Monopoly Price Discrimination and Demand Curvature

Aguirre et. al. (2010)

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Introduction

- Moving from nondiscrimination to discrimination raises the firm’s profits, harms consumers in markets where prices increase and benefits the consumers who face lower prices.
- The overall effect on welfare can be positive or negative.
- The effect of discrimination on welfare can be divided into a misallocation effect and an output effect
- Discussion on the literatures:
  - Pigou (1920) proved that if all demand functions are linear and all markets are served at the nondiscriminatory price then total output remains at the no-discrimination level, in which case *discrimination is bad for welfare*.
  - Schmalensee (1981), showed how the *curvature of demands determines the sign of output effect*.
  - Hal R. Varian (1985) proved very generally that a necessary condition for *welfare to rise* with discrimination is that *total output increases* (see also Marius Schwartz 1990).

Discussion on the literatures

- This paper develops general conditions that determine whether third-degree price discrimination by a monopolist serving all markets reduces or raises output and social welfare (sum of consumer surplus and profit).
  1. Provide conditions based on the shapes of the demand functions to determine the sign of the welfare effect.
  2. Address the classic question of the effect of discrimination on total output, and the paper combines new findings with existing results in a unified framework.

The general model of the paper

- It explores the welfare effect *directly* using the technique developed by Schmalensee (1981) and Thomas J. Holmes (1989) to analyze the output effect
- Assuming at the nondiscriminatory price all markets are served with positive quantities
- Explore the case with two markets
- Using price different (r) as a constraint, it sees how overall welfare and total output vary as the price-difference constraint is relaxed

I The Model of Monopoly Pricing

- A monopolist sells its product in two markets and has a constant marginal cost, $c \geq 0$. Utility are quasi-linear.
- Demand is $q(p)$, is twice-differentiable, decreasing and independent of the price in the other market.
- The price elasticity of demand is $\eta \equiv -pq'/q$.
- The profit function in a market is $\pi = (p - c)q(p)$.
- Assume that

$$\pi''(p) = 2q' + (p-c)q'' = \left[2 + (p-c)\frac{q''}{q'}\right]q' < 0,$$

- Define $\alpha(p) \equiv -pq''/q'$ as the *convexity* (or curvature) *of direct demand* or relative risk aversion for a utility function and is the elasticity of the slope of demand.
- $\sigma(q) \equiv -qp''/p' = qq''/[q']$ is the curvature or *convexity of the inverse demand*
- The Lerner index, is $L(p) \equiv (p - c)/p$ and $2 + (p - c)q''/q' = 2 - L\alpha > 0$ by strict concavity.
- The two curvature measures are related to the price elasticity by $\sigma = \alpha/\eta$.
- The values of σ and of α play key roles in the analysis.

I The Model of Monopoly Pricing

a. When the firm *discriminates*, it chooses p^*, the FOC in each market is:

$$\pi'(p^*) = q(p^*) + (p^* - c)q'(p^*) = 0,$$

Where: $p^* > c$ is optimal price with full discrimination

$L^* = 1/\eta^*$. Thus $L^*\alpha^* = \alpha^*/\eta^* = \sigma^*$ and, with strict concavity, $2 - L^*\alpha^* = 2 - \sigma^* > 0$

b. When the firm *cannot discriminate* it chooses the single price $\bar{p}$.

- The FOC: $\pi'_w(\bar{p}) + \pi'_s(\bar{p}) = 0.$
- Assuming that both markets are served at the nondiscriminatory price imply that:

  $\pi'_w(\bar{p}) = q_w(\bar{p})[1 - L(\bar{p})\eta_w(\bar{p})] < 0$ and $\pi'_s(\bar{p}) = q_s(\bar{p})[1 - L(\bar{p})\eta_s(\bar{p})] > 0$, so $\eta_w(\bar{p}) > \eta_s(\bar{p})$.
- The weak market has the higher elasticity at $\bar{p}$
- With strict concavity of π it follows that $p^*_s > \bar{p} > p^*_w$

- Marginal effect of price on social welfare in a market is $dW/dp = (p - c)q'(p)$

II The Effect of Discrimination on Welfare

- Allowing constraint $p_s - p_w \leq r$ where $r \geq 0$ is the degree of discrimination allowed.
- The objective function is $\pi_w(p_w) + \pi_s(p_w + r)$ and the FOC is $\pi'_w(p_w) + \pi'_s(p_w + r) = 0$ when $r = 0$
- When the constraint binds, $r = 0$, the firm sets the nondiscriminatory price.
- As r rises more discrimination is allowed, the p_w falls and p_s rises:

$$(1) \qquad p'_w(r) = \frac{-\pi''_s}{\pi''_w + \pi''_s} < 0; \; p'_s(r) = \frac{\pi''_w}{\pi''_w + \pi''_s} > 0.$$

