission fees, so its merger with the green firm becomes less attractive.

Figure 7c depicts the case of $d = \frac{1}{4}$ and $\alpha = \frac{1}{2}$, where the merger between a brown and a less polluting green firm can be supported in the shaded regions. Compared to Figure 7b, shifting output to the green firm does not save as much emission fees, so the region denoting profitable mergers from output shifts to the green firm above cutoff $\underline{m}_B^{R}$ and below cutoff $\underline{m}_B^{\pi}$ shrinks. Meanwhile, shifting output to the brown firm does not generate as much additional environmental damages, expanding the region of profitable mergers (above cutoff $\overline{m}_B^{\pi}$ and below cutoff $\overline{m}_B^{R}$).

Overall speaking, environmental regulation takes output competition (coordination) between unmerged (merged) firms into consideration and provides the right incentives for all firms to produce socially optimal aggregate output. This includes, for instance, the merger between one green and one brown firm into a single firm in an industry with two or more firms of each type, which does not exist when firms are not subject to environmental regulation (following the "80%" rule in Salant et al., 1983). Intuitively, emission fees induce the merged firms to shift their output to the less polluting firms, to the more efficient firms, or a combination of both.

Insert Figures 7a, 7b, and 7c here.

Figure 7. Merger incentives under regulation

Figure 8a depicts firms' incentives to merge using a similar approach as in Figure 4, namely, plotting the market share of the merging green firms (vertical axis) against that of the merging brown firms (horizontal axis), assuming parameter values $\alpha = 0$, $\beta = \frac{1}{3}$, $d = \frac{1}{4}$, $m_B = \frac{2}{3}$, and $n_B = n_G = 50$. Relative to Figure 4a (no environmental regulation), the vertical (horizontal) cutoff, to the right of (above) which every brown (green) firm has incentives to merge, shifts outwards (down). Intuitively, the brown (green) firm has less (more) incentives to merge since it faces more stringent emission fees (receives subsidies) to reduce (increase) the production of polluting (clean) goods.[29]

Figure 8b shows that when there are more firms in the market ($n_B = n_G = 250$ in dashed and $n_B = n_G = 1,000$ in dotted lines), the merger is profitable only if more firms from both types join to gain more market power. This result is analogous to Figure 4b without environmental regulation, as an increase in n_B and n_G, keeping the number of insiders k_B and k_G constant, increases the number of outsiders, each of them individually increasing their output level after the merger (output substitution effect), thereby making the merger more difficult to sustain in a more competitive market.[30]

Insert Figures 8a and 8b here.

Figure 8. Market concentration under regulation

Benchekroun et al. (2019) evaluate merger profitability in industries exploiting a natural resource, such as oil. While we do not study natural resource exploitation, we also demonstrate that environmental regulation makes mergers profitable only when a larger number of firms join, but environmental policy can help deter the merger.

4.5 Antitrust overlooks environmental effects

Suppose the AA, because of its directives, focuses on consumer and producer surpluses as in expression (2), overlooking environmental damages. In this context, we consider two cases: (1) AA anticipating the emission fees that the EPA sets in the third stage; and (2) a *naïve* AA that does not.

[29] In the case of non-polluting firms, where $d = \alpha = 0$, the horizontal (vertical) cutoff shifts down (inwards), suggesting that when both brown and green firms are subsidized to produce more units of non-polluting goods, mergers can be sustained even when fewer firms choose to merge. In comparison, in the case of brown (green) firms, where $d = \frac{1}{4}$ and $\alpha = \frac{1}{2}$, the horizontal (vertical) cutoff shifts up (outwards), suggesting that when brown and green firms are taxed to reduce output, mergers can be sustained only when a larger proportion of firms choose to merge.

[30] Similar results are obtained if we assume different values of α, β, d, and m_B.

In case (1), the AA expects that firms, because of emission fees, will produce socially optimal output whether they merge or not. In this setting, the AA can adopt a "laissez-faire" approach and does not block any merger proposals: whether the AA approves or blocks the merger, the same social welfare level arises, thereby reconciling our results to those in Lemma 7. (The same results hold even when we have an AA that considers consumer surplus alone, as in expression (3)). Interestingly, this happens because the AA anticipates that the EPA will, in the third stage, design emission fees to induce socially optimal output regardless of the market structure.[31] This outcome is independent of the AA's specific welfare function and whether it differs from the EPA's or not. If their welfare functions differ, the AA can anticipate that the EPA will set emission fees inducing a welfare level which is suboptimal from its perspective, but this welfare level, which is first-best according to expression (4), occurs regardless of whether the AA approves or blocks the merger.

In case (2), however, the AA *naïvely* assumes that firms are not subject to emission fees in the third stage, so that the AA approves or blocks the merger following the same decision rules as those stated in section 3.2.1. In both cases, since the AA's merger approval decisions only affect the industry structure but not aggregate output levels, the EPA can subsequently charge different emission fees, following those in Proposition 1, to induce socially optimal output from each firm. In this setting, even if the AA overlooks environmental effects, or whether it ignores the EPA's activities in subsequent stages altogether, output levels are eventually corrected via environmental regulation. In this regard, the AA's "mistakes" are eventually corrected by the EPA. However, if the administration of emission fees is costly, or if the EPA does not perfectly observe firms' output levels, emission fees may fall short of inducing first-best output levels. In these contexts, it becomes more necessary for the AA to also consider environmental externalities from the mergers.

To further illustrate our results, let us consider the merger between one brown and one green firm into a single firm in an industry comprised of 10 firms of each type ($n_B = n_G = 10$ and $k_B = k_G = 1$). Figure 9a illustrates that when both firms are non-polluting ($d = \alpha = 0$), mergers are approved only when the relative inefficient brown (green) firm shuts down after the merger, as

[31] Since aggregate output coincides with and without the merger, and for any number of firms that merge, consumer surplus also coincides. In addition, producer surplus is in one case subject to emission fees while in the other case is not. Recalling that total tax collection is returned to the firms in the form of lump-sum transfers, so that emission fees are revenue-neutral, and as a result, social welfare coincides in both settings.

in the shaded regions above cutoff $\underline{m}_B$ ($\frac{9\beta^2+11}{20\beta}$) and below cutoff $\frac{20\beta}{9\beta^2+11}$ ($\overline{m}_B$).[32] Otherwise, when firms are relatively cost-symmetric, as in the unshaded regions above $\frac{20\beta}{9\beta^2+11}$ and $\frac{9\beta^2+11}{20\beta}$, mergers that reduce output of both firms unambiguously reduce welfare, and are thus, rejected by the AA.

However, in the merger between a polluting brown and a completely clean green firm ($d = \frac{1}{4}$ and $\alpha = 0$), Figure 9b indicates that mergers are approved when the brown firm is less efficient than the green firm (below $m_B = 1$), since mergers shifting output to the relatively efficient and non-polluting green firm above cutoff $\underline{m}_B^{SW}$ and below 1 save production costs and mitigate pollution. Whereas, mergers that shift output to the relatively efficient brown firm can still be welfare-enhancing above 1 and below cutoff $\overline{m}_B^{SW}$ when firms produce relatively differentiated goods, so mergers that do not significantly reduce the green firm's output, and thus, consumer surplus, may still be approved.

