Can Incomplete Information Lead to Under-exploitation in the Commons?*

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Abstract

This paper analyzes the protection of a common pool resource (CPR) through the management of information. Specifically, we examine an entry deterrence model between an incumbent perfectly informed about the initial stock of a CPR and an uninformed potential entrant. In our model, the appropriation of the CPR by the incumbent reduces both players' future profits from exploiting the resource. In the case of complete information, we show that the incumbent operating in a high-stock common pool overexploits the CPR during the first period since it does not internalize the negative external effect that its first-period exploitation imposes on the entrant's future profits. This inefficiency, however, is absent when the commons totally regenerate across periods. Under incomplete information, we identify an additional form of inefficiency. In particular, the incumbent operating in a low-stock CPR underexploits the resource in order to signal the low available stock to potential entrants, deterring entry.

Keywords: Common Pool Resources; Signaling games; Externalities.

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1 Introduction

The "tragedy of the commons" has been analyzed by scholars in different disciplines. Specifically, the "tragedy" examines how open access common pool resources (CPR), such as fishing grounds, forests and water systems are prone to overexploitation. Indeed, users do not internalize the external effect that their independent decisions impose on other agents also exploiting the commons, leading to an overuse of the resource.[1] As a result, multiple studies focus on how to prevent the overexploitation of the commons by analyzing the CPR game as part of a larger environment in which agents interact and examining whether agents select socially optimal actions.

This paper follows a similar approach by analyzing the CPR game within a context of incomplete information among players. We investigate under which conditions this informational setting helps prevent the "tragedy of the commons." In particular, we consider an incumbent who privately observes the commons' initial stock and an entrant who infers the level of the stock by observing the incumbent's previous exploitation, deciding then whether or not to join the CPR. This environment describes multiple CPRs which are initially operated by an incumbent, who usually gathers more accurate information about the available stock than potential entrants. For instance, Pinkerton and Ramirez [2] study CPRs in seven coastal fishing communities in Loreto (Mexico), where local fishers have access to more precise information about the state of the stock than those located at different fishing grounds, who base their entry decision upon the incumbents' actions. Our paper analyzes agents' use of the resource in these informational contexts by focusing on how the incumbent's exploitation of the CPR can convey or conceal information about the actual stock to potential entrants. In addition, we investigate under which conditions the incumbent's incentives to deter entry can serve as a tool to actually promote the conservation of the resource.

As our benchmark, we first study equilibrium appropriation under complete information. When the initial stock is low, the entrant does not join the CPR. The incumbent is hence the only agent exploiting the resource across time, fully internalizing the negative effect that an increase in first-period exploitation causes on its own future profits. In this case, the incumbent exploits the resource at the socially optimal level. In contrast, when the initial stock is high the entrant joins the CPR and both incumbent and entrant compete for the resource in the second period of the game, leading to the standard overexploitation result in CPR games, i.e., the "tragedy of the commons" emerges. Furthermore, we identify an *additional* form of inefficiency. In particular, the incumbent does not internalize the negative external effect of its first-period appropriation on the entrant's second-period profits. Hence, the resource is overexploited not only in the second but also in the first period.

We then introduce incomplete information in the CPR game. First, we show that in the separating equilibrium the incumbent's first-period appropriation conveys information about the

[1] Note that agents exploiting a CPR hence share similar incentives with those competing in a prisoner's dilemma game, or in a public good game as in Bergstrom et al. [1]. In particular, the equilibrium of the game does not necessarily coincide with the Pareto optimum for the group.

actual level of the stock to the potential entrant, attracting entry when the stock is high but deterring entry when it is low. In particular, when the initial stock is high entry occurs, alike in the complete information environment, inducing the incumbent to overexploit the resource in both the first and second period. Mason and Polasky [3] describe an example about the Hudson's Bay Company that can support this result. Faced with the threat of entry from French furtraders during the 18th century, the company increased beaver harvests. Rather than dissuading them from entering, French furtraders built an outpost in the area in 1741. Hence, the overexploitation of the resource by the Hudson's Bay Company could be interpreted as a signal of a high initial stock by the French furtraders.

When the initial stock is low, in contrast, we show that the incumbent's appropriation is below that of complete information. Specifically, in the separating equilibrium the incumbent facing low-stock commons *underexploits* the CPR in order to deter entry. The introduction of incomplete information moves this incumbent away from the complete-information equilibrium and thus from the social optimum. The separating equilibrium hence presents the same inefficiencies as the complete information game when the stock is high, but identifies an additional inefficiency —associated with the underexploitation of the commons— when the stock is low. Importantly, this inefficiency is novel in the literature of CPRs and arises from our incomplete information setting, where the incumbent operating in a low-stock common pool conveys the state of the stock to potential entrants in order to prevent entry. The case of the silver hake provides an interesting example of this type of informative signaling. After two decades of intense exploitation by mechanized U.S. and Canadian fishing boats in the North Atlantic from 1960 to 1980, the available stock became significantly depleted. This low stock led to a reduction in the number of vessels and annual catches. More importantly, incumbent fisheries have consistently underexploited the resource below its annual sustainable catch since the late 1990s; see United Nations Food and Agriculture Organization [4]. Such strategy can be interpreted as a signal to potential entrants, informing them that the stock, despite experiencing a mild recovery, has not yet become sufficiently high to support the entry of additional vessels.[2]

When both types of incumbent choose the same first-period exploitation (in the pooling equilibrium) no information is revealed to the entrant deterring entry. This result suggests that the incumbent operating a high-stock commons can deter entry as if it owned a property right for the use of the resource. Therefore, the informational asymmetry among players acts in this case as an "implicit protection right" for the incumbent. We then evaluate the efficiency properties of this equilibrium outcome. In the second period, we find that the "tragedy of the commons" does not emerge since the incumbent is still the only agent exploiting the resource. In the first period we

[2] Underexploitation has also been reported in several other fishing grounds. For instance, Haughton [5] highlights the underuse of blackfin tuna, dolphinfish and diamond back squids, among others, in the Caribbean region. Similarly, a comprehensive study by the United Nations Food and Agriculture Organization [4] indicates the underexploitation of the Argentine anchovy in the Southern Atlantic and the yellowfin sole in the Pacific Northwest. The underexploitation observed in the previous examples could be explained by the difficulty of access or the fishing technology. Our paper suggests that incomplete information can potentially exacerbate this underexploitation.

show that the pooling exploitation level coincides with the social optimum when the initial stock is low. When the initial stock is high, however, the pooling equilibrium lies below the social optimum, and hence the high-stock incumbent *underexploits* the resource during the first period.

We finally compare the efficiency properties of separating and pooling equilibria. When the initial stock is high, we show that the separating equilibrium supports an overexploitation of the commons, while the pooling equilibrium predicts an underexploitation of the resource. A precise policy recommendation would hence depend on which type of inefficiency (under or overexploitation) society prefers to avoid the most. If social preferences assign a larger welfare loss to the overexploitation than to the underexploitation of the commons, then our results imply that environmental regulators would increase social welfare by promoting the pooling equilibrium, e.g., setting a quota. Otherwise, the separating equilibrium becomes welfare improving. This policy makes the separating equilibrium less attractive for the incumbent, inducing it towards pooling equilibrium appropriation levels. Our findings hence provide an additional role for quotas, a policy tool often used to deal with CPRs.

Several studies examine under which circumstances the tragedy of the commons is ameliorated. The main approaches can be grouped into two broad categories, where studies either: (1) modify individual payoffs so that agents' strategic incentives become different from those in a CPR game, Ostrom [6] and Ostrom et al. [7]; or (2) insert the unmodified CPR game into an enlarged structure, e.g., allowing for the game to be repeated along time, Baland and Platteau [8].[3] This paper contributes to the second approach by introducing incomplete information in a CPR game. Other authors have theoretically and experimentally analyzed uncertainty regarding the profitability of the CPR; see Suleiman and Rapoport [15], Suleiman et al. [16] and Apesteguia [17]. Unlike our paper, this literature considers that all players have access to the same information about the resource, thus not allowing for informational asymmetries among players. Our study offers hence two advantages: first, it examines informational settings where an incumbent holds more accurate information about a resource than potential entrants, which might apply to many CPRs such as fisheries. Second, the results show under which conditions the incumbent might choose to actually overprotect the commons, because such overprotection deters entry.

Our paper also contributes to the literature on entry deterrence in the commons, as Mason and Polasky [3], who assume complete information among players. By allowing for incomplete information and signaling, we compare equilibrium behavior under complete and incomplete information. Therefore, this paper relates to the literature on entry deterrence in signaling games. Usual entry deterrence models assume that the incumbent's first-period action (e.g., price setting by a monopolist) does not affect incumbent and entrant's future profits; see Milgrom and Roberts

[3] Building upon the seminal work of Hotelling [9] and Hardin [10], several studies analyze CPR games in a dynamic context under complete information; see Levhari and Mirman [11], Reinganum and Stokey [12] and Dutta and Sundaram [13]. For a comprehensive review of the CPR literature see Faysee [14].

[18], Matthews and Mirman [19] and Bagwell and Ramey [20]. In our model, in contrast, the incumbent's first-period exploitation depletes the CPR, thus affecting its second-period appropriation and reducing its profits. More importantly, it also affects the entrant's second-period profits, thus imposing a negative external effect on the entrant, unlike Polasky and Bin [21] where agents do not compete for the same stock in the commons. This paper hence provides an explicit analysis of signaling games where agents' actions cause external effects, and compares it with signaling models where externalities are absent. Therefore our study is in the line of signaling games in which one player's first-period actions affect another player's second-period profits, such as Spence [22, 23], where a worker's education can raise his second-period productivity, thus increasing the firm's second-period profits afterwards.

The following section describes the model. Section three examines the equilibrium under complete information. Section four introduces the signaling game and compares exploitation levels under both informational contexts. Finally, section five concludes.

2 Model

Consider a common pool resource (CPR), such as fishing grounds or forests, where an incumbent (Firm 1) initially exploits the commons and an entrant (Firm 2) analyzes whether or not to enter. There are no entry barriers and the initial stock of the CPR is either low or high, $\theta_K = \{\theta_L, \theta_H\}$. We first analyze the case where entrant and incumbent are informed about the CPR's initial stock, and afterwards the case in which the entrant is uninformed. For compactness, let us thereafter refer to the incumbent operating in a low (high) stock common pool as the low-stock (high-stock) incumbent. In particular, consider a two-stage game where, in the first stage, the incumbent operates as a monopolist and decides its appropriation level $x_1 > 0$. In the second stage of the game a potential entrant, observing the incumbent's appropriation level in the first period, chooses whether or not to join the incumbent. If entry occurs, agents compete for the CPR and simultaneously select their appropriation levels q_1 and q_2, for the incumbent and entrant, respectively.

First stage. In the first stage of the game, the incumbent appropriates x_1, with an associated total cost of $c(x_1, \theta_K)$, which is increasing and convex in appropriation, i.e., $c_{x_1} > 0$ and $c_{x_1 x_1} > 0$. In addition, the marginal cost of appropriation is decreasing in the available stock θ_K, i.e., $c_{x_1\theta} \leq 0$. Henceforth, we assume that all functions are continuous and differentiable in all arguments. For simplicity, we assume that incumbent and entrant sell their appropriation in an international market, where their sales represent a small share of the total market for the good.[4] Hence, during the first period the incumbent is the only agent exploiting the resource, obtaining monopoly profits of $M_1^K(x_1) \equiv x_1 - c(x_1, \theta_K)$ where $K = \{H, L\}$.

[4] Alternatively, both agents sell their appropriation at a price p, normalized to one during both periods.

Second stage, No entry. During the second period, if entry does not occur, the incumbent appropriates $q_1 > 0$ obtaining the following monopoly profits

$$\overline{M}_1^K(q_1; x_1) \equiv q_1 - c^1(q_1, x_1, \theta_K, \beta) \quad \text{where } K = \{H, L\},$$

where $c^1(q_1, x_1, \theta_K, \beta)$ is the incumbent's second-period total cost, which depends on its appropriation during that period, q_1, first-period appropriation, x_1, the initial stock, θ_K, and the regeneration rate of the CPR, $\beta \in [0, 1]$. On one hand, $\beta < 1$ indicates that the regeneration rate of the CPR does not compensate the reduction of the initial stock (biological regeneration does not offset first-period appropriation). In this case, an increase in first-period appropriation, x_1, reduces the amount of available stock in the second period, increasing as a consequence the incumbent's second-period marginal costs from appropriation, i.e., $c^1_{q_1 x_1} > 0$. On the other hand, $\beta = 1$ illustrates that the regeneration rate exactly compensates the reduction of the initial stock (biological regeneration offsets first-period appropriation),[5] i.e., $c^1_{q_1 x_1} = 0$. Similarly to the first period, appropriation cost is increasing and convex, i.e., $c^1_{q_1} > 0$ and $c^1_{q_1 q_1} > 0$. Furthermore, the marginal product of the first unit of appropriation is larger than its associated marginal cost, i.e., $1 > c^1_{q_1}(0, x_1, \theta_K, \beta)$. Finally, first-period appropriation increases second-period costs, $c^1_{x_1} > 0$, at an increasing rate, $c^1_{x_1 x_1} > 0$, where such increase diminishes in the initial stock, i.e., $c^1_{x_1 \theta} < 0$.