- When $r = 0$, the marginal change in social welfare W as more price discrimination is allowed is

$$(2) \qquad W'(r) = (p_w - c)q'_w(p_w)p'_w(r) + (p_s - c)q'_s(p_s)p'_s(r).$$

II The Effect of Discrimination on Welfare

- For $r > r^* = p\ s^* - p\ {}^*w$, the marginal welfare effect is zero because the prices remain at the discriminatory levels.
- Define $W'(0)$ and $W'(r^*)$ as right- and left-derivatives respectively.
- Since marginal effect on total output is $Q'(r) \equiv q_w'\ p_w' + q_s'\ p_s'$,following Schmalensee (1981), (2) is written as

$$(3) \qquad W'(r) = \underbrace{(p_w - \bar{p})q'_w(p_w)p'_w(r) + (p_s - \bar{p})q'_s(p_s)p'_s(r)}_{\text{Misallocation effect}} + \underbrace{(\bar{p} - c)Q'(r)}_{\text{(Value of) output effect}} .$$

- The first two terms (marginal misallocation effects) are equal zero at $r = 0$ and is negative for $r > 0$.
- When r = 0 or at the $\bar{p}$, the marginal welfare effect is proportional to the marginal change in aggregate output.
- Integrating (3) over $[0, r^*]$ gives the total welfare effect as two negative terms (the total misallocation effect) plus $(\bar{p} - c)$ times the total output change → output increase is necessary for social welfare to rise

II The Effect of Discrimination on Welfare

- Let $z(p)$ is the ratio of W′(r) to π(p)″ such that:

$$z(p) \equiv \frac{(p-c)q'(p)}{2q' + (p-c)q''} = \frac{p-c}{2-L\alpha},$$

- **The increasing ratio condition** (**IRC**): $z(p)$ is increasing in p in each market (for Proposition 1-3). This holds for a very large set of demand functions.
- IRC ensures Welfare varies monotonically with r or else has a peak

- Substituting (1), into (2) and using (4) gives the marginal welfare effect:

$$(5) \qquad W'(r) = \underbrace{\left(\frac{-\pi''_w \pi''_s}{\pi''_w + \pi''_s}\right)}_{>0} \boxed{[z_w(p_w(r)) - z_s(p_s(r))]}.$$

?

- The marginal welfare effect thus *has the same sign* as $[z_w(p_w(r)) - z_s(p_s(r))]$

LEMMA

LEMMA: *Given the IRC, if there exists* $\hat{r}$ *such that* $W'(\hat{r}) = 0$ *then* $W''(\hat{r}) < 0$.

PROOF:

From (5).

$$W''(r) = \left(\frac{-\pi''_w \pi''_s}{\pi''_w + \pi''_s}\right) [z'_w p'_w - z'_s p'_s] + [z_w - z_s] \frac{d}{dr}\left(\frac{-\pi''_w \pi''_s}{\pi''_w + \pi''_s}\right),$$

which is negative if $W' = 0$ because $z'_w p'_w < 0$ and $z'_s p'_s > 0$, and $z_w = z_s$ where $W' = 0$.

- The IRC implies *W*(*r*) is *strictly quasi-concave* and thus is *monotonic in r* or *has a single interior peak*
- Three possible outcomes:
  1. W (r) is everywhere decreasing
  2. W (r) is everywhere increasing, or
  3. W (r) first rises then falls.
- Which holds depends on the signs of $W'(0)$ and $W'(r^*)$.

1. r increases, W drops

- First, if $W'(0) \le 0$, then $W(r)$ is decreasing for $r > 0$ and discrimination therefore reduces welfare.

PROPOSITION 1: *Given the IRC, if the direct demand function in the strong market is at least as convex as that in the weak market at the nondiscriminatory price then discrimination reduces welfare.*

PROOF:
The Lemma implies that discrimination reduces welfare if $W'(0) \le 0$. At the nondiscriminatory price, where $r = 0$, $p_w - c = p_s - c$ and $L_w = L_s$. So from (5), $[z_w(\bar{p}) - z_s(\bar{p})]$ and hence $W'(0)$ have the sign of $[\alpha_w(\bar{p}) - \alpha_s(\bar{p})]$, the difference in curvatures of direct demand, which is non-positive under the condition stated in the proposition.