Figure 9c sketches that when the brown firm is more polluting than the green firm ($d = \frac{1}{4}$ and $\alpha = \frac{1}{2}$), mergers that shift output to the relative efficient green firm below the dotted horizontal line ($m_B = 1$) are still approved under the same parameter conditions as in Figure 9b. However, mergers that shift output to the relative efficient brown firm are approved under larger parameter conditions. In particular, cutoff $\overline{m}_B^{SW}$ rotates clockwisely, indicating that even when goods become more homogenous, mergers can still be approved since cost efficiencies from shifting output to the brown firm can compensate for the additional environmental damages that this output shift creates.

Insert Figures 9a, 9b and 9c here.

Figure 9. Merger incentives under regulation

Finally, Figure 10 depicts the regions of SIM and SEM when the AA considers only consumer and producer surpluses but ignores environmental damages, where we superimpose cutoffs in Figure 9a onto Figure 9b (9c) to obtain Figure 10a (10b).[33]

When the AA ignores the fact that the brown firm generates pollution (while it makes the right assumption that the green firm is non-polluting, where $d = \frac{1}{4}$ and $\alpha = 0$), it approves welfare-reducing mergers above cutoff $\frac{9\beta^2+11}{20\beta}$ and below cutoff $\overline{m}_B$, where the more efficient brown firm shutting down the green firm generates significant environmental damages that should have been

[32]In this example, cutoff $\underline{m}_B = \frac{10\beta}{11}$ and cutoff $\overline{m}_B = \frac{11}{10\beta}$.

[33]When the AA only considers consumer surplus, it ignores profit gains and environmental damages from the merger, expanding (contracting) the region of SIM (SEM) where welfare-enhancing (reducing) mergers are blocked (approved).

blocked, yielding SEM in the purple-shaded region of Figure 10a. Whereas, mergers that should be approved when output shifts to the relatively efficient firm above cutoff $\frac{20\beta}{9\beta^2+11}$ and below cutoff $\overline{m}_B^{SW}$ are blocked by the AA, resulting in SIM in the yellow-shaded region of the same figure.

When the AA ignores the fact that both firms pollute but the green firm is less polluting than the brown firm ($d = \frac{1}{4}$ and $\alpha = \frac{1}{2}$), SEM still occurs when the AA allows the less polluting green firm to shut down, as in the purple-shaded region above cutoff $\max\left\{\overline{m}_B^{SW}, \frac{9\beta^2+11}{20\beta}\right\}$ and below cutoff $\overline{m}_B$ of Figure 10b. In contrast, the yellow-shaded region denoting SIM expands, as shifting output to the brown firm should be approved under wider parameter conditions, but such mergers are blocked by the AA above cutoff $\frac{20\beta}{9\beta^2+11}$ and below cutoff $\min\left\{\overline{m}_B^{SW}, \frac{9\beta^2+11}{20\beta}\right\}$ of the same figure.

Insert Figures 10a and 10b here.

Figure 10. SEM and SIM under regulation

5. Discussion

5.1 Welfare gains from the AA

In this section, we discuss the welfare gains of having an AA in different contexts.

Corollary 8. *When the EPA is present, the presence of the AA does not lead to strict welfare gains.*

When the EPA is present, this welfare gain, as captured by the difference in welfare level when both the AA and EPA are active and when only the EPA is active, $\Delta SW^{EPA} = SW_{AA,EPA} - SW_{EPA}$, is nil as the EPA induces socially optimal output in both settings (whether the AA is present or absent).

When the EPA is absent, however, the welfare gain of having an AA may be positive or negative, as discussed in section 3.2. In particular, when the AA is absent, we consider that firms can merge whenever there is a profit gain from it, that is, mergers are allowed under all parameter values. However, when the AA is present, it may approve or block mergers under different settings. Thus, the AA yields no welfare gain when approving mergers, as they would have happened anyway in its absence. In other words, the AA only gives rise to a welfare gain when it blocks a merger, yielding $SW_{AA}^{NM} - SW^{M}$. In this context, we must separately consider the following scenarios.

5.1.1 AA considers environmental damages

The following corollary summarizes the AA's welfare contribution in the absence of the EPA.

Corollary 9. *When the EPA is absent, the presence of the AA weakly improves social welfare when it blocks mergers.*

Graphically, the AA blocks mergers, and thus generates a welfare gain, in the unshaded regions above cutoff $\underline{m}_B$ ($\overline{m}_B^{SW}$) and below cutoff $\underline{m}_B^{SW}$ ($\overline{m}_B$) in Figures 2a and 2b. Otherwise, the AA generates no welfare gains, relative to the setting without this agency, in the regions above cutoff $\underline{m}_B^{SW}$ and below cutoff $\overline{m}_B^{SW}$ because mergers that enhance welfare will be approved. In addition, welfare gain $SW_{AA}^{NM} - SW^M$ is particularly large, suggesting that the presence of this agency is critical, when firms sell differentiated products (low β, where mergers do not initiate substantial output shifts), costs are relatively asymmetric (m_B close to 1, so that mergers do not generate large cost efficiencies), and asymmetric in pollution intensities (low α, so that mergers may aggravate environmental damages).

5.1.2 AA ignores environmental damages

In this context, as analyzed in section 3.2.1, the AA considers the social welfare function $\widehat{SW}$ rather than SW, and may allow mergers that should have been blocked according to SW (referred to as SEM) and, similarly, block mergers that should have been allowed (denoted as SIM). For simplicity, let us start with the case in which no regulatory mistakes occur (e.g., $d = \alpha = 0$). First, when the AA approves a merger according to both SW and $\widehat{SW}$, the presence of the AA yields no welfare gains since, as discussed above, the merger would have ensued even in the absence of the AA, generating the same social welfare. Second, when the AA blocks a merger according to both SW and $\widehat{SW}$, the presence of the AA yields the same welfare gain because when the AA considers environmental damages, $SW_{AA}^{NM} - SW^M$, capturing the difference in welfare when the merger is prevented (with the AA) and when it ensues (without the AA).

SEM settings. When SEM can be supported, however, the merger is approved when the AA is present, which would happen in the absence of the AA anyway, thus yielding no welfare gains, that is, $\widehat{SW}_{AA}^{M} - SW^M = 0$. According to this welfare measure, the presence of an AA that ignores environmental damages yields no additional benefits. Nonetheless, from section 3.2, such a merger would be blocked had the AA take the environmental effects from the merger into account, implying that this type of AA gives rise to a welfare loss, as captured by $SW_{AA}^{NM} - SW^{M} > 0$. As discussed above, this welfare loss is particularly large in regions where SEM can be sustained, namely, when goods are relatively homogeneous (high β, where mergers initiate large output shifts), costs are relatively symmetric (m_B close to 1, so mergers do not yield large cost savings), and environmental damages are relatively strong (high d, where mergers can shift output to the more polluting firms).