Second stage, Entry. If entry occurs in the second period, incumbent and entrant compete for the common resource. Agents' profits when competing as duopolists are

$$D_i^K(q_i, q_j; x_1) \equiv q_i - z^i(q_i, q_j, x_1, \theta_K, \beta) \quad \text{where } K = \{H, L\},$$

for both players $i = \{1, 2\}$ and $j \neq i$. In particular, when appropriation levels for incumbent and entrant are q_1 and q_2, respectively, each player's total cost becomes $z^i(q_i, q_j, x_1, \theta_K, \beta)$, allowing for different cost efficiencies between the incumbent and the entrant, since costs functions $z^1(\cdot)$ and $z^2(\cdot)$ can differ. Distinct efficiencies can arise, for instance, when the incumbent enjoys a technological advantage from its experience exploiting the commons. Total costs after entry, $z^i(\cdot)$, satisfy the same properties as $c^1(\cdot)$ indicated above. Additionally, every agent's cost of appropriation, $z^i(\cdot)$, increases in the other agent's appropriation level, i.e., $z^i_{q_j} > 0$, illustrating agents' competition for the CPR. Hence, for a given positive appropriation level, the incumbent's second-period profits under monopoly are larger than under duopoly.

3 Complete information

Second stage, No entry. Let us start examining the second period of the game. In the case that no entry occurs, the incumbent chooses an appropriation level q_1 that maximizes its second-period

[5] We do not consider cases of strong biological regeneration. In particular, this could lead the initial stock to grow from θ_L to θ_H. In this case, the second-period stock would be high under all parameter conditions, supporting entry as a result, and hence dissipating the role of signaling in our model.

monopoly profits

$$\overline{M}_1^K(x_1) \equiv \max_{q_1 \geq 0} q_1 - c^1(q_1, x_1, \theta_K, \beta)$$

where $q_1^m(x_1, \theta_K)$ is the profit-maximizing appropriation level.[6] Note that $\overline{M}_1^K(x_1)$ is decreasing in first-period appropriation x_1 since, using the envelope theorem, we find that $\frac{d\overline{M}_1^K(x_1)}{dx_1} = -c_{x_1}^1(q_1^m, x_1, \theta_K, \beta)$, which is negative by definition. Intuitively, the incumbent is negatively affected by its first-period exploitation of the resource, x_1. Indeed, a larger appropriation reduces the available stock at the beginning of the second period, increasing marginal costs and hence decreasing profits. In addition, using the implicit function theorem, $\frac{\partial q_1^m}{\partial x_1} = -\frac{c_{q_1 x_1}^1}{c_{q_1 q_1}^1} < 0$, indicating that the incumbent's equilibrium appropriation level when entry does not occur decreases in its previous exploitation of the CPR.

Second stage, Entry. In the case that entry occurs, both firms compete in a duopoly. Hence, for a given equilibrium appropriation level of agent j, $q_j^d(x_1, \theta_K)$, agent $i \neq j$ appropriates the level q_i that maximizes its second-period profits under duopoly

$$D_i^K(x_1) \equiv \max_{q_i \geq 0} q_i - z^i(q_i, q_j^d(x_1, \theta_K), x_1, \theta_K, \beta)$$

Let $q_i^d(x_1, \theta_K)$ denote the profit-maximizing appropriation level for player $i = \{1, 2\}$. Therefore, the entrant decides to enter if its profits, $D_2^K(x_1)$, are weakly higher than those from staying out, which for simplicity we assume to be zero, i.e., $D_2^K(x_1) \geq 0$. Similarly to the case of no entry, $\frac{\partial q_i^d}{\partial x_1} = -\frac{z_{q_i x_1}^i}{z_{q_i q_i}^i} < 0$. Using the envelope theorem on the incumbent's equilibrium second-period profits after entry,

$$\frac{dD_1^K(x_1)}{dx_1} = -\frac{\partial z^1}{\partial x_1} - \frac{\partial z^1}{\partial q_2}\frac{\partial q_2^d}{\partial x_1}. \tag{1}$$

The above expression describes a negative and positive effect on duopoly profits. The first component illustrates the increase in the incumbent's second-period costs z^1 due to a larger first-period appropriation, x_1, causing a decrease in profits, i.e., direct effect. In contrast, the second component reflects a strategic effect, since an increase in first-period appropriation reduces the available stock at the second period, thus decreasing the entrant's exploitation of the resource, q_2^d. This reduction in the entrant's appropriation produces a decrease in the incumbent's costs, z^1, for a given appropriation level q_1^d, increasing the incumbent's profits as a result. For simplicity, we assume that the negative (direct) effect on profits dominates the positive (strategic) effect. As a consequence, a marginal increase in the incumbent's first-period appropriation, x_1, produces an overall *decrease* in its equilibrium profits when entry ensues, i.e., $D_1^K(x_1)$ decreases in x_1. Intuitively, this occurs when the regeneration rate is low enough, inducing $\frac{\partial z^1}{\partial x_1}$ to be significantly negative.[7] If

[6] Optimal effort $q_1^m(x_1, \theta_K) > 0$ exists since $1 > c_{q_1}^1(0, x_1, \theta_K, \beta)$ by definition. In addition, it is unique given that $0 \leq c_{q_1 q_1}^1$ by convexity of the cost function. Existence and uniqueness is also guaranteed for the case of entry.

[7] Note that if condition (1) were positive, an increase in first-period appropriation would actually *increase* the incumbent's second-period profits. Since we are mainly interested in analyzing the incumbent's trade-off between first and second-period profits when it increases x_1, we only consider the case in which future profits are negatively

instead, the CPR totally regenerates across periods ($\beta = 1$), then second-period costs are unaffected by first-period appropriation, $\frac{\partial z^1}{\partial x_1} = 0$, and the entrant's exploitation of the commons is unaltered either, i.e., $\frac{\partial q_2^d}{\partial x_1} = 0$, making $D_1^K(x_1)$ constant in x_1. A similar analysis can be conducted for the entrant's equilibrium profits, $D_2^K(x_1)$. To make the entry decision interesting, assume that $0 > D_2^L(x_1)$, reflecting that entry does not occur when the initial stock is low, and $D_2^H(x_1) > 0$, illustrating that entry ensues when the stock is high for all x_1.

First stage, No entry. Given equilibrium appropriation levels in the second stage of the game, the incumbent chooses the first-period appropriation x_1 that maximizes its first and second-period profits. In particular, if entry does not occur, the incumbent profits across both periods are $x_1 - c(x_1, \theta_K) + \delta \overline{M}_1^K(x_1)$, where $\delta \in (0, 1)$ denotes the incumbent's discount factor. Hence, a marginal increase in its first-period appropriation x_1 produces, on one hand, a marginal benefit of $MB = 1$ from additional appropriation during the first period (by the envelope theorem). On the other hand, increasing x_1 induces a marginal cost of $MC^m(x_1, \theta_K) = c_{x_1}(x_1, \theta_K) + \delta c_{x_1}^1(q_1^m, x_1, \theta_K, \beta)$ illustrating the increase in first- and second-periods costs. Furthermore, assume that the marginal benefit from the first unit of appropriation is larger than its corresponding marginal cost, i.e., $MB > MC^m(0, \theta_K)$ evaluated at $x_1 = 0$. The marginal cost when no entry occurs, $MC^m(x_1, \theta_K)$, is especially relevant for the low-stock incumbent, who anticipates that entry does not occur since $0 > D_2^L(0) > D_2^L(x_1)$. Therefore, the incumbent with low stocks chooses x_1 such that

$$\max_{x_1 \geq 0} M_1^L(x_1) + \delta \overline{M}_1^L(x_1) \tag{2}$$

Let $x_1^{L,NE}$ denote the solution to the above maximization problem with low stocks, where NE represents that no entry occurs.[8]

First stage, Entry. If instead entry occurs, the incumbent faces profits of $x_1 - c(x_1, \theta_K) + \delta D_1^K(x_1)$. In this case, an increase in first period appropriation x_1 produces a marginal benefit of $MB = 1$ which arises from an additional first-period appropriation. Furthermore, an increase in x_1 generates a marginal cost of $MC^d(x_1, \theta_K) = c_{x_1}(x_1, \theta_K) - \delta \frac{dD_1^K(x_1)}{dx_1}$ in the first and second period, where $\frac{dD_1^K(x_1)}{dx_1}$ indicates the loss in duopoly profits. We consider that an increase in x_1 produces a significant increase in first- and second-period marginal costs, but only a relatively small increase in the strategic effect.[9] Hence, $MC^d(x_1, \theta_K)$ is increasing in x_1. In addition, we assume that a given increase in first-period appropriation produces a larger increase in second-period cost in the case of no entry, where the incumbent bears all the negative effect of first-period appropriation, than in the case of entry, where such appropriation affects both agents, i.e., $c_{x_1}^1 > z_{x_1}^1$, which

affected by first-period appropriation.

[8] Existence of $x_1^{L,NE}$ is guaranteed since $MB > MC^m(0, \theta_K)$, where both expressions are evaluated at $x_1 = 0$. Uniqueness is satisfied since a given increase in x_1 produces $MB_{x_1} = 0 \leq MC_{x_1}^m = c_{x_1 x_1} + \delta c_{x_1 x_1}^1$.

[9] Specifically, this implies that $MC_{x_1}^d(x_1, \theta_K) = c_{x_1 x_1} + z_{x_1 x_1}^1 + \frac{\partial z^1}{\partial q_2 \partial x_1} \frac{\partial q_2^d}{\partial x_1} + \frac{\partial z^1}{\partial q_2} \frac{\partial^2 q_2^d}{\partial x_1^2}$ is positive for all x_1, where the first two components represent the increase in first- and second-period marginal costs whereas the last two elements denote the change in the strategic effect due to higher levels of x_1.

implies $MC^d(x_1, \theta_K) < MC^m(x_1, \theta_K)$. Similarly, an increase in the initial stock θ produces a larger decrease in the incumbent's costs under no entry, since the incumbent fully benefits from a more abundant resource, than under entry, i.e., $\left|c_\theta^1\right| > \left|z_\theta^1\right|$.

Provided that entry occurs when the initial stock is high, the incumbent chooses x_1 such that

$$\max_{x_1 \geq 0} M_1^H(x_1) + \delta D_1^H(x_1) \tag{3}$$

Let $x_1^{H,E}$ denote the solution to (3), where E represents that entry occurs.[10] When the stock totally regenerates across periods, $\beta = 1$, first-period actions do not affect second-period profits. Specifically, both $MC^m(x_1, \theta_K)$ and $MC^d(x_1, \theta_K)$ coincide with $c_{x_1}(x_1, \theta_K)$, and $x_1^{K,E}$ and $x_1^{K,NE}$ increase to $x_{1,monop}^K$.

3.1 Discussion

We next evaluate the efficiency properties of our equilibrium results. In this spirit, let us first describe the social welfare function for the social planner. In particular, we assume that the social planner only considers aggregate profits across both periods. This assumption allows us to isolate the external effects that the incumbent's exploitation of the commons imposes on the potential entrant. We hence focus on those CPRs in which appropriation does not induce significant ecological costs associated to the loss of species or biodiversity.[11] The social planner thus selects an aggregate second-period appropriation of[12] $q_1^m(\cdot)$, which yields second-period aggregate profits of $\overline{M}^K(x_1)$, for all x_1. In the first period, the social planner chooses $x_1^{K,NE}$, which solves $\max_{x_1}$ $M_1^K(x_1) + \delta\overline{M}^K(x_1)$.

Let us now assess the efficiency of our equilibrium results. When no entry occurs, the incumbent is the only agent exploiting the resource during both periods, and hence it fully internalizes the effect that first-period appropriation causes on its future profits. Therefore, the resource is exploited at its socially optimal level, and the "tragedy of the commons" does not apply. The following table describes our findings under complete information when $\beta < 1$.

[10] Existence of $x_1^{H,E}$ is guaranteed since $MB > MC^d(0, \theta_H)$ after entry, evaluated at $x_1 = 0$. Uniqueness is satisfied since a given increase in x_1 produces $MB_{x_1} = 0 \leq MC_{x_1}^d$.