- $\alpha_s(\bar{p}) \ge \alpha_w(\bar{p})$ means that $\alpha_s(\bar{p})$ should be greater (more convex) than $\alpha_w(\bar{p})$, or equal (as convex)
- In this case, at $\bar{p}$, where the marginal misallocation effect is 0, locally output sign is ≤ 0 (same as welfare effect)

- Proposition 1 encompasses the results of Cowan (2007), who has demand in the strong market being an affine transformation of demand in the weak market, i.e., $q_s(p) = M + Nq_w(p)$ where M and N are positive (and demand in both markets is zero at a sufficiently high price)

Proposition 1

- Proposition 1 is more general because it allows the demand functions to have different parameters or different functional forms, as in the following example

Example 1: Exponential and linear demands.

- Demand in market 1 is $q_1(p) = Be^{-p/b}$ *where* $B, b > 0$, so:

  $\sigma_1 = 1$, $\alpha_1 = \eta_1 = \frac{p}{b} > 0 \rightarrow p_1^* = b + c$

- Demand in market 2 is $q_2(p) = a - p$, so:

  $\eta_2(p) = \frac{p}{(a-p)}$, $\alpha_2 = \sigma_2 = 0$ and $p_2^* = \frac{(a+c)}{2}$

- Proposition 1 applies if $p_2^* > p_1^* \rightarrow b > \frac{(a+c)}{2}$, the condition for market 1 to be the strong one.
- The weak market is served with nondiscriminatory pricing if (but not only if) $a > b + c$.
- If discrimination is to raise welfare, given the IRC, direct demand in the weak market must be strictly more convex than demand in the strong market at the nondiscriminatory price → total output to rise