SIM settings. When SIM can be sustained, the AA blocks a merger that would occur in its absence, yielding a welfare loss $\widehat{SW}_{AA}^{NM} - SW^{M}$. However, had the AA considered the environmental effects from the merger, this welfare loss would have been avoided since $SW_{AA}^{NM} - SW^{M} < 0$. This loss is particularly large when goods are relatively differentiated (low β, where mergers do not cause a sharp reduction in output), costs are relatively asymmetric (m_B not equal to 1, so mergers can be justified on cost-efficiency grounds), and environmental damages are relatively strong (high d, so that firms can coordinate the production of more environmental-friendly goods). Therefore, in industries with these parameter conditions, we can expect that SIM yield particularly large welfare losses, implying that the AA's consideration of environmental damages is especially important in this type of markets. Alternatively, the presence of an AA that ignores environmental damages is, in this setting, welfare-reducing, which implies that society would be better off if the AA was absent. This result suggests that either the AA should consider environmental effects from the merger or, instead, simply approves all merger requests from firms satisfying the above parameter conditions.

5.2 Welfare gains from the EPA

In the absence of the AA, the introduction of the EPA yields an unambiguous welfare gain since the latter induces firms to produce socially optimal output levels. When the AA is present, as discussed in section 4, the EPA keeps inducing socially optimal output, thus yielding the first-best outcome. This result holds when the AA considers environmental damages and the merger approval decision is correct, but the EPA provides firms with incentives to increase or decrease their output to approach the social optimum, ultimately producing a welfare gain. Similarly, this result holds even when the AA ignores the environmental effects from the merger because, while the merger approval decision may be incorrect, the EPA ultimately induces firms to produce the same socially optimal output, whether or not they should have merged according to the welfare definition SW.

The following corollary summarizes the EPA's contribution to social welfare.

Corollary 10. *The EPA makes the largest contribution to social welfare when SEM and SIM occur in equilibrium.*

Therefore, the welfare gain attributed to the EPA is the largest when it needs to use emission fees to rectify an incorrect merger approval decision of the AA (which occurs when SEM and SIM are sustained in equilibrium), second largest in settings where the AA is absent (where all profitable mergers are approved, yielding only SEM but not SIM), and followed by contexts where the AA is present and made correct merger approval decisions (where only welfare-enhancing mergers are approved, resulting in no SEM or SEM). In other words, the EPA's role is the most important when the AA overlooks the environmental effects from the merger, but less necessary when the AA considers these effects when processing firms' merger requests.

6. Conclusion

Our results indicate that, while mergers of symmetric firms under no environmental regulation can lead to a welfare loss, as in Tirole (1988) and Lambertini (2013), mergers can improve welfare when firms are relatively asymmetric in costs and pollution intensities. In these settings, mergers should be promoted. Furthermore, if the EPA can charge emission fees, then first-best outcomes can be achieved whether firms are allowed to merge or not. Our paper then sheds light on the Horizontal Merger Guidelines (US Department of Justice and Federal Trade Commission, 2010), suggesting that the AA can approve mergers of differentiated firms if they can save costs and reduce pollution. These criteria may offset the anticompetitive effects of mergers, going beyond the conventional thinking of consumer surplus and efficiency gains in merger evaluation, as in the merger between Boeing Company and McDonnell Douglas Corporation. Examples where a broader perspective in merger evaluations may be useful include mergers between food processing companies Sysco and US Foods (Woodall and Shannon, 2018, p.220), and between nuclear waste disposal companies EnergySolutions and Waste Control Specialists (US Department of Justice, 2017).

Our results further describe that, when the AA does not consider the environmental effects of the mergers, socially excessive mergers can arise when output shifts to firms which environmental damages more than compensate the associated efficiency gains. These occur when firms producing relatively homogeneous goods are relatively asymmetric in their pollution intensities but are relatively cost symmetric. In contrast, socially insufficient mergers that were originally blocked may be reconsidered in light of their reduction of environmental externalities, which happen when firms produce relatively differentiated goods, are relatively asymmetric in costs, and environmental damages are strong.

Our model can be extended in several directions. First, one could assume that the EPA does not set emission fees to maximize social welfare but, instead, Pigouvian taxes that coincide with the marginal environmental damage from each firm's emissions. Second, we could consider that the EPA cannot set firm-specific emission fees, but instead, firm-uniform emission fees. This second-best environmental policy would yield lower welfare levels than those in the first-best scenario we consider in this paper.[34] Finally, our model can be extended to allow for the AA, the EPA, or both agencies, not to observe firms' production costs or pollution intensities, and how merger decisions are affected, when emission fees are used to induce socially optimal output levels (Sawaki, 2015).

7. Appendices

7.1 Appendix - Technical proofs

7.1.1 Proof of Lemma 1

No merger. Let N_i denote the set of all type-i firms. Every type-i firm chooses q_i to solve

$$\max_{q_i \geq 0} \pi\left(q_i\right) = \left(1 - q_i - \sum_{k \in N_i, k \neq i} q_k - \beta \sum_{j \in N_j} q_j - c_i\right) q_i$$

where $i, j \in \{B, G\}$ denotes the type of firm i and j, respectively.

Taking the first order condition with respect to q_i, and assuming interior solutions,

$$1 - 2q_i - \sum_{k \in N_i, k \neq i} q_k - \beta \sum_{j \in N_j} q_j - c_i = 0$$

[34]In this setting, the AA's role at the beginning of the game may become more relevant, since the AA no longer anticipates that the EPA will induce socially optimal outcomes in the third stage following its merger approval decisions.

Since all type-i firms are symmetric, in equilibrium we must have $q_i = q_k$, yielding

$$q_i\left(q_j\right) = \frac{1 - \beta \sum_{j \in N_j} q_j - c_i}{n_i + 1}$$

Intersecting the best response functions $q_i\left(q_j\right)$ and $q_j\left(q_i\right)$, equilibrium output becomes

$$q_i^{NM} = \frac{\left(n_j + 1\right)\left(1 - c_i\right) - \beta n_j \left(1 - c_j\right)}{\left(n_i + 1\right)\left(n_j + 1\right) - \beta^2 n_i n_j}$$

In this context, every firm i produces positive units before the merger if and only if

$$m_i > \frac{\beta n_j}{n_j + 1}$$

Merger. Let K_i (K_j) denote the subset of merged firms within the set of all type-i (-j) firms, N_i (N_j). When $k_i \leq n_i$ type-i and $k_j \leq n_j$ type-j firms merge, the merged firm solves

$$\begin{aligned}\max_{q_i^M, q_j^M \geq 0} \pi\left(q_i^M, q_j^M\right) = &\left(1 - q_i^M - \sum_{i \in \{N_i \backslash K_i\}} q_i^{UM} - \beta q_j^M - \beta \sum_{j \in \{N_j \backslash K_j\}} q_j^{UM} - c_i\right) q_i^M \\ &+ \left(1 - q_j^M - \sum_{j \in \{N_j \backslash K_j\}} q_j^{UM} - \beta q_i^M - \beta \sum_{i \in \{N_i \backslash K_i\}} q_i^{UM} - c_j\right) q_j^M\end{aligned}$$

Taking the first order condition with respect to q_i^M, and assuming interior solutions,

$$1 - 2q_i^M - \sum_{i \in \{N_i \backslash K_i\}} q_i^{UM} - 2\beta q_j^M - \beta \sum_{j \in \{N_j \backslash K_j\}} q_j^{UM} - c_i = 0$$