[11] In addition, the social planner does not consider consumer surplus since the agent/s exploiting the resource sell a relatively small share of their production in local markets.

[12] Note that the the social planner can assign the socially optimal appropriation $q_1^m(\cdot)$ to a single agent, or instead, distribute it between the incumbent and the entrant. Furthermore, such appropriation can be equally shared among the incumbent and the entrant in the case that cost functions are symmetric, i.e., each agent extracts $\frac{q_1^m(\cdot)}{2}$ when $z^i = z^j$, or unequally shared among agents when their cost functions are asymmetric, as long as aggregate appropriation sums to $q_1^m(\cdot)$.

		Complete Information
Low stock (no entry)	1^{st} *Period*	$x_1^{L,NE}$, socially optimal
	2^{nd} *Period*	q_1^m, socially optimal
High stock (entry)	1^{st} *Period*	$x_1^{H,E}$, overexploitation
	2^{nd} *Period*	$q_1^d + q_2^d$, overexploitation

Table I. Equilibrium under complete information.

In contrast, when entry occurs, appropriation levels are not socially optimal. In the second period, both agents compete for exploiting the CPR, not internalizing the external effect that their appropriation levels impose on other players in the form of lower profits. Hence, the resource is overexploited, and the "tragedy of the commons" emerges. In the first period, the high-stock incumbent's appropriation is not socially optimal either. Specifically, the high-stock incumbent selects the first-period appropriation that maximizes its profits across periods; as indicated in (3). However, the social planner would select $x_1^{H,NE}$, which lies below the first-period appropriation selected by the high-stock incumbent in equilibrium, $x_1^{H,E}$. In particular, the social planner considers the effect that a marginal increase in x_1 imposes on the entrant's equilibrium profits during the second period, while the incumbent does not.

Full regeneration. In the particular case in which the stock totally regenerates across periods, the incumbent's first-period appropriation does not impose a negative externality on the entrant's second-period profits. As a consequence, the incumbent's appropriation is socially optimal in the first period, not overexploiting the CPR. Specifically, both incumbent and social planner solve $\max_{x_1} \quad M_1^K(x_1) + \delta \overline{M}_1^K$ by selecting $x_1 = x_{1,monop}^K$. During the second period, however, every agent's appropriation still imposes a negative effect on the other agent's profits, and hence aggregate equilibrium exploitation is beyond the social optimum, reflecting the presence of overexploitation after entry. Therefore, when $\beta = 1$ the first-period's inefficiency disappears, whereas that of the second period is still present.

4 Signaling the CPR's stock

This section investigates the case where the incumbent is privately informed about the CPR's initial stock, while the entrant only observes the incumbent's first-period appropriation level using it to infer the stock's level. The time structure of this signaling game is as follows.

1. Nature decides the realization of the CPR's stock, either high or low, θ_H or θ_L, with probabilities $p \in (0, 1)$ and $1 - p$, respectively. The incumbent privately observes this realization but the entrant does not.

2. The incumbent chooses its first-period appropriation level, x_1.

3. Observing the incumbent's first-period appropriation level, the entrant forms beliefs about the initial stock of the CPR. Let $\mu(\theta_H|x_1)$ denote the entrant's posterior belief about the initial stock being high after observing x_1.

4. Given the above beliefs, the entrant decides whether or not to enter the CPR.

5. If entry does not occur, the incumbent remains the only agent exploiting the CPR, whereas if entry occurs, both agents compete for the CPR.

The following subsection examines the separating equilibrium of the game. We then investigate pooling equilibria and compare our equilibrium results according to their efficiency properties.

4.1 Separating equilibrium

Let us next analyze the separating equilibrium where the incumbent selects a particular first-period appropriation level when the stock is high, but chooses a different appropriation when the stock is low. Let x_1^H (x_1^L) denote the first-period appropriation level that the high (low, respectively) stock incumbent selects in the separating equilibrium. We assume that the separating appropriation level x_1^L does not coincide with the low-stock incumbent's appropriation under complete information, $x_1^{L,NE}$. Otherwise, the high-stock incumbent could be tempted to pool with the low-stock incumbent by selecting[13] $x_1^{L,NE}$. Entrant's equilibrium beliefs after observing equilibrium appropriation levels x_1^H and x_1^L are $\mu(\theta_H|x_1^H) = 1$ and $\mu(\theta_H|x_1^L) = 0$, respectively. The entrant enters when it infers that the initial stock is high, but stays out when it interprets that the stock is low. First, we investigate the incentive compatibility conditions that guarantee the existence of a separating equilibrium. When the stock is high, the incumbent selects the appropriation level that maximizes its profits across both periods given that entry occurs, i.e., $x_1^{H,E}$ arising from the profit maximization problem in (3), with an associated equilibrium profit of $M_1^H(x_1^{H,E})+\delta D_1^H(x_1^{H,E})$. If the high-stock incumbent deviates towards the low-stock incumbent's appropriation level x_1^L, it deters entry. Hence, the high-stock incumbent selects its equilibrium appropriation $x_1^{H,E}$ if $M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E}) \geq M_1^H(x_1^L) + \delta \overline{M}_1^H(x_1^L)$ or equivalently,[14]

$$M_1^H(x_1^{H,E}) - M_1^H(x_1^L) \geq \delta \left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E})\right] \qquad (IC_H)$$

Likewise, if the low-stock incumbent chooses the equilibrium appropriation x_1^L, it deters entry, yielding profits of $M_1^L(x_1^L) + \delta\overline{M}_1^L(x_1^L)$. If instead the incumbent deviates towards the high-stock incumbent's appropriation level, $x_1^{H,E}$, it attracts entry obtaining a profit of $M_1^L(x_1^{H,E})+\delta D_1^L(x_1^{H,E})$. Conditional on entry, the low-stock incumbent can select an off-the-equilibrium appropriation $x_1 \neq x_1^{H,E} \neq x_1^L$ that achieves a higher profit than that associated to $x_1^{H,E}$. In particular, the incumbent

[13] We analyze both players' incentives to pool using the same first-period appropriation, including $x_1^{L,NE}$, in the following section about the pooling equilibrium of the game.

[14] Incentive compatibility condition IC_H also guarantees that the high-stock incumbent does not have incentives to deviate towards any off-the-equilibrium appropriation x_1 such that $x_1 \neq x_1^{H,E} \neq x_1^L$; see proof of Proposition 1.

selects an appropriation level $x_1^{L,E}$ which maximizes its profits after entry, yielding $M_1^L(x_1^{L,E}) + \delta D_1^L(x_1^{L,E})$. Thus, the low-stock incumbent selects the equilibrium appropriation of x_1^L if $M_1^L(x_1^L) + \delta \overline{M}_1^L(x_1^L) \geq M_1^L(x_1^{L,E}) + \delta D_1^L(x_1^{L,E})$, or equivalently,

$$M_1^L(x_1^{L,E}) - M_1^L(x_1^L) \leq \delta \left[\overline{M}_1^L(x_1^L) - D_1^L(x_1^{L,E}) \right] \qquad (IC_L)$$

The following proposition describes the separating equilibrium.

Proposition 1. *The following strategy profile describes the set of separating Perfect Bayesian Equilibria (PBE) in the CPR signaling game:*

1. *In the first period, the high-stock incumbent selects $x_1^{H,E}$ and the low-stock chooses $x_1^L \in [x_1^A, x_1^B]$, where x_1^A and x_1^B solve the incentive compatibility condition for the low and high-stock incumbent, respectively, and $x_1^L < x_1^{L,NE}$;*
2. *The entrant enters only after observing an appropriation level of $x_1^{H,E}$, given equilibrium beliefs $\mu(\theta_H|x_1^{H,E}) = 1$ and $\mu(\theta_H|x_1^L) = 0$ for any $x_1^L \in [x_1^A, x_1^B]$, and off-the-equilibrium beliefs $\mu(\theta_H|x_1) = 1$ for all $x_1 \neq x_1^{H,E} \neq x_1^L$; and*
3. *In the second period of the game, the incumbent selects an appropriation $q_1^m(x_1, \theta_K)$ if entry does not occur, and every agent $i = \{1, 2\}$ chooses $q_i^d(x_1, \theta_K)$ if entry occurs.*

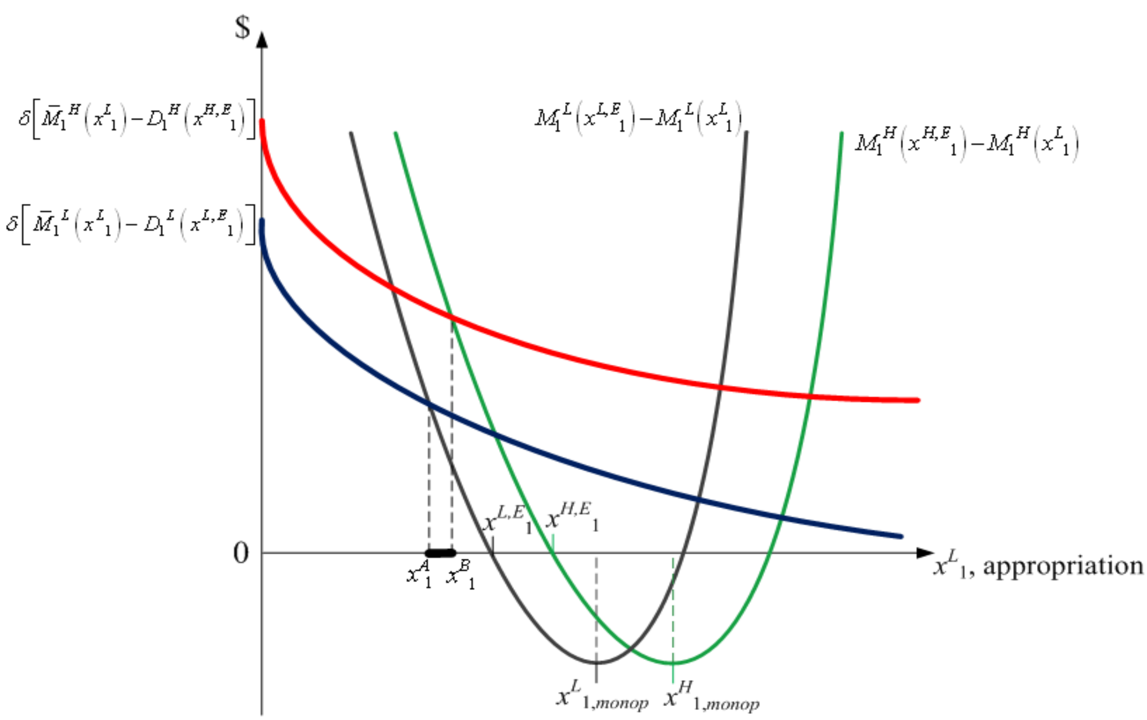

Figure 1. Separating equilibria under $\beta < 1$.

The figure above depicts the set of separating equilibria where the high-stock incumbent selects a first-period appropriation $x_1^{H,E}$, which coincides with its appropriation level under complete information. The figure also shows the low-stock incumbent's first-period appropriation $x_1^L \in [x_1^A, x_1^B]$, which is strictly below its appropriation level in the complete information context.

Specifically, the low-stock incumbent reduces its first-period appropriation level, relative to complete information, in order to convey its private information to the entrant, deterring it from exploiting the resource. The curve $\delta\left[\overline{M}_1^K(x_1^L) - D_1^K(x_1^{K,E})\right]$ represents the incumbent's entry deterrence benefits. Specifically, since $\overline{M}_1^K(x_1^L)$ is decreasing in x_1^L (due to depletion) and convex, then curve $\delta\left[\overline{M}_1^K(x_1^L) - D_1^K(x_1^{K,E})\right]$ is also decreasing and convex in x_1^L for $K = \{H, L\}$.[15] In addition, curve $M_1^K(x_1^{K,E}) - M_1^K(x_1^L)$ depicts the incumbent's loss in first-period profits from selecting appropriation levels away from that maximizing profits across both periods given entry, $x_1^{K,E}$. Note that $x_1^{K,E} < x_{1,monop}^K$, and since first-period profits are maximal at $x_{1,monop}^K$, $M_1^K(x_1^{K,E}) < M_1^K(x_{1,monop}^K)$, as represented in the negative region of the figure.

