Campaign Contributions and Policy Convergence: Asymmetric Agents and Donations Constraints

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Abstract

We extend previous work on the role of politically motivated donors who contribute to candidates in an election with single dimension policy preferences. In a two-stage game where donors observe candidate policy positions and then allocate funding accordingly, we find that reducing the cost of donations incentivizes candidates to position closer to one another, reducing policy divergence. Furthermore, we find that as donations become more effective at influencing voter decisions, candidates respond less to voter preferences and more to those of donors. In addition, we analyze the presence of asymmetries in this model using numerical analysis techniques. We also extend our model by allowing for public funding from governments. By implementing stringent campaign contributions limits, candidate positions align with voter preferences at the cost of increased policy divergence. In contrast, unlimited campaign contributions lead to candidate positions moving away from voters to donors' preferences, but increases policy convergence.

Keywords: Asymmetries; Policy Preferences; Policy Divergence; Donors; Political Contributions; Campaign Limits.

JEL Codes: C63, C72, D72

1 Introduction

The landscape of American campaign finance has changed drastically over the past decade. With the Supreme Court of the United States of America's landmark 2010 decision in *Citizens United v. FEC*, the ability to contribute to political organizations by corporations, unions, and other similar organizations was declared protected as a form of free speech. Following that ruling, in 2014, the Supreme Court in *McCutcheron v. FEC* ruled that donations by private individuals to political candidates cannot be limited. These two rulings coincided with a drastic increase in campaign donations, and spending, by American politicians.[1] In this paper, we examine whether setting no limits on political contributions to candidates can bring their policies closer to the median voter or, instead, move them further away, hindering political representation. Specifically, we demonstrate that setting no limit on donations, as currently in the U.S., induces candidates to converge towards one another in a race to deny donations to their opponent. While more policy convergence arises in this setting, it occurs towards the donors' preferred policies, which may often differ from those of voters. We also analyze the opposite scenario, where regulations set stringent limits on donations (where each individual may donate a maximum amount, and political parties are limited to what they can spend). In this context, our results show that, while donors still have incentives to contribute to either candidate, candidates behave considering their own ideal policies and those of voters, but ignoring those of the donors. As a consequence, more policy divergence emerges than when donations are severely limited.

We consider a two-stage game where, in the first period, every candidate simultaneously announces his policy position; and in the second period, donors observe candidates' positions, and respond contributing to either (or both) candidates. Given political positions and donations, every candidate has a probability of winning the election which depends on the underlying distribution of voter preferences, and on the amount of campaign funding of each candidate (e.g., voters' decisions are affected by advertising, house visits, etc.). For generality, we assume that, when choosing his policy position, every candidate considers: (1) his probability of winning the election, as in Downs (1953); and (2) the difference between the policy that is elected and his ideal, as in Wittman (1983).

[1] Campaign spending levels in the US Presidential election increased by 45% ($687 million to $1 billion) from 2004 to 2008, and again by 40% ($1 billion to $1.409 billion) from 2008 to 2012. In the UK parliamentary elections, in contrast, donations decreased 32% ($36.5 million to $24.9 million) from 2005 to 2010 and a subsequent increase of 12% ($24.9 million to $28 million) from 2010 to 2015.

Our setting thus allows for a linear combination of these two polar incentives, which also lets us examine the models of Downs or Wittman as special cases.

Candidates face three separate forces when choosing their policy positions. First, as in the original Downs model (without contributions), candidates select their positions by considering their probability of winning the election alone (i.e., locating as close as possible to the median voter). Adding expected policy payoffs (i.e., motive (2) from above, but still without donations) would reduce candidates' incentives to locate at the median voter, as they must also consider the utility they obtain from the implemented policy. Finally, if contributions are allowed, candidates must also consider both donors' ideal policies in order to maximize their share of donations.

In the equilibrium of the second stage, we show that donors contribute at most to one candidate. Intuitively, if a donor contributing to one candidate donates to the other, he would be reducing the former's probability of winning, which is contrary to his original objective. In the first stage, candidates anticipate the effect that a change in their political position has on donations during the second stage before choosing their policies, thus considering the three forces mentioned above. Furthermore, we demonstrate the existence of a "donation stealing" incentive (as originally explained in Ball (1999a)), whereby every candidate, by locating closer to his opponent's donor, reduces his rival's donations more significantly than the decrease he experiences in his own contributions. This plays a role when donations become less costly, where more policy convergence can be sustained in equilibrium as candidates try to deny donations to their rivals.

For presentation purposes, we first analyze a symmetric setting, where voters, donors, and candidates are symmetrically distributed around the midpoint of the policy spectrum. In that context, we show that adding donations has no effect on the Downsian results, regardless of their cost and effectiveness. In contrast, allowing for political contributions to the Wittman model gives rise to the donation stealing effect, ultimately increasing policy convergence. In addition, we show that as donations become more effective at determining the outcome of the election, candidates move away from the median voter toward the (average of) donors' ideal policies.

In the asymmetric version of the model, we first demonstrate that equilibrium results are qualitatively unaffected when candidates assign a sufficiently high weight on the utility from the implemented policy. However, when such weight is low, candidates can gain a donation advantage relative to their opponent by positioning closer to one of the donors and further away from the median voter. If a candidate chooses such a position, his opponent mimics his strategy, leading the former to move back to a position close to the median voter; ultimately yielding that no stable policy profile emerges in equilibrium. In the special case where donors have similar ideal policies, we find that the candidate whose ideal policy is closest to those of the donors receives all donations, while his opponent chooses a position favorable to the voter distribution.

To build a more complete picture of international elections, we model public funding as an extension to our model. When one candidate receives a funding advantage prior to announcing his position (as is common in many publicly funded elections where funding is allocated based on votes in the previous election), we find that he positions closer to his ideal policy, while his opponent also positions closer to him in order to balance the advantage in public funding.