Since $\sum_{j \in \{N_j \backslash K_j\}} q_j^{UM} = \left(n_j - k_j\right) q_j^{UM}$ and $\sum_{i \in \{N_i \backslash K_i\}} q_i^{UM} = \left(n_i - k_i\right) q_i^{UM}$, we obtain that

$$q_i^M\left(q_i^{UM}\right) = \frac{1 - c_i - \beta\left(1 - c_j\right) - \left(1 - \beta^2\right)\left(n_i - k_i\right) q_i^{UM}}{2\left(1 - \beta^2\right)}$$

On the other hand, the unmerged firm i chooses q_i^{UM} to solve

$$\max_{q_i^{UM} \geq 0} \pi\left(q_i^{UM}\right) = \left(1 - q_i^{UM} - \sum_{k \in \{N_i \backslash K_i\}, k \neq i} q_k^{UM} - q_i^M - \beta \sum_{j \in \{N_j \backslash K_j\}} q_j^{UM} - \beta q_j^M - c_i\right) q_i$$

Taking the first order condition with respect to q_i^{UM}, and assuming interior solutions,

$$1 - 2q_i^{UM} - \sum_{k \in \{N_i \backslash K_i\}, k \neq i} q_k^{UM} - q_i^M - \beta \sum_{j \in \{N_j \backslash K_j\}} q_j^{UM} - \beta q_j^M - c_i = 0$$

Since $\sum\limits_{k\in\{N_i\setminus K_i\},k\neq i} q_k^{UM} = (n_i - k_i - 1)\, q_i^{UM}$ and $\sum\limits_{j\in\{N_j\setminus K_j\}} q_j^{UM} = \left(n_j - k_j\right) q_j^{UM}$, we have that

$$1 - (n_i - k_i + 1)\, q_i^{UM} - q_i^{M} - \beta\left(n_j - k_j\right) q_j^{UM} - \beta q_j^{M} - c_i = 0$$

Solving for q_i^{UM}, we obtain the best response function of the unmerged firm i, as follows

$$q_i^{UM}\left(q_i^M, q_j^M\right) = \frac{\left(n_j - k_j + 1\right)(1 - c_i) - \beta\left(n_j - k_j\right)\left(1 - c_j\right) - \left[(1-\beta^2)\left(n_j - k_j\right) + 1\right] q_i^M - \beta q_j^M}{(n_i - k_i + 1)\left(n_j - k_j + 1\right) - \beta^2 (n_i - k_i)\left(n_j - k_j\right)}$$

Simultaneously solving for the equilibrium output of merged and unmerged firms, we find

$$q_i^M = \frac{\left[\beta^2 (n_i - k_i) + n_j - k_j + 2\right](1 - c_i) - \beta\left(n_i - k_i + n_j - k_j + 2\right)\left(1 - c_j\right)}{(1-\beta^2)\left[(n_i - k_i + 2)\left(n_j - k_j + 2\right) - \beta^2 (n_i - k_i)\left(n_j - k_j\right)\right]}$$

$$q_i^{UM} = \frac{\left(n_j - k_j + 2\right)(1 - c_i) - \beta\left(n_j - k_j\right)\left(1 - c_j\right)}{(n_i - k_i + 2)\left(n_j - k_j + 2\right) - \beta^2 (n_i - k_i)\left(n_j - k_j\right)}$$

After the merger, every merged firm i produce positive units if and only if

$$\frac{\beta\left(n_i - k_i + n_j - k_j + 2\right)}{\beta^2 (n_i - k_i) + n_j - k_j + 2} \leq m_i < \frac{n_i - k_i + \beta^2\left(n_j - k_j\right) + 2}{\beta\left(n_i - k_i + n_j - k_j + 2\right)}$$

while every unmerged firm i produces positive units if and only if

$$m_i \geq \frac{\beta\left(n_j - k_j\right)}{n_j - k_j + 2}$$

When $\frac{n_i - k_i + \beta^2(n_j - k_j) + 2}{\beta(n_i - k_i + n_j - k_j + 2)} \leq m_i \leq \frac{n_i - k_i + 2}{\beta(n_i - k_i + 1)}$, the merged firm i (j) produces positive (zero) units, that is, $q_j^M = 0$. Setting $n_i - k_i + 1$ $(n_j - k_j)$ for the number of type-i (-j) firms, every firm i produces

$$q_i^{UM} = \frac{\left(n_j - k_j + 1\right)(1 - c_i) - \beta\left(n_j - k_j\right)\left(1 - c_j\right)}{(n_i - k_i + 2)\left(n_j - k_j + 1\right) - \beta^2 (n_i - k_i + 1)\left(n_j - k_j\right)}$$

which is positive since $\frac{n_i - k_i + \beta^2(n_j - k_j) + 2}{\beta(n_i - k_i + n_j - k_j + 2)} > \frac{\beta(n_j - k_j)}{n_j - k_j + 1}$, and the merged firm produces the same units of output as every unmerged firm i, that is, $q_i^M = q_i^{UM}$. In this context, every unmerged firm j produces

$$q_j^{UM} = \frac{(n_i - k_i + 2)\left(1 - c_j\right) - \beta\,(n_i - k_i + 1)(1 - c_i)}{(n_i - k_i + 2)\left(n_j - k_j + 1\right) - \beta^2 (n_i - k_i + 1)\left(n_j - k_j\right)}$$

7.1.2 Proof of Corollary 2

When $\frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z}$, the inequalities $q_i^M < k_i q_i^{NM}$ and $q_i^{UM} > q_i^{NM}$ reduce to

$$\frac{\beta\left[y\left(n_j - k_j\right) - n_j v_i\right]}{y\left(n_j - k_j + 1\right) - v_i\left(n_j + 1\right)} < m_i < \frac{\beta\left[yz - (1-\beta^2)\, k_i n_j x\right]}{w_i y - (1-\beta^2)\, k_i\left(n_j + 1\right) x}$$

that is satisfied; and when $\frac{w_j}{\beta z} \leq m_i \leq \frac{n_i - k_i + 2}{\beta(n_i - k_i + 1)}$, it becomes $q_i^{NM} < q_i^{UM} < k_i q_i^{NM}$ that holds.

7.1.3 Proof of Lemma 3

Before the merger, aggregate output of type-i all firms is

$$Q_i^{NM} = n_i q_i^{NM} = \frac{n_i\left(n_j+1\right)(1-c_i) - \beta n_i n_j\left(1-c_j\right)}{y}$$

After the merger, aggregate output of all type-i firms is

$$Q_i^M = q_i^M + (n_i - k_i)\, q_i^{UM} = \begin{cases} \frac{(n_i-k_i)[(n_j-k_j+2)(1-c_i)-\beta(n_j-k_j+1)(1-c_j)]}{v_j} & \text{if } \underline{m}_i \le m_i < \frac{\beta z}{w_i} \\ \frac{v_j(1-c_i)-\beta(x-z)(1-c_j)}{(1-\beta^2)x} & \text{if } \frac{\beta z}{w_i} \le m_i < \frac{w_j}{\beta z} \\ \frac{(n_i-k_i+1)[(n_j-k_j+1)(1-c_i)-\beta(n_j-k_j)(1-c_j)]}{v_i} & \text{if } \frac{w_j}{\beta z} \le m_i \le \overline{m}_i \end{cases}$$