The following corollary examines the particular case in which the CPR is totally regenerated across periods, $\beta = 1$. Note that the incentive compatibility condition for the high-stock incumbent becomes

$$M_1^H(x_{1,monop}^H) - M_1^H(x_1^L) \geq \delta\left[\overline{M}_1^H - D_1^H\right] \qquad (IC'_H)$$

where second-period profits are unaffected by first-period appropriation, both under entry and no entry. As a consequence, the first-period profit-maximizing appropriation level when entry occurs becomes $x_{1,monop}^H$ rather than $x_1^{H,E}$. Similarly, the incentive compatibility condition for the low-stock incumbent is

$$M_1^L(x_{1,monop}^L) - M_1^L(x_1^L) \leq \delta\left[\overline{M}_1^L - D_1^L\right]. \qquad (IC'_L)$$

Corollary 1. *The following strategy profile describes the set of separating PBE in the CPR signaling game when $\beta = 1$:*

1. *In the first period, the high-stock incumbent selects $x_{1,monop}^H$ and the low-stock chooses $x_1^L \in [\widetilde{x}_1^A, \widetilde{x}_1^B]$, where $\widetilde{x}_1^A$ and $\widetilde{x}_1^B$ solve the incentive compatibility condition for the low and high-stock incumbent, respectively, and $x_1^L < x_{1,monop}^L$;*

2. *The entrant enters only after observing an appropriation level of $x_{1,monop}^H$ in the first period, given equilibrium beliefs $\mu(\theta_H | x_{1,monop}^H) = 1$ and $\mu(\theta_H | x_1^L) = 0$ for any $x_1^L \in [\widetilde{x}_1^A, \widetilde{x}_1^B]$, and off-the-equilibrium beliefs $\mu(\theta_H | x_1) = 1$ for all $x_1 \neq x_{1,monop}^H \neq x_1^L$; and*

[15] Appendix 2 (see technical appendix) shows this result. In addition, we demonstrate that $\delta\left[\bar{M}_1^H(x_1^L) - D_1^H(x_1^{H,E})\right]$ is above $\delta\left[\bar{M}_1^L(x_1^L) - D_1^L(x_1^{L,E})\right]$ under all parameter values. Finally, note that the negatively sloped curve $\delta\left[\overline{M}_1^L(x_1^L) - D_1^L(x_1^{L,E})\right]$ is strictly positive when evaluated at $x_1^L = x_1^{L,E}$, since $\overline{M}_1^L(x_1^{L,E}) > D_1^L(x_1^{L,E})$.

3. *In the second period of the game, the incumbent selects an appropriation $q_1^m(\theta_K)$ if entry does not occur, and every agent $i = \{1, 2\}$ chooses $q_i^d(\theta_K)$ if entry occurs.*

The total regeneration of the initial stock makes second-period profits independent on first-period appropriation. As a consequence, second-period equilibrium appropriation levels, $q_1^m(\theta_K)$ and $q_i^d(\theta_K)$, are also independent on the previous exploitation of the resource. Therefore, the high-stock incumbent selects a first-period appropriation $x_{1,monop}^H$, which maximizes its first-period monopoly profits given that second-period profits are unaffected by previous exploitation. This appropriation level coincides with the high-stock incumbent's appropriation under complete information and $\beta = 1$. The low-stock incumbent chooses an appropriation level in the interval $x_1^L \in [\widetilde{x}_1^A, \widetilde{x}_1^B]$, which is lower than the first-period appropriation that this incumbent selects under complete information, $x_{1,monop}^L$. Similarly to the case under $\beta < 1$, the low-stock incumbent reduces its first-period appropriation in order to communicate the low initial stock to the potential entrant, deterring entry as a result. The following figure illustrates the set of separating equilibria for $\beta = 1$.

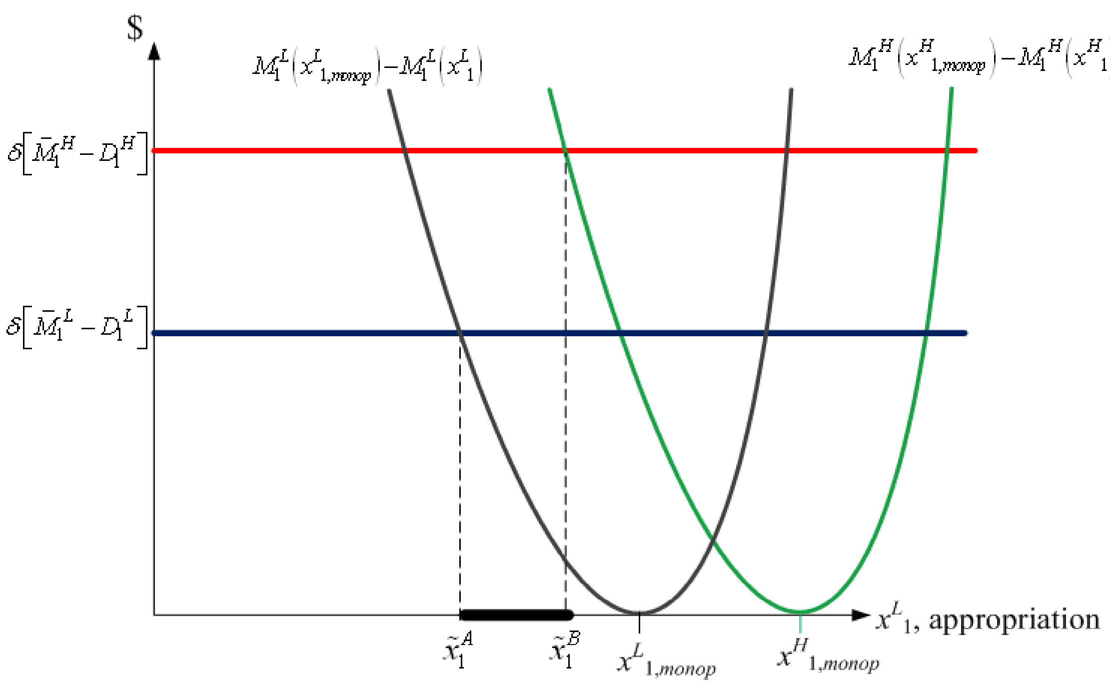

Figure 2. Separating equilibria under $\beta = 1$.

First, note that the incumbent's benefit from maintaining its monopolistic power is unaffected by its first-period appropriation x_1^L when $\beta = 1$, but it becomes decreasing when $\beta < 1$. This benefit —measured by $\delta\left[\overline{M}_1^K - D_1^K\right]$ in figure 2— is constant in x_1^L, but the equivalent benefit under $\beta < 1$, $\delta\left[\overline{M}_1^K(x_1^L) - D_1^K(x_1^{K,E})\right]$, becomes decreasing in x_1^L (see figure 1). Intuitively, an increase in first-period appropriation does not affect the incumbent's second-period profits when $\beta = 1$, since the CPR fully regenerates, both after entry and no entry. However, a lower regeneration rate ($\beta < 1$) reduces second-period monopoly profits, since fewer stock is available for exploitation, ultimately reducing the incumbent's benefit from protecting its monopolistic position. In addition, note that curve $M_1^K(x_{1,monop}^K) - M_1^K(x_1^K)$ in figure 2 experiences a downward shift when $\beta < 1$ (see equivalent curve in figure 1). Intuitively, the incumbent is willing to give up first-period profits in order to preserve the future profitability of the resource.

Before comparing the set of separating equilibria under different regeneration rates, let us apply the Cho and Kreps' [24] Intuitive Criterion in order to eliminate any "unreasonable" separating equilibria.

Proposition 2. *All separating PBE identified in Proposition 1 in which the low-stock incumbent selects $x_1^L \in [x_1^A, x_1^B)$ violate the Cho and Kreps' Intuitive Criterion. The least-costly separating equilibrium whereby the low-stock incumbent chooses $x_1^L = x_1^B$ survives the Intuitive Criterion if $p > \overline{p}(x_1)$, where $\overline{p}(x_1) \equiv \frac{-D_2^L(x_1)}{D_2^H(x_1) - D_2^L(x_1)}$. Similarly, all separating PBE described in Corollary 1 in which the low-stock incumbent selects $x_1^L \in [\widetilde{x}_1^A, \widetilde{x}_1^B)$ violate the Intuitive Criterion. The least-costly separating equilibrium where the low-stock incumbent chooses $x_1^L = \widetilde{x}_1^B$ survives the Intuitive Criterion if $p > \overline{p}$.*

Hence, the low-stock incumbent can signal its type by appropriating the highest possible level x_1^B during the first period. All other appropriation levels (those in the interval $x_1^L \in [x_1^A, x_1^B)$ when $\beta < 1$ or those in $x_1^L \in [\widetilde{x}_1^A, \widetilde{x}_1^B)$ when $\beta = 1$) would never be selected by the low-stock incumbent. Indeed, starting from an appropriation x_1^A, any higher level also deters entry, and increases the low-stock incumbent's profits. This argument can be repeated for all appropriation levels higher than x_1^A, inducing the incumbent to raise its first-period appropriation until x_1^B, since it still signals a low stock and thus deters entry.[16] A similar argument holds for the case in which the commons totally regenerate across periods, $\beta = 1$, whereby the low-stock incumbent selects the highest appropriation inducing separation, $\widetilde{x}_1^B$.

Let us next analyze how incomplete information affects first-period appropriation for the low-stock incumbent.[17] In particular, when the initial stock totally regenerates across periods, the low-stock incumbent's first-period appropriation under complete information, $x_{1,monop}^L$, is higher than the least-costly appropriation that deters entry, $\widetilde{x}_1^B$. Intuitively, the entrant's lack of information about the available stock induces the incumbent to give up first-period profits (extracting a lower

[16] Note that if the low-stock incumbent deviates towards a first-period appropriation x_1^L such that $x_1^L > x_1^B$, then the high-stock incumbent might have incentives to deviate from the equilibrium appropriation of $x_1^{H,E}$ and pool with the low-stock incumbent when such deviation deters entry. This occurs when the prior probability of the stock being high is sufficiently small, i.e., $p < \overline{p}(x_1)$; as we explain in the next section about the pooling equilibrium. Otherwise, the high-stock incumbent does not deviate from $x_1^{H,E}$ and the low-stock incumbent deters entry by selecting the least-costly separating appropriation level x_1^B.

[17] The high-stock incumbent's appropriation in the first period of the game, $x_1^{H,E}$, coincides with that under complete information. For this reason, we focus on the low-stock incumbent.

appropriation level) in order to deter entry. When the stock does not regenerate across time, the low-stock incumbent also decreases its first-period appropriation from $x_1^{L,NE}$ under complete information to x_1^B in the signaling game. A similar intuition as above applies to this case. Therefore, the presence of incomplete information serves as a tool to promote the incumbent's own conservation of the CPR when the initial stock is relatively low. In addition, if the regeneration rate of the resource is low, entry deterrence benefits sharply decrease in first-period appropriation, x_1^L, inducing a small conservation effort by the incumbent, $x_1^{L,NE} - x_1^B$. In this case, the conservation effort under full regeneration, $x_{1,monop}^L - \widetilde{x}_1^B$, is larger than under partial regeneration. That is, the low-stock incumbent underexploits the resource more when the stock fully regenerates in order to convey the characteristics of the CPR to the entrant than when the regeneration rate is low. For simplicity, we thereafter focus on this case.

Efficiency properties. Let us next evaluate the efficiency properties of the separating equilibrium. When the initial stock is high, the incumbent's first-period appropriation coincides with that under complete information. As shown in the previous section, this appropriation level is not socially optimal, since the incumbent does not internalize the future negative effect that an increase in first-period appropriation has on the entrant's profits. Therefore, the high-stock incumbent overexploits the CPR both under complete and incomplete information. When the initial stock is low, we just demonstrated that the incumbent's first-period appropriation lies below that under complete information. Since the low-stock incumbent's appropriation under complete information coincides with the socially optimal level, the introduction of incomplete information leads to an underexploitation of the CPR. Thus, incomplete information raises two forms of inefficiency: the high-stock's incumbent's overexploitation of the CPR (that arises under both informational contexts) and the underexploitation of the resource by the low-stock incumbent. The following table summarizes our results.

		Separating PBE	*Complete Information*
Low stock	1^{st} *Period*	x_1^B, underexploitation	$x_1^{L,NE}$, socially optimal
	2^{nd} *Period*	q_1^m, socially optimal	q_1^m, socially optimal
High stock	1^{st} *Period*	$x_1^{H,E}$, overexploitation	$x_1^{H,E}$, overexploitation
	2^{nd} *Period*	$q_1^d + q_2^d$, overexploitation	$q_1^d + q_2^d$, overexploitation

Table II. Separating equilibrium.

Full regeneration. In the particular case in which the resource totally regenerates across periods, $\beta = 1$, the first type of inefficiency is absent, as suggested in our discussion of the model under complete information. The second type of inefficiency, however, does not disappear but instead, becomes larger as the stock totally regenerates. In particular, the underexploitation of the low-stock CPR is more significant when the resource fully regenerates across time than when it does not. Our result would recommend no need of government intervention when asymmetric information is present, since the low-stock incumbent already has incentives to conserve the CPR.