2. r increases, W rises

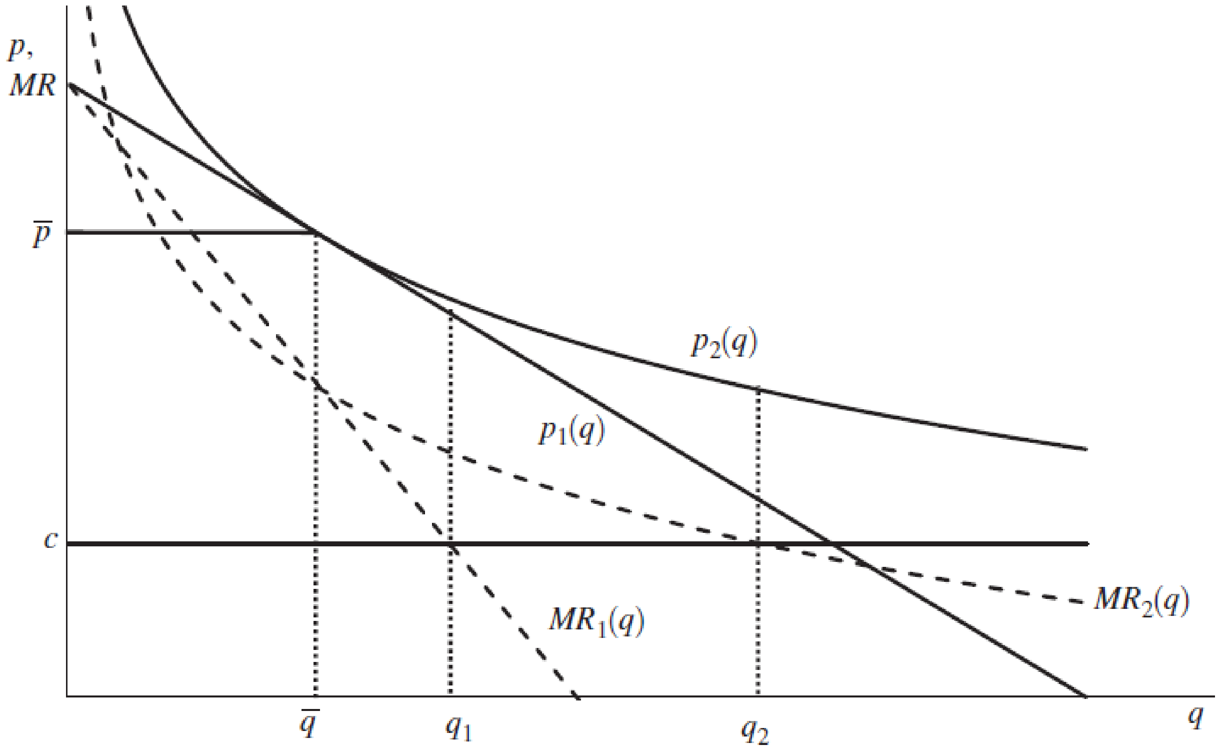

FIGURE 1. THE EFFECT OF INCREASED DEMAND CURVATURE IN THE WEAK MARKET

- At $q > \overline{q}$:
  - $MR_2(q) > MR_1(q)$ because $p_2 > p_1$
  - Given q increases, p_2 experiences a smaller price reduction

- Changes in W when $r > 0$ is larger with the transformed function because:
  1. The output increase is greater at every $q > \overline{q}$ Marginal social value of output is higher

- Summary: discrimination raises Welfare overall when the *demand* in the weak market becomes *more convex*

as r increases, W rises

PROPOSITION 2: *Given the IRC, if* $(p_w^* - c)/(2 - \sigma_w^*) \geq (p_s^* - c)/(2 - \sigma_s^*)$ *(so inverse demand in the weak market is more convex than that in the strong market at the discriminatory prices, which are close together) then welfare is higher with discrimination.*

PROOF:

The Lemma implies that discrimination increases welfare if $W'(r^*) \geq 0$. From (5) and the fact that $L^*\alpha^* = \alpha^*/\eta^* = \sigma^*$ the left-derivative of $W(r)$ at r^* has the same sign as

$$z_w(p_w^*) - z_s(p_s^*) = \frac{p_w^* - c}{2 - \sigma_w^*} - \frac{p_s^* - c}{2 - \sigma_s^*},$$

which is non-negative under the stated condition.

For discrimination to raise welfare, we should have $\sigma_w^*(p_w^*) \geq \sigma_s^*(p_s^*)$ and with a small price discrimination.

W rises, then falls

PROPOSITION 3: *Given the IRC, if* (i) *direct demand in the weak market is more convex than demand in the strong market at the nondiscriminatory price, and* (ii) *inverse demand in the strong market is at least as convex as that in the weak market at the discriminatory prices, then welfare rises initially as the degree of discrimination increases, and then falls.*

PROOF:

Condition (i), which is the opposite of the condition in Proposition 1, implies $W'(0) > 0$. Condition (ii), which negates the condition in Proposition 2, implies $W'(r^*) < 0$. From the Lemma, there is a unique $\hat{r} \in (0, r^*)$ at which $W'(\hat{r}) = 0$. For $r < \hat{r}$ we have $W'(r) > 0$, while $W'(r) < 0$ for $r > \hat{r}$.

III The Effect of Discrimination on Output

- Use approach of Holmes (1989), which follows Schmalensee (1981).
- The IRC is no longer necessary.
- Total output, as a function of the allowed amount of discrimination, is $Q(r) = q_w(p_w(r)) + q_s(p_s(r))$.
- Using the comparative statics formulate for prices, (1), the marginal output effect is:

$$(6) \qquad Q'(r) = \underbrace{\left(\frac{-q'_w q'_s}{\pi''_w + \pi''_s}\right)}_{>0} [L_w \alpha_w - L_s \alpha_s],$$

- So, the sign of Q'(r) is the same as the sign of $[L_w \alpha_w - L_s \alpha_s]$

- which has the sign of:

$$(7) \qquad L_w \alpha_w - L_s \alpha_s = L_w \eta_w (\sigma_w - \sigma_s) + (L_w \eta_w - L_s \eta_s) \sigma_s.$$

At the nondiscriminatory price, $Q'(0) \overset{sgn}{=} \alpha_w(\overline{p}) - \alpha_s(\overline{p})$. With discriminatory pricing $L\eta = 1$ in each market so $Q'(r^*) \overset{sgn}{=} \sigma_w(q_w^*) - \sigma_s(q_s^*)$.

III The Effect of Discrimination on Output

PROPOSITION 4: (i) *If demand is concave in the strong market and less concave, or convex, in the weak market then output increases with discrimination.* (ii) *If demand is convex in the strong market and concave, or less convex, in the weak market then output decreases with discrimination.* (iii) *If inverse demands are convex, and more so in the weak market, then output increases with discrimination, while* (iv) *if inverse demands are concave, and more so in the weak market, output falls with discrimination.*

- The four results in Proposition 4 are usefully summarized in the two following statements.
    1. If *both direct demand and inverse demand* are *more convex in the weak market* than in the strong market, so (i) or (iii) holds, *total output rises*.
    2. If *both direct demand and inverse demand* are *more (or equally) convex in the strong market* than in the weak market, so (ii) or (iv) applies, *total output does not increase*.