Consumer surplus from consuming Q_i units of good i and Q_j units of good j is

$$\begin{aligned} CS\left(Q_i, Q_j\right) &= Q_i + Q_j - \frac{1}{2}\left(Q_i^2 + 2\beta Q_i Q_j + Q_j^2\right) - \left(1 - Q_i - \beta Q_j\right) Q_i - \left(1 - Q_j - \beta Q_i\right) Q_j \\ &= \frac{1}{2}\left(Q_i^2 + 2\beta Q_i Q_j + Q_j^2\right) \end{aligned}$$

Social welfare is, in this context, given by

$$\begin{aligned} SW\left(Q_i, Q_j\right) &= \underbrace{\frac{1}{2}\left(Q_i^2 + 2\beta Q_i Q_j + Q_j^2\right)}_{CS} + \underbrace{(p_i - c_i)\, Q_i + \left(p_j - c_j\right) Q_j}_{PS} - \underbrace{\frac{1}{2}\left(\sqrt{s_i - 1} Q_i + \sqrt{s_j - 1} Q_j\right)^2}_{Env} \\ &= \left(1 - \frac{s_i}{2} Q_i - c_i\right) Q_i + \left(1 - \frac{s_j}{2} Q_j - c_j\right) Q_j - r Q_i Q_j \end{aligned}$$

Specifically, social welfare before the merger is

$$SW^{NM} = \frac{E_i(1-c_i)^2 - 2F(1-c_i)\left(1-c_j\right) + E_j\left(1-c_j\right)^2}{2y^2}$$

Whereas, social welfare after the merger becomes

$$SW^M = \begin{cases} \frac{I_j(1-c_i)^2 - 2H_j(1-c_i)(1-c_j) + G_j(1-c_j)^2}{2v_j^2} & \text{if } \underline{m}_i \le m_i < \frac{\beta z}{w_i} \\ \frac{J_i(1-c_i)^2 - 2L(1-c_i)(1-c_j) + J_j(1-c_j)^2}{2(1-\beta^2)^2 x^2} & \text{if } \frac{\beta z}{w_i} \le m_i < \frac{w_j}{\beta z} \\ \frac{G_i(1-c_i)^2 - 2H_i(1-c_i)(1-c_j) + I_i(1-c_j)^2}{2v_i^2} & \text{if } \frac{w_j}{\beta z} \le m_i \le \overline{m}_i \end{cases}$$

Therefore, $SW^M \ge SW^{NM}$ if and only if $\underline{m}_i^{SW} \le m_i \le \overline{m}_i^{SW}$, where m_i solves

$$\begin{cases} (y^2 I_j - v_j^2 E_i) m_i^2 - 2(y^2 H_j - v_j^2 F) m_i + (y^2 G_j - v_j^2 E_j) \ge 0 & \text{if } \underline{m}_i \le m_i < \frac{\beta z}{w_i} \\ [y^2 J_i - (1-\beta^2)^2 x^2 E_i] m_i^2 - 2[y^2 L - (1-\beta^2)^2 x^2 F] m_i + [y^2 J_j - (1-\beta^2)^2 x^2 E_j] \ge 0 & \text{if } \frac{\beta z}{w_i} \le m_i < \frac{w_j}{\beta z} \\ (y^2 G_i - v_i^2 E_i) m_i^2 - 2(y^2 H_i - v_i^2 F) m_i + (y^2 I_i - v_i^2 E_j) \ge 0 & \text{if } \frac{w_j}{\beta z} \le m_i \le \overline{m}_i \end{cases}$$

where $E_i \equiv n_i\{[2y - s_i n_i(n_j + 1) + 2\beta r n_i n_j](n_j + 1) - \beta^2 s_j n_i n_j^2\}$, $F \equiv n_i n_j\{r[(n_i + 1)(n_j + 1) + \beta^2 n_i n_j] - \beta[s_i n_i(n_j + 1) + s_j n_j(n_i + 1) - 2y]\}$, $G_i \equiv (n_i - k_i + 1)\{[2v_i - s_i(n_i - k_i + 1)(n_j - k_j + 1) + 2\beta r(n_i - k_i + 1)(n_j - k_j)](n_j - k_j + 1) - \beta^2 s_j(n_i - k_i + 1)(n_j - k_j)^2\}$, $H_i \equiv (n_i - k_i + 1)(n_j - k_j)\{r[(n_i - k_i + 2)(n_j - k_j + 1) + \beta^2(n_i - k_i + 1)(n_j - k_j)] - \beta[s_i(n_i - k_i + 1)(n_j - k_j + 1) + s_j(n_i - k_i + 2)(n_j - k_j) - 2v_i]\}$, $I_i \equiv (n_j - k_j)\{[2v_i - s_j(n_i - k_i + 2)(n_j - k_j) + 2\beta r(n_i - k_i + 1)(n_j - k_j)](n_i - k_i + 2) - \beta^2 s_i(n_i - k_i + 1)^2(n_j - k_j)\}$, $J_i \equiv [2(1 - \beta^2)x - s_i v_j]v_j + 2\beta r v_j(x - z) - \beta^2 s_j(x - z)^2$, and $L \equiv r[v_i v_j + \beta^2(x - z)^2] - \beta(x - z)[s_i v_j + s_j v_i - 2(1 - \beta^2)x]$.

7.1.4 Proof of Lemma 4

Substituting $p_i = 1 - Q_i - \beta Q_j$ into π_i, firm i earns a profit of $\pi_i^{NM} = \left(q_i^{NM}\right)^2$ before the merger; and after the merger, firm i obtains $\pi_i^M = q_i^{UM} q_i^M$ if merged and $\pi_i^{UM} = \left(q_i^{UM}\right)^2$ if not merged.

Aggregate profits of all the firms that merge, $\pi^{NM} \equiv k_i q_i^{NM} + k_j q_j^{NM}$, before the merger are

$$\frac{\left[k_i(n_j+1)^2+\beta^2 k_j n_i^2\right](1-c_i)^2-2\beta[k_i n_j(n_j+1)+k_j n_i(n_i+1)](1-c_i)(1-c_j)+\left[k_j(n_i+1)^2+\beta^2 k_i n_j^2\right](1-c_j)^2}{y^2}$$

When $\frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z}$, total profits of the merged firm, $\pi^M \equiv \pi_i^M + \pi_j^M$, after the merger are

$$\frac{\left[(n_j-k_j+2)w_i+\beta^2(n_i-k_i)z\right](1-c_i)^2-\beta[(n_i-k_i+n_j-k_j+4)z+(n_i-k_i)w_j+(n_j-k_j)w_i](1-c_i)(1-c_j)+\left[(n_i-k_i+2)w_j+\beta^2(n_j-k_j)z\right](1-c_j)^2}{(1-\beta^2)x^2}$$

When $\frac{w_j}{\beta z} \leq m_i \leq \overline{m}_i$, profits of the merged firm, $\pi^M \equiv \pi_i^M$, after the merger become

$$\frac{\left(n_j - k_j + 1\right)^2 (1 - c_i)^2 - 2\beta\left(n_j - k_j\right)\left(n_j - k_j + 1\right)(1 - c_i)\left(1 - c_j\right) + \beta^2\left(n_j - k_j\right)^2\left(1 - c_j\right)^2}{v_i^2}$$