Note that this incumbent would actually favor a regulation that prescribes socially optimal appropriation levels across periods. Specifically, socially optimal levels for the low-stock incumbent coincide with equilibrium appropriation under complete information, which are higher than those in the separating equilibrium and yield higher profits. Hence, under such regulation the low-stock incumbent would not need to reduce first-period appropriation in order to deter entry, yielding higher profits than under the threat of entry.[18]

4.2 Pooling equilibrium

Let us now examine the pooling equilibrium of the game, where both types of incumbent select the same first-period appropriation level.

Proposition 3. *The following strategy profile describes a pooling PBE in the CPR signaling game that survives the Cho and Kreps' Intuitive Criterion:*

1. *In the first period, both incumbents select the same first-period appropriation* $x_1^{L,NE}$*;*
2. *The entrant does not enter after observing the equilibrium appropriation* $x_1^{L,NE}$*, but enters after observing any off-the-equilibrium appropriation* x_1'*, given beliefs* $\mu(\theta_H|x_1^{L,NE}) = p < \overline{p}(x_1^{L,NE})$ *and* $\mu(\theta_H|x_1') = 1$*; and*
3. *In the second period of the game, the incumbent selects* $q_1^m(x_1, \theta_K)$ *if entry does not occur, and every agent* $i = \{1, 2\}$ *chooses* $q_i^d(x_1, \theta_K)$ *if entry occurs.*

Therefore, in the pooling equilibrium both types of incumbent selects the same first-period appropriation, which reveals no additional information about the initial stock to the entrant, deterring entry. This is a positive result in terms of overexploitation since entry does not occur; unlike the complete information context when the initial stock is high, leading to overexploitation during the second period. Note that incomplete information provides the high-stock incumbent with an "implicit protection right," since it helps the incumbent protect the resource from entry.

Let us next evaluate the efficiency properties of the pooling equilibrium. First, in the case of no entry, first-period socially optimal appropriation is $x_1^{K,NE}$, as described in section 3.1. Hence, both types of incumbent (weakly) underexploit the resource during the first period. Specifically, the high-stock incumbent underexploits the resource, given that $x_1^{L,NE} \leq x_1^{H,NE}$, whereas the low-stock incumbent selects the socially optimal appropriation level $x_1^{L,NE}$. In contrast, in the second period both types of incumbent choose socially optimal levels given that no entry occurs. A similar intuition applies to the case in which the resource totally regenerates across periods, $\beta = 1$, where both types of incumbent select $x_{1,monop}^L$.

[18] The regulator could, instead, prescribe a distribution of the second-period socially optimal appropriation levels between the incumbent and the entrant. In such case, the low-stock incumbent's profits are not necessarily higher than those under the threat of entry.

4.3 Efficiency comparison

The following table compares the efficiency properties of the separating and pooling equilibria when $\beta < 1$.

		Separating PBE	*Pooling PBE*
Low stock	1^{st} *Period*	x_1^B, underexploitation	$x_1^{L,NE}$, socially optimal
	2^{nd} *Period*	q_1^m, socially optimal	q_1^m, socially optimal
High stock	1^{st} *Period*	$x_1^{H,E}$, overexploitation	$x_1^{L,NE}$, underexploitation
	2^{nd} *Period*	$q_1^d + q_2^d$, overexploitation	q_1^m, socially optimal

Table III. Efficiency properties in the separating and pooling equilibria.

When the initial stock is high, the pooling equilibrium induces the incumbent to conserve the commons by underexploiting it during the first period and by selecting the socially optimal appropriation level in the second period. In contrast, in the separating equilibrium the CPR is overexploited along both periods. Therefore when the CPR's stock is high, our results do not prescribe a precise policy recommendation. If, however, social preferences assign a greater welfare loss to overexploitation than to underexploitation, then environmental agencies holding private information about the commons' stock being high should promote parameter conditions under which the pooling equilibrium emerges. In particular, the regulator can promote the pooling equilibrium by setting a first-period quota that specifies significant penalties for those incumbents exceeding $x_1^{L,NE}$. Intuitively, these penalties make the separating equilibrium appropriation less attractive for the high-stock incumbent.

5 Conclusions

We examine the exploitation of a common pool resource (CPR) under complete and incomplete information, and investigate how the presence of incomplete information can lead to different degrees of conservation. In particular, we show that the lack of information about the initial stock promotes the preservation of the resource when its initial stock is low, relative to complete information. This conservation of the CPR is especially significant when the commons fully regenerate across periods. Under complete information we demonstrate that only one type of inefficiency arises, due to the overexploitation of the CPR by the high-stock incumbent both in the first and second period of the game. However, under incomplete information an additional type of inefficiency exists, due to the underexploitation of the CPR by the low-stock incumbent in the separating equilibrium. Specifically, this type of incumbent uses underexploitation as a tool to signal a low available stock to potential entrants, who are thus deterred from entering. This last form of inefficiency might be observed in different CPRs where the entrant is uninformed about the available initial stock of the resource.

In the pooling equilibrium the overexploitation of the CPR in the second period is absent but the high-stock incumbent underexploits the resource in the first period. Hence, appropriation levels in the pooling equilibrium coincide with the social optimum when the initial stock is low. The high-stock incumbent's overexploitation observed in the separating equilibrium during the second period disappears in the pooling equilibrium, whereas its overexploitation in the first period reverts to underexploitation, suggesting that environmental agencies should promote the pooling equilibrium. Thus, the tragedy of the commons –present in the separating equilibrium– dissipates in the pooling equilibrium when the initial stock is high.

This paper considers a single entrant in a two-period model. If, instead, multiple entrants sequentially choose whether to enter the commons, our separating equilibrium still applies. In particular, the low-stock incumbent deters entry in the first period, and selects its monopoly appropriation level in the second period, which reveals the state of the stock to potential entrants, further deterring entry. The high-stock incumbent attracts entry, and chooses its duopoly appropriation in the second period, which also conveys information to future entrants, attracting entry. In the pooling equilibrium, however, our model predicts that the high-stock incumbent selects its second-period monopoly appropriation level, which might not be sensible if entry is still possible in future periods. Indeed, this incumbent chooses a monopoly exploitation level when no future entry exists, but could choose a different appropriation level in order to keep potential entrants uninformed about the stock. Another venue of potential research considers the presence of more than one incumbent in a context of complete information, as in Gilbert and Vives [25], and how an increase in the number of incumbents affects the first-period overexploitation of the resource. The introduction of multiple incumbents in an incomplete information setting, however, facilitates the entrant's access to more accurate information about the available stock, hampering the role of appropriation as a signaling device. Finally, it could be interesting to study policy instruments that reduce the extent of the two types of inefficiencies identified in this paper.

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Can Incomplete Information Lead to Under-exploitation in the Commons?
TECHNICAL APPENDIX

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1 Appendix 1 - Single crossing property

In the following lemma we describe under which conditions the single-crossing property holds. Let second-period equilibrium costs be denoted by $z^i(x_1, \theta_K)$ and $c^1(x_1, \theta_K)$ after entry and no entry, respectively.

Lemma A. *When entry does not occur, incumbent's profits satisfy the single-crossing property for all parameter values. When entry occurs, the single-crossing property holds if*

$$\frac{\partial z^1(x_1,\theta_L)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_L)}{\partial x_1} > \frac{\partial z^1(x_1,\theta_H)}{\partial q_2}\frac{\partial q_2^d(x_1,\theta_H)}{\partial x_1}$$

Intuitively, the incumbent's payoff structure satisfies the single-crossing property if an additional unit of first-period appropriation x_1 produces a larger strategic effect when the stock is low than when it is high.

Proof. If entry does not occur, the high-stock incumbent's profits are $M_1^H(x_1)+\delta\overline{M}_1^H(x_1)$, for a given first-period appropriation x_1, and for a given appropriation level $q_1^m(x_1,\theta_H)$ that maximizes profits in the second period of the game. If the high-stock incumbent marginally increases first period appropriation, it experiences an increase in profits of $1 - c_{x_1}(x_1,\theta_H) - \delta c^1_{x_1}(x_1,\theta_H)$, where $c^1(x_1,\theta_H)$ denotes the high-stock incumbent's second-period cost, given that no entry occurs and that the incumbent selects the monopoly profit-maximizing appropriation in the second period. The previous derivative can be alternatively expressed as $MB - MC^m(x_1,\theta_H)$. Similarly for the low-stock incumbent. Hence, under no entry, the single-crossing property holds if $MB -$

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$MC^m(x_1, \theta_H) \geq MB - MC^m(x_1, \theta_L)$, or alternatively, if $MC^m(x_1, \theta_H) \leq MC^m(x_1, \theta_L)$, i.e., the incumbent's marginal costs from raising x_1 are decreasing in the initial stock. Note that this condition implies

$$\delta \left[c^1_{x_1}(x_1, \theta_H) - c^1_{x_1}(x_1, \theta_L) \right] < c_{x_1}(x_1, \theta_L) - c_{x_1}(x_1, \theta_H)$$

where the right-hand side is positive since first-period costs satisfy $c_{x_1\theta} < 0$ by definition. In addition, the left-hand side is negative since second-period costs satisfy $c^1_{x_1\theta} < 0$ by definition. Hence, the single-crossing property holds under all parameter values if no entry follows.

If entry occurs, the high-stock incumbent's profits are $M^H_1(x_1) + \delta D^H_1(x_1)$, for a given first-period appropriation x_1, and for a given appropriation level $q^d_1(x_1, \theta_H)$ that maximizes profits in the second period of the game. If the high-stock incumbent marginally increases first-period appropriation, it experiences an increase in profits given by $1 - c_{x_1}(x_1, \theta_H) - \delta \left[\frac{\partial z^1(x_1,\theta_H)}{\partial x_1} + \frac{\partial z^1(x_1,\theta_H)}{\partial q_2} \frac{\partial q^d_2(x_1,\theta_H)}{\partial x_1} \right]$. The previous condition can be alternatively expressed as $MB - MC^d(x_1, \theta_H)$. Similarly for the low-stock incumbent. Hence, under entry, the single-crossing property is satisfied if $MB - MC^d(x_1, \theta_H) \geq MB - MC^d(x_1, \theta_L)$, or alternatively, if $MC^d(x_1, \theta_H) \leq MC^d(x_1, \theta_L)$, i.e., the incumbent's marginal costs from raising x_1 are decreasing in the initial stock. Note that this condition implies

$$\begin{aligned} & c_{x_1}(x_1, \theta_H) + \delta \left[\frac{\partial z^1(x_1, \theta_H)}{\partial x_1} + \frac{\partial z^1(x_1, \theta_H)}{\partial q_2} \frac{\partial q^d_2(x_1, \theta_H)}{\partial x_1} \right] \\ < \;\; & c_{x_1}(x_1, \theta_L) - \delta \left[\frac{\partial z^1(x_1, \theta_L)}{\partial x_1} + \frac{\partial z^1(x_1, \theta_L)}{\partial q_2} \frac{\partial q^d_2(x_1, \theta_L)}{\partial x_1} \right] \end{aligned}$$

rearranging,

$$\begin{aligned} & \delta \left[\frac{\partial z^1(x_1, \theta_H)}{\partial x_1} - \frac{\partial z^1(x_1, \theta_L)}{\partial x_1} \right] + \delta \left[\frac{\partial z^1(x_1, \theta_H)}{\partial q_2} \frac{\partial q^d_2(x_1, \theta_H)}{\partial x_1} - \frac{\partial z^1(x_1, \theta_L)}{\partial q_2} \frac{\partial q^d_2(x_1, \theta_L)}{\partial x_1} \right] \\ < \;\; & c_{x_1}(x_1, \theta_L) - c_{x_1}(x_1, \theta_H) \end{aligned}$$

where the right-hand side of the inequality is positive since first-period costs satisfy $c_{x_1\theta} < 0$ by definition. Furthermore, the first term in the left-hand side is negative since $z^1_{x_1\theta} < 0$ by definition. Therefore, the single-crossing property holds if

$$\frac{\partial z^1(x_1, \theta_H)}{\partial q_2} \frac{\partial q^d_2(x_1, \theta_H)}{\partial x_1} < \frac{\partial z^1(x_1, \theta_L)}{\partial q_2} \frac{\partial q^d_2(x_1, \theta_L)}{\partial x_1}$$

