EconS 594 - Industrial Organization
Homework #2 - Due date: September, 23rd 2021

1. **Merger that generate economies of scope.** Consider two firms selling differentiated goods and competing à la Cournot. The inverse demand function of every firm i is

$$p(q_i) = a - bq_i - dq_j,$$

where $i \in \{1, 2\}$, $j \neq i$, and $b > d \geq 0$ indicates that own-price effects dominate cross-price effects. When $d = 0$ products are completely differentiated, whereas when $d \to b$ products are completely homogeneous. The total cost function of every unmerged firm i is

$$C(q_i) = \frac{c}{2}q_i^2$$

When firms merge and coordinate output, total cost of the merged firm becomes

$$C(q_1, q_2) = \frac{c}{2}\left(q_1^2 + q_2^2\right) - \beta q_1 q_2$$

where $a > c > \beta$. When $\beta > 0$, the merged firm enjoys cost efficiencies in jointly producing the goods, which is commonly known as "economies of scope." However, when $\beta < 0$, the merged firm suffers cost inefficiencies, which is referred to as "diseconomies of scope." Finally, when $\beta = 0$, the merged firm does not enjoy any cost complementarities.

   (a) *Before the merger.* Set up firm i's profit maximization problem and solve for equilibrium output and profits. How do they change in parameters a, b, c, and d? Explain.

   (b) *After the merger.* Find equilibrium output and profits when firms merge into a monopoly. Compared to $\beta = 0$, how are your results affected when $\beta < 0$ and when $\beta > 0$?

   (c) *Merger incentives.* Do firms have incentives to merge into a monopoly? Explain.

2. **Mergers between firms selling differentiated products, based on Gelves (2014).**[1] Consider two firms producing differentiated products facing an inverse demand function

$$p_i(q_i) = 1 - q_i - \gamma q_j,$$

where $\gamma \in (0, 1)$ represents the degree of product differentiation. When $\gamma = 0$, products are completely differentiated, but when $\gamma = 1$, products are completely homogeneous. Assume that firm 1 has a higher marginal production cost than firm 2, that is, $c_1 > c_2 \geq 0$.

   (a) Find equilibrium output, profits, and social welfare if firms compete à la Cournot.

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[1] Gelves, J. A. (2014). Differentiation and Cost Asymmetry: Solving the Merger Paradox. *International Journal of the Economics of Business, 21*(3), 321-340.

(b) Find equilibrium output, profits, and social welfare if firms merge.

(c) *Cost symmetry.* Assume that costs satisfy $c_1 = c_2 = 0$. How does welfare change with γ? Is the merger welfare improving? Explain.

(d) Assume that $\gamma = 1$ and $c_2 = 0$. When will the merger be welfare improving? Explain.