In this context, firm i has incentives to merge if and only if $\pi^M \geq \pi^{NM}$, where m_i solves

$$\begin{cases} \frac{\beta R_j - \sqrt{\beta^2 R_j^2 - O_j S_j}}{S_j} \equiv \underline{m}_i^{NR} \leq m_i \leq \overline{m}_i^{NR} \equiv \frac{\beta R_j + \sqrt{\beta^2 R_j^2 - O_j S_j}}{S_j} & \text{if } \underline{m}_i \leq m_i < \frac{\beta z}{w_i} \\ \frac{\beta P - \sqrt{\beta^2 P^2 - 4M_i M_j}}{2M_i} \equiv \underline{m}_i^{NR} \leq m_i \leq \overline{m}_i^{NR} \equiv \frac{\beta P + \sqrt{\beta^2 P^2 - 4M_i M_j}}{2M_i} & \text{if } \frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z} \\ \frac{\beta R_i - \sqrt{\beta^2 R_i^2 - O_i S_i}}{O_i} \equiv \underline{m}_i^{NR} \leq m_i \leq \overline{m}_i^{NR} \equiv \frac{\beta R_i + \sqrt{\beta^2 R_i^2 - O_i S_i}}{O_i} & \text{if } \frac{w_j}{\beta z} \leq m_i \leq \overline{m}_i \end{cases}$$

where $M_i \equiv (1 - \beta^2)x^2[k_i(n_j + 1)^2 + \beta^2 k_j n_i^2] - [w_i(n_j - k_j + 2) + \beta^2(n_i - k_i)z]y^2$, $O_i \equiv v_i^2[k_i(n_j + 1)^2 + \beta^2 k_j n_i^2] - (n_j - k_j + 1)^2 y^2$, $P \equiv 2(1 - \beta^2)x^2[k_i n_j(n_j + 1) + k_j n_i(n_i + 1)] - [(n_i - k_i + n_j - k_j + 4)z + (n_i - k_i)w_j + (n_j - k_j)w_i]y^2$, $R_i \equiv v_i^2[k_i n_j(n_j + 1) + k_j n_i(n_i + 1)] - (n_j - k_j + 1)(n_j - k_j)y^2$, and $S_i \equiv v_i^2[k_j(n_i + 1)^2 + \beta^2 k_i n_j^2] - \beta^2(n_j - k_j)^2 y^2$.

7.1.5 Proof of Lemma 5

Substituting the after-tax per-unit cost, $c_i + t_i$, for the pre-tax per-unit cost of every type-i firm, c_i, in the proof of Lemma 1, we derive firm i's equilibrium output as a function of emission fees.

7.1.6 Proof of Lemma 6

Differentiating $SW\left(Q_i, Q_j\right)$ with respect to Q_i, and assuming interior solutions, we obtain

$$1 - s_i Q_i - c_i - r Q_j = 0$$

Rearranging and solving for Q_i, socially optimal output becomes

$$Q_i^{SO} = \frac{s_j\left(1 - c_i\right) - r\left(1 - c_j\right)}{s_i s_j - r^2}$$

which is positive if and only if $\frac{r}{s_j} \equiv \underline{m}_i^R \leq m_i \leq \overline{m}_i^R \equiv \frac{s_i}{r}$.

When $m_i < \overline{m}_i^R$, the EPA sets $Q_i^{SO} = 0$ but when $m_i > \overline{m}_i^R$, it solves $\max_{Q_i > 0} SW\left(Q_i, 0\right)$, yielding

$$Q_i^{SO} = \frac{1 - c_i}{s_i}$$

Next, substituting Q_i^{SO} and Q_j^{SO} into expression (4), social welfare becomes

$$SW^{SO} = \frac{\left(1 - c_i\right)^2 - 2\beta\left(1 - c_i\right)\left(1 - c_j\right) + \left(1 - c_j\right)^2 + \left[\sqrt{s_i - 1}\left(1 - c_j\right) - \sqrt{s_j - 1}\left(1 - c_i\right)\right]^2}{2\left(s_i s_j - r^2\right)}$$

We find that social welfare is concave in aggregate output, given the Hessian matrix of

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial^2 SW(Q_i,Q_j)}{\partial Q_i^2} & \frac{\partial^2 SW(Q_i,Q_j)}{\partial Q_i Q_j} \\ \frac{\partial^2 SW(Q_i,Q_j)}{\partial Q_j Q_i} & \frac{\partial^2 SW(Q_i,Q_j)}{\partial Q_j^2} \end{bmatrix} \\ &= -\begin{bmatrix} s_i & r \\ r & s_j \end{bmatrix} \\ &= -\left(s_i s_j - r^2\right) < 0 \end{aligned}$$

7.1.7 *Proof of Corollary 5*

When $\underline{m}_B^R \leq m_B \leq \overline{m}_B^R$, differentiating Q_i^{SO} with respect to β, we obtain

$$\frac{\partial Q_i^{SO}}{\partial \beta} = \frac{2rs_j(1-c_i) - \left(s_i s_j + r^2\right)\left(1-c_j\right)}{\left(s_i s_j - r^2\right)^2}$$

which is positive if and only if $m_i \geq \overline{m}_i^\beta \equiv \frac{s_i s_j + r^2}{2rs_j}$. Specifically, $\underline{m}_B^\beta \equiv \frac{2rs_B}{s_B s_G + r^2} < \frac{s_B s_G + r^2}{2rs_G} \equiv \overline{m}_B^\beta$ reduces to $\left(r^2 - s_B s_G\right)^2 > 0$ that holds, and it is straightforward to verify that $\overline{m}_i^\beta < \overline{m}_i^R$, which reduces to $s_i s_j - r^2 > 0$, holds, yielding $\underline{m}_B^R < \underline{m}_B^\beta$ and $\overline{m}_B^\beta < \overline{m}_B^R$. Combining, we have $\underline{m}_B^R < \underline{m}_B^\beta < \overline{m}_B^\beta < \overline{m}_B^R$.

7.1.8 *Proof of Corollary 6*

When $\underline{m}_B^R \leq m_B \leq \overline{m}_B^R$, differentiating Q_i^{SO} with respect to d, we find

$$\frac{\partial Q_i^{SO}}{\partial d} = -\frac{\left(\sqrt{s_i-1} - \beta\sqrt{s_j-1}\right)^2(1-c_i) + \left[\sqrt{s_i-1}\sqrt{s_j-1}\,(1-\beta)^2 - \beta\left(\sqrt{s_i-1} - \sqrt{s_j-1}\right)^2\right]\left(1-c_j\right)}{d\left(s_i s_j - r^2\right)^2}$$

which is negative if and only if $m_i \geq \overline{m}_i^d \equiv \frac{\beta\left(\sqrt{s_i-1}-\sqrt{s_j-1}\right)^2 - \sqrt{s_i-1}\sqrt{s_j-1}(1-\beta)^2}{\left(\sqrt{s_i-1}-\beta\sqrt{s_j-1}\right)^2}$. Since $\frac{\partial \overline{m}_i^d}{\partial s_j} = -\frac{\sqrt{s_i-1}\left(1-\beta^2\right)}{2\sqrt{2d(s_j-1)}\left(\sqrt{s_i-1}-\beta\sqrt{s_j-1}\right)^2} < 0$, $\overline{m}_i^d$ attains the maximum value at $s_j = 1$. In particular, when $s_j = 1$, we obtain that $\overline{m}_i^d = \underline{m}_i^R = \beta$, so that $\frac{\partial Q_i^{SO}}{\partial d} < 0$ holds for all parameter values.