2 Appendix 2

First, note that $\delta \left[\overline{M}^H_1(x^L_1) - D^H_1(x^{H,E}_1) \right]$ from IC_H and $\delta \left[\overline{M}^L_1(x^L_1) - D^L_1(x^{L,E}_1) \right]$ from IC_L are both decreasing and convex in x^L_1, since $\overline{M}^K_1(x^L_1)$ is decreasing and convex in x^L_1, i.e., $\frac{\partial \overline{M}^K_1(x^L_1)}{\partial x^L_1} = -\delta c^1_{x_1} > 0$ and $\frac{\partial^2 \overline{M}^K_1(x^L_1)}{\partial {x^L_1}^2} = -\delta c^1_{x_1 x_1} > 0$. We next investigate the conditions under which $\delta \left[\overline{M}^H_1(x^L_1) - D^H_1(x^{H,E}_1) \right]$ is above $\delta \left[\overline{M}^L_1(x^L_1) - D^L_1(x^{L,E}_1) \right]$. For compactness, let $\overline{M}_1(x_1, \theta)$ denote monopoly second-period equilibrium profits as a function of first-period appropriation, x_1, and the initial stock, θ. Similarly, let $D_1(x_1^E(\theta), \theta)$ represent duopoly second-period equilibrium profits as a function of the first-period equilibrium appropriation that maximizes the incumbent's profits given that entry follows, $x_1^E(\theta)$, for a given initial stock θ. We next show that the difference $\overline{M}_1(x_1, \theta) - D_1(x_1^E(\theta), \theta)$ is increasing in θ. In particular, differentiating with respect to θ and using the envelope theorem, we obtain

$$-c_\theta^1(x_1, \theta) + z_{q_2}^1 \left[\frac{\partial q_2^d(x_1, \theta)}{\partial x_1} \frac{\partial x_1^E(\theta)}{\partial \theta} + \frac{\partial q_2^d(x_1, \theta)}{\partial \theta} \right] + z_{x_1}^1 \frac{\partial x_1^E(\theta)}{\partial \theta} + z_\theta^1(x_1, \theta)$$

which is positive since $\left|c_\theta^1(x_1, \theta)\right| > \left|z_\theta^1(x_1, \theta)\right|$ and the second and third term are positive by definition.

Finally, note that the negative slope of $\delta \left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E}) \right]$ is $-\delta c_{x_1}^1(x_1, \theta_H)$, which is smaller (in absolute value) than that of $\delta \left[\overline{M}_1^L(x_1^L) - D_1^L(x_1^{L,E}) \right]$, $-\delta c_{x_1}^1(x_1, \theta_L)$, where $c^1(x_1, \theta_K)$ denotes the incumbent's second-period cost, given that no entry occurs and that the incumbent selects the appropriation level $q_1^m(x_1, \theta_K)$ that maximizes its monopoly second-period profits. Therefore, $\delta \left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E}) \right]$ is flatter than $\delta \left[\overline{M}_1^L(x_1^L) - D_1^L(x_1^{L,E}) \right]$, guaranteeing that the former does not cross the latter. ■

3 *Proof of Proposition 1*

First, note that entrant beliefs become $\mu(\theta_H | x_1^H) = 1$ after observing the equilibrium appropriation level x_1^H and $\mu(\theta_H | x_1^L) = 0$ after observing the equilibrium level x_1^L for any $x_1^L \in [x_1^A, x_1^B]$. If the entrant observes an off-the-equilibrium appropriation level of $x_1 \neq x_1^H \neq x_1^L$, then Bayes' rule does not specify a particular posterior off-the-equilibrium belief, i.e., $\mu(\theta_H | x_1) \in [0, 1]$, and for simplicity we assume $\mu(\theta_H | x_1) = 1$. Given these beliefs, the entrant enters after observing an appropriation level of x_1^H since $D_2^H(x_1^H) > 0$, but stays out after observing an appropriation of x_1^L given that $0 > D_2^L(0) > D_2^L(x_1^L)$. After observing an off-the-equilibrium level $x_1 \neq x_1^H \neq x_1^L$, the entrant enters if and only if its expected profits from entering satisfy

$$\mu(\theta_H | x_1) \times D_2^H(x_1) + (1 - \mu(\theta_H | x_1)) D_2^L(x_1) > 0, \text{ or } \mu(\theta_H | x_1) > \frac{-D_2^L(x_1)}{D_2^H(x_1) - D_2^L(x_1)} \equiv \overline{\mu}(x_1)$$

where $D_2^H(x_1) > 0$, implying $D_2^H(x_1) - D_2^L(x_1) > -D_2^L(x_1)$, and since both sides of the inequality are positive, we can conclude that $\overline{\mu}(x_1) \in (0, 1)$. In this case, the entrant enters if its off-the-equilibrium beliefs $\mu(\theta_H | x_1)$ satisfy $\mu(\theta_H | x_1) > \overline{\mu}(x_1)$, which holds since $\mu(\theta_H | x_1) = 1$.

Let us now examine the high-stock incumbent's incentives. By selecting the equilibrium appropriation level $x_1^{H,E}$, the high-stock incumbent obtains profits of $M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$. First, note that $x_1^{H,E}$ maximizes $M_1^H(x_1) + \delta D_1^H(x_1)$. Second, first-period appropriation $x_1^{H,E}$ coincides with the equilibrium level that the high-stock incumbent selects under complete information, yielding the same profits. By deviating towards the low-stock incumbent's equilibrium appropriation, x_1^L, the high-stock incumbent deters entry, yielding profits of $M_1^H(x_1^L) + \delta \overline{M}_1^H(x_1^L)$. Hence, the high-stock incumbent prefers to select an equilibrium first-period appropriation of $x_1^{H,E}$ rather than deviating towards x_1^L if $M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E}) \geq M_1^H(x_1^L) + \delta \overline{M}_1^H(x_1^L)$, or alternatively,

$$M_1^H(x_1^{H,E}) - M_1^H(x_1^L) \geq \delta \left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E}) \right] \qquad (IC_H)$$

If instead the high-stock incumbent deviates towards an off-the-equilibrium level of $x_1 \neq x_1^{H,E} \neq x_1^L$ then entry follows, yielding profits of $M_1^H(x_1) + \delta D_1^H(x_1)$, which cannot exceed equilibrium profits of $M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$.

Let us now turn to the low-stock incumbent. Selecting the equilibrium first-period appropriation level of x_1^L yields $M_1^L(x_1^L) + \delta \overline{M}_1^L(x_1^L)$. By deviating towards the high-stock incumbent's equilibrium appropriation level, $x_1^{H,E}$, the low-stock incumbent attracts entry, obtaining profits of $M_1^L(x_1^{H,E}) + \delta D_1^L(x_1^{H,E})$. Therefore, the low-stock incumbent selects an equilibrium first-period appropriation level of x_1^L rather than deviating towards $x_1^{H,E}$ if

$$M_1^L(x_1^L) + \delta \overline{M}_1^L(x_1^L) \geq M_1^L(x_1^{H,E}) + \delta D_1^L(x_1^{H,E}) \qquad (A.1)$$

If instead the low-stock incumbent deviates towards any off-the-equilibrium level $x_1 \neq x_{1,monop}^H \neq x_1^L$ then entry follows, and therefore the incumbent selects the value of x_1 that maximizes $M_1^L(x_1) + \delta D_1^L(x_1)$. Let $x_1^{L,E}$ denote the solution to this maximization problem, yielding profits of $M_1^L(x_1^{L,E}) + \delta D_1^L(x_1^{L,E})$. Hence, the low-stock incumbent chooses its equilibrium appropriation level of x_1^L rather than deviating towards $x_1^{L,E}$ if

$$M_1^L(x_1^L) + \delta \overline{M}_1^L(x_1^L) \geq M_1^L(x_1^{L,E}) + \delta D_1^L(x_1^{L,E}) \qquad (A.2)$$

Note that condition A.2 implies A.1 since $M_1^L(x_1^{L,E}) + \delta D_1^L(x_1^{L,E}) > M_1^L(x_1^{H,E}) + \delta D_1^L(x_1^{H,E})$, given that $x_1^{L,E}$ maximizes the low-stock incumbent's profits (across both periods) given entry, whereas $x_1^{H,E}$ does not. Therefore, condition A.2 becomes the incentive compatibility condition that must be satisfied in order to guarantee that the low-stock incumbent does not deviate from its equilibrium level of x_1^L. Let us denote this incentive compatibility condition as follows

$$M_1^L(x_1^{L,E}) - M_1^L(x_1^L) \leq \delta \left[\overline{M}_1^L(x_1^L) - D_1^L(x_1^{L,E}) \right] \qquad (IC_L)$$

4 *Proof of Corollary 1*

In this case the CPR totally regenerates, $\beta = 1$. First, entrant beliefs become $\mu(\theta_H | x_{1,monop}^H) = 1$ after observing the equilibrium appropriation level of $x_{1,monop}^H$ and $\mu(\theta_H | x_1^L) = 0$ after observing the equilibrium level of x_1^L for any $x_1^L \in [\widetilde{x}_1^A, \widetilde{x}_1^B]$. If the entrant observes an off-the-equilibrium level of $x_1 \neq x_{1,monop}^H \neq x_1^L$, then Bayes' rule does not specify a particular posterior off-the-equilibrium belief, i.e., $\mu(\theta_H | x_1) \in [0, 1]$, and for simplicity we take $\mu(\theta_H | x_1) = 1$. Given these beliefs, the entrant enters after observing an appropriation level of $x_{1,monop}^H$ since $D_2^H(0) \equiv D_2^H > 0$, but stays out after observing an appropriation of x_1^L given that $D_2^L(0) \equiv D_2^L < 0$. After observing an off-the-equilibrium level $x_1 \neq x^H_{1,monop} \neq x^L_1$, the entrant enters if and only if its expected profits from entering satisfy $\mu(\theta_H|x_1) \geq \frac{-D^L_2}{D^H_2 - D^L_2} \equiv \overline{\mu}$, which holds as shown in Proposition 1.

Let us now examine the high-stock incumbent's incentives. By selecting the equilibrium appropriation level of $x^H_{1,monop}$, the high-stock incumbent obtains profits of $M^H_1(x^H_{1,monop}) + \delta D^H_1$, where $M^H_1(x^H_{1,monop})$ represents the highest monopoly profit that the high-stock incumbent can obtain during the first period, and where second-period profits are independent on x_1 since the resource is totally regenerated ($\beta = 1$). Note that $x^H_{1,monop}$ coincides with the first-period appropriation level that the high-stock incumbent selects in the complete information context. By deviating towards the low-stock incumbent's equilibrium appropriation, x^L_1, the high-stock incumbent deters entry, yielding profits of $M^H_1(x^L_1) + \delta \overline{M}^H_1$. Hence, the high-stock incumbent prefers an equilibrium first-period appropriation level of $x^H_{1,monop}$ rather than deviating towards x^L_1 if $M^H_1(x^H_{1,monop}) + \delta D^H_1 \geq M^H_1(x^L_1) + \delta \overline{M}^H_1$, or alternatively,

$$M^H_1(x^H_{1,monop}) - M^H_1(x^L_1) \geq \delta \left[\overline{M}^H_1 - D^H_1 \right] \tag{IC_H}$$

Note that if the high-stock incumbent deviates towards an off-the-equilibrium level of $x_1 \neq x^H_{1,monop} \neq x^L_1$, entry follows, yielding profits of $M^H_1(x_1) + \delta D^H_1$, which cannot exceed equilibrium profits of $M^H_1(x^H_{1,monop}) + \delta D^H_1$ given that $x^H_{1,monop}$ is the profit-maximizing appropriation level under monopoly.