IV Constant Curvature of Inverse Demand

- Welfare effects of discrimination can be gained using two additional techniques that are useful when inverse demand curvature is constant.

1. Using a quantity restriction technique (t)

- Let $\bar{q}_i \equiv q_i(\bar{p})$ be the quantity sold in market *i* when the uniform price is charged
- Define $\Pi_i(q_i) = \pi_i(p_i(q_i))$ as profit as a function of quantity.
- To ensure profit function is concave, assume MR is declining in q in each market, which holds when $\sigma < 2$.
- Problem of maximizing $\Sigma_i \Pi_i(q_i)$ subject to a limit $t \geq 0$ on how much quantities can vary relative to their nondiscriminatory levels:

$$-p'_w(\bar{q}_w)(q_w - \bar{q}_w) \leq -p'_s(\bar{q}_s)(q_s - \bar{q}_s) + t.$$

- Constrained-optimal quantities then satisfy

$$\frac{\Pi'_w(q_w(t))}{p'_w(\bar{q}_w)} + \frac{\Pi'_s(q_s(t))}{p'_s(\bar{q}_s)} = 0. \tag{8}$$

IV Constant Curvature of Inverse Demand

- When $t = 0$ the firm chooses the quantities that are sold at the uniform price $\overline{p}$.
- As t rises the firm increases the quantity in the weak market and cuts supply to the strong market so prices move toward their discriminatory levels
- From (8) it follows that, as more quantity variation is allowed, $W'(t)$ has the sign of:

$$(9) \qquad \delta(t) \equiv \left(\frac{p_w - c}{2 - \sigma_w}\right) \frac{p_w'(\overline{q}_w)}{p_w'(q_w)} - \left(\frac{p_s - c}{2 - \sigma_s}\right) \frac{p_s'(\overline{q}_s)}{p_s'(q_s)}.$$

- Note that $\delta(0)$ simply has the sign of $\sigma_w - \sigma_s$ evaluated at the quantities at price $\overline{p}$
- For $t > 0$, we assume that the curvatures of inverse demand σ_i are constant

PROPOSITION 5: *With constant curvature of inverse demand:* (i) *If* $1 > \sigma_s \geq \sigma_w$ *and* $L(\overline{p})\eta_w(\overline{p})\sigma_w \leq 1$, *discrimination reduces welfare.* (ii) *If* $\sigma_w \geq \sigma_s > 1$ *and* $L(\overline{p})\eta_s(\overline{p})\sigma_s \geq 1$, *discrimination raises welfare.*

Proposition 5 shows that, if the discriminatory prices are not far apart, whether welfare falls or rises depends on whether σ is below or above 1. If σ < 1 the misallocation effect outweighs the positive output effect, while if σ > 1 the output effect is strong enough to exceed the misallocation effect.

IV Constant Curvature of Inverse Demand

2. Using a quantity restriction technique (t)

- Ippolito (1980) finding in his examples that output always rises with discrimination but welfare can fall or rise
- Aguirre (2006) proves that total output rises with discrimination using an inequality due to Bernoulli
- Using Aguirre (2006) proof, we define the "harmonic mean" price ρ_i by:

$$\frac{1}{\rho_i} = \frac{m}{p_i^*} + \frac{1-m}{\overline{p}}, \tag{10}$$

  - for $i \in \{w, s\}$ and for $0 \leq m \leq 1$.
  - ρ_i is a weighted harmonic mean of the p* and , with weight m on the former.
  - Total output and welfare are respectively convex and (under the stated condition) concave in m, with zero derivatives at $m = 0$, allowing us to state:

PROPOSITION 6. *When demand functions have constant elasticities:* (i) *total output is higher with discrimination, and* (ii) *social welfare is lower with discrimination if the difference between the elasticities is at most* 1.

V Conclusions

1. The welfare effects depend on simple curvature properties of demand functions
2. Discrimination reduces welfare when the direct demand function is more convex in the high-price market
3. Welfare is higher with discrimination when inverse demand in the low-price market is more convex than that in the other market and the price difference with discrimination is small
4. New analysis of how discrimination affects total output and a synthesis of existing results on welfare and output are provided