7.1.9 *Proof of Corollary 7*

When $\underline{m}_B^R \leq m_B \leq \overline{m}_B^R$, differentiating Q_G^{SO} with respect to α, we have

$$\frac{\partial Q_G^{SO}}{\partial \alpha} = \frac{2d\left\{2(\beta-\alpha)(1+2d)(1-c_G) - [(1+2d)(1-\alpha\beta) + (\beta+2\alpha d)(\beta-\alpha)](1-c_B)\right\}}{\left(s_B s_G - r^2\right)^2}$$

which is negative if and only if $m_B \geq \underline{m}_B^\alpha \equiv \frac{2(\beta-\alpha)(1+2d)}{(1+2d)(1-\alpha\beta)+(\beta+2\alpha d)(\beta-\alpha)}$, yielding $\frac{\partial \underline{m}_B^\alpha}{\partial \alpha} = -\frac{2\left(1-\beta^2\right)(1+2d)^2}{[(1+2d)(1-\alpha\beta)+(\beta+2\alpha d)(\beta-\alpha)]^2} < 0$.

When $\underline{m}_B^R \leq m_B \leq \overline{m}_B^R$, differentiating Q_B^{SO} with respect to α, we have

$$\frac{\partial Q_B^{SO}}{\partial \alpha} = \frac{2d\left\{2(1-\alpha\beta)(\beta+2\alpha d)(1-c_B) - [(1+2d)(1-\alpha\beta) + (\beta+2\alpha d)(\beta-\alpha)](1-c_G)\right\}}{\left(s_B s_G - r^2\right)^2}$$

which is positive if and only if $m_B \geq \overline{m}_B^\alpha \equiv \frac{(1+2d)(1-\alpha\beta)+(\beta+2\alpha d)(\beta-\alpha)}{2(1-\alpha\beta)(\beta+2\alpha d)}$, yielding $\frac{\partial \overline{m}_B^\alpha}{\partial \alpha} = -\frac{\left(1-\beta^2\right)(\beta+2\alpha d)^2 + 2d(1+2d)(1-\alpha\beta)^2}{2(1-\alpha\beta)^2(\beta+2\alpha d)^2} < 0$, and it is straightforward to verify that $\underline{m}_B^\alpha < \overline{m}_B^\alpha$.

7.1.10 Proof of Proposition 1

To find the optimal emission fees t_i^{NM} and t_i^M for every firm i, we align the firms' aggregate output with the socially optimal aggregate output, solving $Q_i^{NM} = \frac{n_i(n_j+1)(1-c_i-t_i)-\beta n_i n_j(1-c_j-t_j)}{y} = \frac{s_j(1-c_i)-r(1-c_j)}{s_i s_j - r^2} = Q_i^{SO}$ before the merger and $Q_i^M = \frac{v_j(1-c_i-t_i)-\beta(x-z)(1-c_j-t_j)}{(1-\beta^2)x} = \frac{s_j(1-c_i)-r(1-c_j)}{s_i s_j - r^2} = Q_i^{SO}$ after the merger, when $\underline{m}_i^R \leq m_i \leq \overline{m}_i^R$. Otherwise, when $m_i > \overline{m}_i^R$, the EPA shuts down all type-j firms, setting an emission fee t_i^M for every firm i that solves $Q_i^{NM} = \frac{n_i(1-c_i-t_i)}{n_i+1} = \frac{1-c_i}{s_i} = Q_i^{SO}$ before the merger and $Q_i^M = \frac{(n_i-k_i+1)(1-c_i-t_i)}{n_i-k_i+2} = \frac{1-c_i}{s_i} = Q_i^{SO}$ after the merger.

7.1.11 Proof of Lemma 8

Substituting t_i^{NM} into the profit function before the merger, every firm i generates a profit of

$$\pi_i^{NM,R} = \begin{cases} 0 & \text{if } m_i < \underline{m}_i^R \\ \left(\frac{s_j(1-c_i)-r(1-c_j)}{n_i(s_i s_j - r^2)}\right)^2 & \text{if } \underline{m}_i^R \leq m_i \leq \overline{m}_i^R \\ \left(\frac{1-c_i}{s_i(n_i+1)}\right)^2 & \text{if } m_i > \overline{m}_i^R \end{cases}$$

Substituting t_i^M into the profit function after the merger, every merged firm i earns a profit of

$$\pi_i^{M,R} = \begin{cases} 0 & \text{if } m_i < \underline{m}_i^R \\ \frac{(1-\beta^2)\left[T_i V_i(1-c_i)^2-(T_i W_i+U_i V_i)(1-c_i)(1-c_j)+U_i W_i(1-c_j)^2\right]}{[v_i v_j-\beta^2(x-z)^2]^2(s_i s_j-r^2)^2} & \text{if } \underline{m}_i^R \leq m_i \leq \overline{m}_i^R \\ \left(\frac{1-c_i}{s_i(n_i-k_i+2)}\right)^2 & \text{if } m_i > \overline{m}_i^R \end{cases}$$

In this context, firm i has incentives to merge if and only if $\pi_i^{M,R} + \pi_j^{M,R} \geq k_i \pi_i^{NM,R} + k_j \pi_j^{NM,R}$, where m_i solving $X_i m_i^2 - Y m_i + X_j \geq 0$ satisfies $m_i \leq \underline{m}_i^\pi \equiv \frac{Y-\sqrt{Y^2-4X_iX_j}}{2X_i}$ or $m_i \geq \overline{m}_i^\pi \equiv \frac{Y+\sqrt{Y^2-4X_iX_j}}{2X_i}$. For compactness, define $T_i \equiv (n_j - k_j + 2)(s_j v_i - \beta r(x-z)) + \beta(n_j - k_j)(r v_j - \beta s_j(x-z))$, $U_i \equiv (n_j - k_j + 2)(r v_i - \beta s_i(x-z)) + \beta(n_j - k_j)(s_i v_j - \beta r(x-z))$, $V_i \equiv (s_j v_i - \beta r(x-z))w_i + \beta(r v_j - \beta s_j(x-z))z$, $W_i \equiv (r v_i - \beta s_i(x-z))w_i + \beta(s_i v_j - \beta r(x-z))z$, $X_i \equiv (1-\beta^2)n_i^2 n_j^2(T_i V_i + U_j W_j) - [v_i v_j - \beta^2(x-z)^2]^2(k_i n_j^2 s_j^2 + k_j n_i^2 r^2)$, and $Y \equiv (1-\beta^2)n_i^2 n_j^2(T_i W_i + T_j W_j + U_i V_i + U_j V_j) - 2r[v_i v_j - \beta^2(x-z)^2]^2(k_i n_j^2 s_j + k_j n_i^2 s_i)$. In the setting of $n_i = n_j = k_i = k_j = 1$, we obtain that $T_i = 4(s_j - \beta r)$, $U_i = 4(r - \beta s_i)$, $V_i = 4(1-\beta^2)s_j$, $W_i = 4(1-\beta^2)r$, $X_i = 32\beta(1-\beta^2)^2 r s_j$, and $Y = 32\beta(1-\beta^2)^2(s_i s_j + r^2)$, so the above inequality is rearranged to yield $\frac{r}{s_j} \leq m_i \leq \frac{s_i}{r}$ that holds by assumption. Otherwise, firm i has incentives to merge if and only if $k_i \geq \frac{2n_i+3-\sqrt{4n_i+5}}{2}$, as in Salant et al. (1983).