Let us now analyze the low-stock incumbent. Selecting the equilibrium first-period appropriation level of x^L_1 yields $M^L_1(x^L_1) + \delta \overline{M}^L_1$. By deviating towards the high-stock incumbent's equilibrium appropriation level, $x^H_{1,monop}$, the low-stock incumbent attracts entry, with associated profits of $M^L_1(x^H_{1,monop}) + \delta D^L_1$. Therefore, the low-stock incumbent selects an equilibrium first-period appropriation level of x^L_1 rather than deviating towards $x^H_{1,monop}$ if

$$M^L_1(x^L_1) + \delta \overline{M}^L_1 \geq M^L_1(x^H_{1,monop}) + \delta D^L_1 \tag{A.4}$$

If instead the low-stock incumbent deviates towards any off-the-equilibrium level $x_1 \neq x^H_{1,monop} \neq x^L_1$, it attracts entry, and therefore the incumbent selects the value of x_1 that maximizes

$$\max_{x_1} M^L_1(x_1) + \delta D^L_1 \quad \text{subject to } x_1 \neq x^H_{1,monop} \neq x^L_1$$

But note that this maximization problem is equivalent to $\max_{x_1} M^L_1(x_1)$, with solution given by the appropriation level that maximizes the first-period monopoly profits, i.e., $x^L_{1,monop}$, yielding profits of $M^L_1(x^L_{1,monop}) + \delta D^L_1$. Hence, the low-stock incumbent chooses its equilibrium appropriation level of x^L_1 rather than deviating towards $x^L_{1,monop}$ if

$$M^L_1(x^L_1) + \delta \overline{M}^L_1 \geq M^L_1(x^L_{1,monop}) + \delta D^L_1 \tag{A.5}$$

Condition A.5 implies A.4 since $M^L_1(x^L_{1,monop}) + \delta D^L_1 > M^L_1(x^H_{1,monop}) + \delta D^L_1$, given that

$M_1^L(x_{1,monop}^L) > M_1^L(x_{1,monop}^H)$. Therefore, condition A.5 becomes the incentive compatibility condition that must be satisfied in order to guarantee that the low-stock incumbent does not deviate from its equilibrium appropriation of x_1^L. Let us denote this incentive compatibility condition as follows

$$M_1^L(x_{1,monop}^L) - M_1^L(x_1^L) \leq \delta \left[\overline{M}_1^L - \delta D_1^L\right] \qquad (IC_L)$$

Note that cutoffs $\delta \left[\overline{M}_1^H - D_1^H\right]$ from IC_H and $\delta \left[\overline{M}_1^L - \delta D_1^L\right]$ from IC_L are both independent on x_1^L. We next investigate the conditions under which the former cutoff is above the latter. For compactness, let $\overline{M}_1(\theta)$ and $D_1(\theta)$ denote monopoly and duopoly second-period equilibrium profits as a function of the initial stock, θ. Hence, we want to show that the difference $\overline{M}_1(\theta) - D_1(\theta)$ is increasing in θ. In particular, differentiating with respect to θ and using the envelope theorem, we obtain

$$-c_\theta^1(\theta) + z_{q_2}^1 \frac{\partial q_2^d(\theta)}{\partial \theta} + z_\theta^1(\theta)$$

which is positive since $z_{q_2}^1 \frac{\partial q_2^d(\theta)}{\partial \theta} > 0$ and $\left|c_\theta^1(\theta)\right| > \left|z_\theta^1(\theta)\right|$ by definition, guaranteeing that $\delta \left[\overline{M}_1^H - D_1^H\right]$ is above $\delta \left[\overline{M}_1^L - \delta D_1^L\right]$. ■

5 *Proof of Proposition 2*

Case in which $\beta < 1$. Suppose that the low-stock incumbent appropriates $x_1^L = x_1^A$. Let us first check if a deviation towards $x_1 \in (x_1^A, x_1^B]$ is equilibrium dominated for either type of incumbent.

On one hand, the highest profit that the high-stock incumbent can obtain deviating towards $x_1 \in (x_1^A, x_1^B]$ occurs when entry does not ensue. In such case, the high-stock incumbent obtains $M_1^H(x_1) + \delta\overline{M}_1^H(x_1)$. Hence, it deviates only if $M_1^H(x_1) + \delta\overline{M}_1^H(x_1) > M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$. But IC_H guarantees that this inequality cannot hold for any $x_1 \in (x_1^A, x_1^B]$. Hence the high-stock incumbent does not have incentives to deviate from $x_1^{H,E}$ to $x_1 \in (x_1^A, x_1^B]$.

On the other hand, the highest profit that the low-stock incumbent can obtain from deviating towards $x_1 \in (x_1^A, x_1^B]$ occurs when entry does not ensue. In such case, the low-stock incumbent's payoff is $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$ which exceeds its equilibrium profit of $M_1^L(x_1^A) + \delta\overline{M}_1^L(x_1^A)$ since $x_1^A < x_1 \leq x_1^{L,NE}$ and $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$ reaches its maximum at $x_1^{L,NE}$. Therefore, the low-stock incumbent has incentives to deviate from x_1^A to x_1.

Hence, after observing a first-period appropriation of $x_1 \in (x_1^A, x_1^B]$, the entrant concentrates its posterior beliefs on the initial stock being low, i.e., $\mu(\theta_H|x_1) = 0$, and does not enter. Given this updated off-the equilibrium beliefs, the low-stock incumbent appropriates x_1 and deters entry, yielding payoff $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$, which exceeds its equilibrium profit from appropriating x_1^A. Thus, the low-stock incumbent deviates from x_1^A, and the separating equilibrium in which it selects x_1^A violates the Intuitive Criterion. A similar argument is applicable to all separating equilibria in which the low-stock incumbent selects $x_1 \in (x_1^A, x_1^B)$, all of them also violating the Intuitive Criterion.

Finally, let us check that the separating equilibrium in which the low-stock incumbent chooses $x_1^L = x_1^B$ survives the Intuitive Criterion. If the low-stock incumbent deviates towards $x_1 \in (x_1^A, x_1^B)$ the highest profit that it can obtain is $M_1^L(x_1)+\delta\overline{M}_1^L(x_1)$, which is lower than its equilibrium payoff of $M_1^L(x_1^B) + \delta\overline{M}_1^L(x_1^B)$. If, instead, it deviates towards $x_1 > x_1^B$, the highest payoff that it can obtain is $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$, which exceeds its equilibrium payoff for all $x_1 \in (x_1^B, x_1^{L,NE}]$. Hence, the low-stock incumbent has incentives to deviate. Let us now check if the high-stock incumbent also has incentives to deviate towards $x_1 \in (x_1^B, x_1^{L,NE}]$. The highest profit that it can obtain is $M_1^H(x_1) + \delta\overline{M}_1^H(x_1)$, which exceeds its equilibrium profits if $M_1^H(x_1) + \delta\overline{M}_1^H(x_1) > M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$. This condition can be rewritten as $\delta\left[\overline{M}_1^H(x_1) - D_1^H(x_1^{H,E})\right] > M_1^H(x_1^{H,E}) - M_1^H(x_1)$, which is satisfied for all $x_1 > x_1^B$ (see figure 1). Hence, the high-stock incumbent also has incentives to deviate towards $x_1 \in (x_1^B, x_1^{L,NE}]$. This implies that, after observing a deviation x_1, the entrant cannot update his prior beliefs, and chooses to enter if the expected profit from entering satisfies $pD_2^H(x_1) + (1-p)D_2^L(x_1) > 0$ or

$$p > \frac{-D_2^L(x_1)}{D_2^H(x_1) - D_2^L(x_1)} \equiv \overline{p}(x_1)$$

where $D_2^H(x_1) > 0$, implying that $D_2^H(x_1) - D_2^L(x_1) > -D_2^L(x_1)$, and since both sides of the inequality are positive, then $\overline{p}(x_1) > 0$. Hence, if $p > \overline{p}(x_1)$, entry occurs, yielding profits $M_1^L(x_1) + \delta D_1^L(x_1)$ for the low-stock incumbent. Such profits are lower than its equilibrium profits $M_1^L(x_1^B) + \delta D_1^L(x_1^B)$. Indeed, from IC_L we know that $M_1^L(x_1^B)+\delta\overline{M}_1^L(x_1^B) \geq M_1^L(x_1^{L,E})+\delta D_1^L(x_1^{L,E})$. Since, in addition, $M_1^L(x_1^{L,E}) + \delta D_1^L(x_1^{L,E}) \geq M_1^L(x_1) + \delta D_1^L(x_1)$ given that $x_1^{L,E}$ is the argmax of $M_1^L(x_1) + \delta D_1^L(x_1)$, then $M_1^L(x_1^B) + \delta\overline{M}_1^L(x_1^B) \geq M_1^L(x_1) + \delta D_1^L(x_1)$ for any deviation x_1, and therefore the low-stock incumbent does not deviate. Regarding the high-stock incumbent, it obtains profits of $M_1^H(x_1) + \delta D_1^H(x_1)$ by deviating towards x_1, which are below its equilibrium profits $M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$ since $x_1^{H,E}$ is the argmax of $M_1^H(x_1) + \delta D_1^H(x_1)$. Hence, the high-stock incumbent does not deviate towards x_1 either, and the separating equilibrium survives the Intuitive Criterion if $p > \overline{p}(x_1)$.

If $p < \overline{p}(x_1)$, then entry does not occur, yielding profits of $M_1^L(x_1) + \delta\overline{M}_1^L(x_1)$ for the low-stock incumbent, which exceed its equilibrium profits $M_1^L(x_1^B) + \delta\overline{M}_1^L(x_1^B)$ since $x_1 \in (x_1^B, x_1^{L,NE}]$. Then the separating equilibrium violates the Intuitive Criterion if $p < \overline{p}(x_1)$.

Case in which $\beta = 1$. Suppose that the low-stock incumbent appropriates $x_1^L = \widetilde{x}_1^A$. Let us first check if a deviation towards $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B]$ is equilibrium dominated for either type of incumbent.

On one hand, the highest profit that the high-stock incumbent can obtain deviating towards $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B]$ occurs when entry does not ensue. In such case, the high-stock incumbent obtains $M_1^H(x_1) + \delta\overline{M}_1^H$. These profits, however, are lower than the high-stock incumbent equilibrium profits of $M_1^H(x_{1,monop}^H) + \delta D_1^H$ since from IC_H we know that

$$M_1^H(x_{1,monop}^H) + \delta D_1^H \geq M_1^H(x_1) + \delta\overline{M}_1^H \quad \text{for all } x_1 \in [\widetilde{x}_1^A, \widetilde{x}_1^B]$$

Hence, the high-stock incumbent does not have incentives to deviate. On the other hand, the highest profit that the low-stock incumbent can obtain from deviating towards $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B]$ occurs when entry does not ensue. In such case, the low-stock incumbent's payoff is $M_1^L(x_1)+\delta\overline{M}_1^L$ which exceeds its equilibrium profit of $M_1^L(\widetilde{x}_1^A) + \delta\overline{M}_1^L$ since $x_1 > \widetilde{x}_1^A$. Therefore, the low-stock incumbent has incentives to deviate from $\widetilde{x}_1^A$ to x_1.

Hence, after observing a first period appropriation of $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B]$, the entrant concentrates its posterior beliefs on the initial stock being low, i.e., $\mu(\theta_H|x_1) = 0$, and does not enter. Given this updated off-the equilibrium beliefs, the low-stock incumbent appropriates x_1 and deters entry, yielding payoff $M_1^L(x_1) + \delta\overline{M}_1^L$, which exceeds its equilibrium profit from appropriating $\widetilde{x}_1^A$. Thus, the low-stock incumbent deviates from $\widetilde{x}_1^A$, and the separating equilibrium in which it selects $\widetilde{x}_1^A$ violates the Intuitive Criterion. A similar argument is applicable for all separating equilibria in which the low-stock incumbent selects $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B)$, all of them also violating the Intuitive Criterion.

Finally, let us check that the separating equilibrium in which the low-stock incumbent chooses $\widetilde{x}_1^B$ survives the Intuitive Criterion. If the low-stock incumbent deviates towards $x_1 \in (\widetilde{x}_1^A, \widetilde{x}_1^B)$ the highest profit that it can obtain is $M_1^L(x_1) + \delta\overline{M}_1^L$, which is lower than its equilibrium payoff of $M_1^L(\widetilde{x}_1^B) + \delta\overline{M}_1^L$. If, instead, it deviates towards $x_1 > \widetilde{x}_1^B$, the highest payoff that it can obtain is $M_1^L(x_1) + \delta\overline{M}_1^L$, which exceeds its equilibrium payoff for all $x_1 \in (x_1^B, x_{1,monop}^L]$. Hence, the low-stock incumbent has incentives to deviate. Let us now check if the high-stock incumbent also has incentives to deviate towards $x_1 \in (x_1^B, x_{1,monop}^L]$. The highest profit that it can obtain is $M_1^H(x_1) + \delta\overline{M}_1^H$, which exceeds its equilibrium profits if $M_1^H(x_1) + \delta\overline{M}_1^H > M_1^H(x_{1,monop}^H) + \delta D_1^H$. This condition can be rewritten as $\delta\left[\overline{M}_1^H - D_1^H\right] > M_1^H(x_{1,monop}^H) - M_1^H(x_1)$, which is satisfied for all $x_1 > \widetilde{x}_1^B$ (see figure 2). Hence, the high-stock incumbent also has incentives to deviate towards $x_1 \in (\widetilde{x}_1^B, x_{1,monop}^L]$. This implies that, after observing a deviation x_1, the entrant cannot update its prior beliefs, and chooses to enter if the expected profit from entering satisfies $pD_2^H + (1-p)D_2^L > 0$ or

$$p > \frac{-D_2^L}{D_2^H - D_2^L} \equiv \overline{p}$$

where $D_2^H > 0$, implying that $D_2^H - D_2^L > -D_2^L$, and since both sides of the inequality are positive, then $\overline{p} > 0$. Hence, if $p > \overline{p}$, entry occurs, yielding profits of $M_1^L(x_1) + \delta D_1^L$ for the low-stock incumbent. Such profits are lower than the low-stock incumbent's equilibrium profits of $M_1^L(\widetilde{x}_1^B)+\delta\overline{M}_1^L$. Indeed, from IC_L we know that $M_1^L(\widetilde{x}_1^B)+\delta\overline{M}_1^L \geq M_1^L(x_{1,monop}^L)+\delta D_1^L$. Since, in addition, $M_1^L(x_{1,monop}^L)+\delta D_1^L \geq M_1^L(x_1)+\delta D_1^L$ given that $x_{1,monop}^L$ is the argmax of $M_1^L(x_1)+\delta D_1^L$, then $M_1^L(\widetilde{x}_1^B) + \delta\overline{M}_1^L \geq M_1^L(x_1) + \delta D_1^L$ and therefore the low-stock incumbent does not deviate. Regarding the high-stock incumbent, it obtains profits of $M_1^H(x_1) + \delta D_1^H$ by deviating towards x_1 which are below its equilibrium profits of $M_1^H(x_{1,monop}^H) + \delta D_1^H$ since $x_{1,monop}^H$ is the argmax of $M_1^H(x_1) + \delta D_1^H$. Hence, the high-stock incumbent does not deviate towards x_1 either, and the separating equilibrium survives the Intuitive Criterion if $p > \overline{p}$.