8. References

Bayer AG. (2019). *Annual report 2018*. https://www.bayer.com/sites/default/files/2020-04/bayer_ar18_entire.pdf

Benchekroun, H., Breton, M., & Ray Chaudhuri, A. (2019). Mergers in nonrenewable resource oligopolies and environmental policies. *European Economic Review, 111*, 35-52.

Benchekroun, H., & Ray Chaudhuri, A. (2008). *Collusion inducing taxation of a polluting oligopoly* (Center for Economic Research Discussion Paper Series No. 2008-80). Tilburg University.

Berchicci, L., Dowell, G., & King, A. A. (2012). Environmental capabilities and corporate strategy: Exploring acquisitions among US manufacturing firms. *Strategic Management Journal, 33*(9), 1053-1071.

Berchicci, L., Dowell, G., & King, A. A. (2017). Environmental performance and the market for corporate assets. *Strategic Management Journal, 38*(12), 2444-2464.

Canton, J., David, M., & Sinclair-Desgagné, B. (2012). Environmental regulation and horizontal mergers in the eco-industry. *Strategic Behavior and the Environment, 2*(2), 107-132.

Creti, A., & Sanin, M.-E. (2017). Does environmental regulation create merger incentives? *Energy Policy, 105*, 618-630.

Davidson, C., & Mukherjee, A. (2007). Horizontal mergers with free entry. *International Journal of Industrial Organization, 25*(1), 157-172.

Erkal, N., & Piccinin, D. (2010). Cooperative R&D under uncertainty with free entry. *International Journal of Industrial Organization, 28*(1), 78-85.

Espínola-Arredondo, A., & Muñoz-García, F. (2015). Can poorly informed regulators hinder competition? *Environmental and Resource Economics, 61*(3), 433-461.

Fauli-Oller, R. (2002). Mergers between asymmetric firms: Profitability and welfare. *The Manchester School, 70*(1), 77-87.

Fikru, M. G., & Gautier, L. (2016). Mergers in Cournot markets with environmental externality and product differentiation. *Resource and Energy Economics, 45*, 65-79.

Fikru, M. G., & Gautier, L. (2017). Environmental taxation and mergers in oligopoly markets with product differentiation. *Journal of Economics, 122*(1), 45-65.

Fikru, M. G., & Gautier, L. (2020). Are big mergers welfare enhancing when there is environmental externality? *Energy Economics, 87*, 104718.

Filtration + Separation. (2018). Nederman buys Auburn FilterSense. *Filtration + Separation*. http://www.filtsep.com/hvac/news/nederman-buys-auburn-filtersense

Forslid, R., Okubo, T., & Ulltveit-Moe, K. H. (2018). Why are firms that export cleaner? International trade, abatement and environmental emissions. *Journal of Environmental Economics and Management, 91*, 166-183.

Gelves, J. A. (2014). Differentiation and cost asymmetry: Solving the merger paradox. *International Journal of the Economics of Business, 21*(3), 321-340.

Hennessy, D. A., & Roosen, J. (1999). Stochastic pollution, permits, and merger incentives. *Journal of Environmental Economics and Management, 37*(3), 211-232.

Hsu, J., & Wang, X. H. (2005). On welfare under Cournot and Bertrand competition in differentiated oligopolies. *Review of Industrial Organization, 27*, 185-191.

Johansson, P. O. (1999). Theory of economic valuation of environmental goods and services. In J. C. van den Bergh (Ed.), *Handbook of Environmental and Resource Economics* (p. 748). Edward Elgar Publishing.

Kao, T., & Menezes, F. (2010). Welfare-enhancing mergers under product differentiation. *The Manchester School, 78*(4), 290-301.

Lambertini, L. (2013). Mergers, vertical relations and collusion. In L. Lambertini (Ed.), *Oligopoly, the Environment and Natural Resources* (pp. 76-101). Routledge.

Lambertini, L., & Tampieri, A. (2012). *Efficient horizontal mergers in polluting industries with Cournot competition* (Working Paper DSE No. 813). Department of Economics, University of Bologna.

Lambertini, L., & Tampieri, A. (2014). *Efficient horizontal mergers in polluting industries with green R&D and endogenous taxation*. Keio Economic Society, Keio University.

Martin, C. (2012). Enbridge buys first solar's 50MW Nevada power plant. *Financial Post*. https://business.financialpost.com/commodities/energy/enbridge-buys-first-solars-50mw-nevada-power-plant

Martinez-Lopez, M. (2000). Commission approves an agreement to improve efficiency in washing machines. *Competition Policy Newsletter, 1*, 13-14.

Meglio, O., & Park, K. (2019). *Strategic Decisions and Sustainability Choices*. Springer International Publishing.

Najjar, N., & Cherniwchan, J. (2021). Environmental regulations and the clean-up of manufacturing: Plant-level evidence. *Review of Economics and Statistics, 103*(3), 476-491.

Pelaez, V., & Mizukawa, G. (2017). Diversification strategies in the pesticide industry: From seeds to biopesticides. *Ciência Rural, 47*(2). https://dx.doi.org/10.1590/0103-8478cr20160007

Perry, M. K., & Porter, R. H. (1985). Oligopoly and the incentive for horizontal merger. *The American Economic Review, 75*(1), 219-227.

Salant, S. W., Switzer, S., & Reynolds, R. J. (1983). Losses from horizontal merger: The effects of an exogenous change in industry structure on Cournot-Nash equilibrium. *The Quarterly Journal of Economics, 98*(2), 185-199.

Sawaki, H. (2015). Horizontal mergers under asymmetric information about synergies. *Australian Economic Papers, 54*(3), 167-184.

Singh, N., & Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *The RAND Journal of Economics, 15*(4), 546-554.

Tirole, J. (1988). *The theory of industrial organization*. The MIT Press.

US Department of Justice. (2017). *U.S. district court blocks EnergySolutions' acquisition of Waste Control Specialists*. https://www.justice.gov/opa/pr/us-district-court-blocks-energysolutions-acquisition-waste-co

US Department of Justice and Federal Trade Commission. (2010). *Horizontal merger guidelines*. https://www.justice.gov/atr/horizontal-merger-guidelines-08192010

Waste 360. (2018). Covanta acquires Quantex Environmental, expands presence in Canada. *Waste 360*. https://www.waste360.com/mergers-and-acquisitions/covanta-acquires-quantex-environmental-expands-pre

Woodall, P., & Shannon, T. L. (2018). Monopoly power corrodes choice and resiliency in the food system. *The Antitrust Bulletin, 63*(2), 198-221.

Zanchettin, P. (2006). Differentiated duopoly with asymmetric costs. *Journal of Economics and Management Strategy, 15*(4), 999-1015.