If $p < \overline{p}$, then entry does not occur, yielding profits of $M_1^L(x_1) + \delta\overline{M}_1^L$ for the low-stock incumbent, which exceed its equilibrium profits $M_1^L(\widetilde{x}_1^B) + \delta\overline{M}_1^L$ since $x_1 \in (\widetilde{x}_1^B, x_{1,monop}^L]$. Then the separating equilibrium violates the Intuitive Criterion if $p < \overline{p}$. ■

6 *Proof of Proposition 3*

In a pooling strategy profile where both types of incumbent select x_1, equilibrium beliefs are $\mu(\theta_H|x_1) = p$ and $\mu(\theta_L|x_1) = 1 - p$, which coincide with the prior probability distribution over types. In addition, off-the-equilibrium beliefs cannot be identified using Bayes' rule, and for simplicity let us assume that, after observing $x_1' \neq x_1$, $\mu(\theta_H|x_1') = 1$. As shown in the proof of Proposition 1, these beliefs induce the entrant to enter after observing x_1'. Otherwise the entrant stays out. On the other hand, after observing x_1, the entrant enters if and only if $pD_2^H(x_1) + (1-p)D_2^L(x_1) > 0$ or

$$p > \frac{-D_2^L(x_1)}{D_2^H(x_1) - D_2^L(x_1)} \equiv \overline{p}(x_1)$$

where $D_2^H(x_1) > 0$, implying that $D_2^H(x_1) - D_2^L(x_1) > -D_2^L(x_1)$, and since both sides of the inequality are positive, we can conclude that the entrant enters if $p > \overline{p}(x_1)$, and stays out otherwise. Note that if entry occurs after x_1, this induces every type of incumbent to select $x_1^{K,E}$. But since $x_1^{H,E} \neq x_1^{L,E}$ this strategy profile cannot be a pooling equilibrium. Hence, it must be that $p < \overline{p}(x_1)$ inducing the entrant to stay out. Let us start by checking under which conditions the high-stock incumbent does not deviate from x_1. By selecting x_1, it deters entry obtaining $M_1^H(x_1)+\delta\overline{M}_1^H(x_1)$. By deviating towards $x_1' \neq x_1$ it attracts entry, yielding a payoff of $M_1^H(x_1') + \delta D_1^H(x_1')$, which is maximized at $x_1^{H,E}$. Hence, the high-stock incumbent does not deviate from x_1 if,

$$M_1^H(x_1) + \delta\overline{M}_1^H(x_1) \geq M_1^H(x_1^{H,E}) + \delta D_1^H(x_1^{H,E})$$

or equivalently,

$$M_1^H(x_1^{H,E}) - M_1^H(x_1) \leq \delta\left[\overline{M}_1^H(x_1) - D_1^H(x_1^{H,E})\right] \tag{IC_H}$$

and similarly, for the low-stock incumbent,

$$M_1^L(x_1^{L,E}) - M_1^L(x_1) \leq \delta\left[\overline{M}_1^L(x_1) - D_1^L(x_1^{L,E})\right] \tag{IC_L}$$

Hence, any x_1 simultaneously satisfying IC_H and IC_L constitutes a pooling equilibrium first-period appropriation of the signaling game.

Intuitive Criterion. *Case 1.* Let us analyze if the pooling first-period appropriation $x_1 = x_1^{L,NE}$ survives the Cho and Kreps' (1987) Intuitive Criterion. We first check if such appropriation level is equilibrium dominated for either type of incumbent. On one hand, the low-stock incumbent obtains an equilibrium profit of $M_1^L\left(x_1^{L,NE}\right) + \delta\overline{M}_1^L(x_1^{L,NE})$. By deviating towards $x_1' \neq x_1^{L,NE}$ the highest payoff that it obtains occurs when entry is deterred, yielding payoffs of $M_1^L(x_1') + \delta \overline{M}_1^L(x_1')$, which lie below its equilibrium profits since $M_1^L(x_1') + \delta \overline{M}_1^L(x_1')$ reaches its maximum at exactly $x_1' = x_1^{L,NE}$. Hence, the low-stock incumbent does not have incentives to deviate from the pooling appropriation level $x_1 = x_1^{L,NE}$. On the other hand, the high-stock incumbent obtains an equilibrium profit of $M_1^H\left(x_1^{L,NE}\right) + \delta \overline{M}_1^H(x_1^{L,NE})$. By deviating towards $x_1' \neq x_1^{L,NE}$ the highest payoff that it obtains occurs when entry is deterred, yielding payoffs of $M_1^H(x_1') + \delta \overline{M}_1^H(x_1')$. Therefore, the high-stock incumbent does not have incentives to deviate if $M_1^H\left(x_1^{L,NE}\right) + \delta \overline{M}_1^H(x_1^{L,NE}) \geq M_1^H(x_1') + \delta \overline{M}_1^H(x_1')$, which only holds for $x_1' \in (x_1^{L,NE}, x_1^{H,NE}]$. Hence, the entrant assigns full probability to the stock being high for every deviation $x_1' \in (x_1^{L,NE}, x_1^{H,NE}]$, i.e., $\mu(\theta_H|x_1') = 1$, whereas its updated beliefs are unaffected after observing any other deviation. Thus, after observing $x_1' \in (x_1^{L,NE}, x_1^{H,NE}]$, the entrant believes that such deviation can only come from a high-stock incumbent and enters. The high-stock incumbent's profits from deviating towards x_1' are $M_1^H(x_1') + \delta D_1^H(x_1')$, which are lower than its equilibrium profits if

$$M_1^H\left(x_1^{L,NE}\right) + \delta \overline{M}_1^H(x_1^{L,NE}) \geq M_1^H(x_1') + \delta D_1^H(x_1'). \tag{A.6}$$

Note that deviation profits, $M_1^H(x_1') + \delta D_1^H(x_1')$, are maximal at $x_1' = x_1^{H,E}$, yielding profits of $M_1^H\left(x_1^{H,E}\right) + \delta D_1^H(x_1^{H,E})$. Hence, if $M_1^H\left(x_1^{L,NE}\right) + \delta \overline{M}_1^H(x_1^{L,NE}) \geq M_1^H\left(x_1^{H,E}\right) + \delta D_1^H(x_1^{H,E})$, then condition A.6 holds for all deviations $x_1' \in (x_1^{L,NE}, x_1^{H,NE}]$. Rearranging the last inequality,

$$M_1^H\left(x_1^{H,E}\right) - M_1^H\left(x_1^{L,NE}\right) \leq \delta\left[\overline{M}_1^H(x_1^{L,NE}) - D_1^H(x_1^{H,E})\right]$$

which graphically implies that the height of the $M_1^H\left(x_1^{H,E}\right) - M_1^H\left(x_1^L\right)$ curve evaluated at $x_1^L = x_1^{L,NE}$ is below the height of the $\delta\left[\overline{M}_1^H(x_1^L) - D_1^H(x_1^{H,E})\right]$ curve also evaluated at $x_1^L = x_1^{L,NE}$. This condition is hence satisfied since $x_1^{L,NE} > x_1^B$. Therefore, the high-stock incumbent does not have incentives to deviate either, and the pooling PBE in which $x_1 = x_1^{L,NE}$ survives the Intuitive Criterion.

Case 2. Let us next check if the pooling first-period appropriation level $x_1 > x_1^{L,NE}$ survives the Cho and Kreps' (1987) Intuitive Criterion. On one hand, the low-stock incumbent obtains $M_1^L(x_1) + \delta \overline{M}_1^L(x_1)$ in equilibrium. By instead deviating towards $x_1' \neq x_1$, the highest profit that it can obtain is $M_1^L(x_1') + \delta \overline{M}_1^L(x_1')$, which exceeds its equilibrium profit of $M_1^L(x_1) + \delta \overline{M}_1^L(x_1)$ if $x_1' \in (x_1^{L,NE}, x_1)$ given the concavity of the $M_1^L(x_1') + \delta \overline{M}_1^L(x_1')$ function with respect to x_1'. On the other hand, the high-stock incumbent obtains $M_1^H(x_1) + \delta \overline{M}_1^H(x_1)$ in equilibrium. By instead deviating towards $x_1' \neq x_1$, the highest profit that it can obtain is $M_1^H(x_1') + \delta \overline{M}_1^H(x_1')$, which exceeds its equilibrium profit of $M_1^H(x_1) + \delta \overline{M}_1^H(x_1)$ if $x_1' \in (x_1^{H,NE}, x_1)$. Hence, beliefs can be restricted to $\mu(\theta_H|x_1') = 0$ after observing a deviation $x_1' \in (x_1^{L,NE}, x_1^{H,NE})$. (Otherwise, entrant's beliefs are unaffected, since either both types of incumbent have incentives to deviate or none of them has.) Therefore, after observing a deviation $x_1' \in (x_1^{L,NE}, x_1^{H,NE})$, the entrant believes that the stock must be low, and chooses not to enter. Under these updated beliefs, the low-stock incumbent's profit exceeds its pooling equilibrium profits. Hence, the low-stock incumbent deviates towards $x_1' \in (x_1^{L,NE}, x_1^{H,NE})$. Therefore, the pooling PBE where $x_1 > x_1^{L,NE}$ violates the Intuitive Criterion.

Case 3. Let us finally check if the pooling first-period appropriation level $x_1 < x_1^{L,NE}$ survives the Cho and Kreps' (1987) Intuitive Criterion. On one hand, the low-stock incumbent obtains $M_1^L(x_1) + \delta \overline{M}_1^L(x_1)$ in equilibrium. By instead deviating towards $x_1^{L,NE}$, the highest profit it can obtain occurs when entry is deterred, yielding profits of $M_1^L\left(x_1^{L,NE}\right) + \delta \overline{M}_1^L(x_1^{L,NE})$, which exceeds its equilibrium profits if $M_1^L\left(x_1^{L,NE}\right) + \delta \overline{M}_1^L(x_1^{L,NE}) \geq M_1^L(x_1) + \delta \overline{M}_1^L(x_1)$, which is true since $x_1 < x_1^{L,NE}$. On the other hand, the high-stock incumbent obtains $M_1^H(x_1) + \delta \overline{M}_1^H(x_1)$ in equilibrium. By instead deviating towards $x_1^{L,NE}$, the highest profit it can obtain occurs after no entry, yielding profits of $M_1^H\left(x_1^{L,NE}\right) + \delta \overline{M}_1^H(x_1^{L,NE})$, which exceeds its equilibrium profits since $M_1^H(x_1) + \delta \overline{M}_1^H(x_1) \leq M_1^H\left(x_1^{L,NE}\right) + \delta \overline{M}_1^H(x_1^{L,NE})$ given that $x_1 < x_1^{L,NE} < x_1^{H,NE}$ and by concavity. Therefore, both types of incumbent have incentives to deviate towards $x_1^{L,NE}$ and entrant's beliefs cannot be updated, i.e., $\mu\left(\theta_H | x_1^{L,NE}\right) = p$, inducing no entry. Given these beliefs, both types of incumbent deviate towards $x_1^{L,NE}$, obtaining higher profits than in equilibrium. Hence, the pooling strategy profile in which both types select $x_1 < x_1^{L,NE}$ violates the Intuitive Criterion. ■