We also model the effect of campaign contribution limits, as seen in the US until 2014 (for a list of countries setting contribution or spending limits, see Appendix 4). We find that when donation constraints are set low, candidates behave as if donations were absent, leading to maximal policy divergence, but their incentives align with voter preferences. Intermediate constraints yield situations where no equilibrium in pure strategies exists, and unbinding high constraints (or no limits) yield the least amount of policy divergence, but allow equilibrium candidate positions to be distorted towards the ideal policies of the donors rather than the voters.

Our results thus contribute to the discussion about the costs and benefits of limiting campaign contributions. On one hand, a social planner can implement stringent donations, which aligns candidate behavior with voter preferences at the cost of increased policy divergence. On the other hand, a social planner can leave donations unlimited, which allows candidate behavior to skew in favor of donor preferences, but increases policy convergence. Depending on donor preferences relative to those of voters and how effective donations are at influencing the election, either stance of the social planner could be optimal.

Related Literature. Our model extends previous work done by Ball (1999a), which considers a spatial setting where policy positions are ordered along a Hotelling line, which borrows from the works of Hotelling (1929) and d'Aspremont, et al (1979). The seminal work by Downs (1957) establishes that, when every candidate only seeks to maximize his own probability of winning an election, he positions at the median voter, thus achieving perfect policy convergence.[2] McKelvey (1975) and Wittman (1983) build upon the work of Downs but assume that candidates care about which policy is implemented (and have different desired policies), rather than their probability of winning the election.[3] In this setting, policy divergence can be sustained in equilibrium, where each candidate positions closer to his own ideal policy and away from the median voter.[4] Our model encompasses both approaches as special cases, adds the role of political contributions, and the effect of limiting campaign funding.[5]

From the perspective of general campaign spending, Welch (1974) develops an initial economic model of campaign spending. Adamany (1977) and Welch (1978) later explore the effect of striking down the Federal Election Campaign Act of 1976. In the campaign finance literature, the seminal work of Austen-Smith (1987) establishes that candidates announce their policy positions anticipating how their potential donors react to such an announcement.[6] Ball (1999a) builds upon this initial model and shows that candidates not only choose their policy positions based on how they expect their own potential donors to react, but they also take into consideration the effect that their position has on the amount of donations that their rival receives. Our work extends Ball's by allowing for asymmetries, such as when both candidates or donors prefer a policy position that is above or below that of the median voter. In Ball's original model, symmetry was required in order to guarantee a closed form solution, as described in Ball (1999b). By using numerical analysis, we examine asymmetries and analyze the cases where donors, candidates, or voters favor policies that are not perfectly balanced against one another. In addition, Ball's work focused solely on the case in which candidates obtain utility from implemented policies (i.e., based on Wittman's model). We consider candidates that are interested in winning the election (as in Downs), in the implemented

[2] For an example of an election with three candidates, see Evrenk and Kha (2011).

[3] Additional work by Barro (1973) examines how candidate motivations may not coincide with those of their electoral base.

[4] These results were extended in Calvert (1985) where he demonstrated under what assumptions policy convergence occurs. Empirical work by Morton (1993) tests these predictions and suggests that policy divergence occurs, but to less than the theoretical prediction. Work by Zakharov and Sorokin (2014) suggests that these results also hold for a wide array of probability functions for voters.

[5] Candidates' announcements of policy positions might not be credible to donors or voters. Work by Alessina (1987) and Aragones. et al. (2007), however, suggest that when candidates face repeated elections, voters (and by extension, donors) recall when implemented and announced policy positions differ, and form their beliefs on a candidate's true position accordingly. As a result, candidates announce the policy that they intend to implement and can be considered credible. We assume that candidates cannot shirk on their announced positions, but our setting can be extended to models where candidates build their reputation.

[6] Work by Hinich and Munger (1994) explores campaign contributions as a hinderance to a rival rather than a benefit to a preferred candidate.

policy (as in Wittman), or in both.

Regarding advertising in political elections, Kaid (1982) suggests that spending money on political advertisements increases the name recognition for that candidate. In addition, Bowen (1994), and West (2005) conclude that advertising brings other positive effects to a candidate, which may not be directly related to vote shares, such as likability. Regarding vote shares, Shaw (1999), Goldstein and Freedman (2000), Stratmann (2009) and Gordon and Hartmann (2013) demonstrate that advertising in an election directly leads to an increase in vote share received by the advertising candidate. We follow this line of reasoning and treat advertising expenditure as a method to increase the probability that the advertising candidate wins the election.

Several studies estimate ideal policy positions of elected officials. Poole and Rosenthal (1984) measure policy divergence in the U.S. Senate from 1959 to 1980 and find that policy divergence increased over that time period. Hare and Poole (2014) follow up on this initial study, updating the data to the Obama administration and find that, while the Democrat party became slightly more liberal since 1980, the Republican party faced a large conservative shift. Our paper models these shifts in ideal policy positions and their resulting change in equilibrium policies and donations.[7]

In real life, special interest groups can offer donations at any stage of the electoral competition. While several studies consider donors offering contribution menus at the beginning of the game, as in Grossman and Helpman (2001), our setting allows contributions after candidates choose policy positions. Therefore, our model closely fits recent elections such as the 2012 U.S. Presidential election.[8] When donors act first, contributions must specify a rather involved contract defining donations as a function of both candidates' positions, which is typically illegal and thus not enforceable. Our setting, however, allows donors to observe candidates' positions and respond to them without the need for a contract.

Our model's time structure is therefore similar to that in Herrera et al. (2008), which considers two parties choosing binding policy positions during the first stage, and their campaign effort (funds)

[7] Empirical work has been done to estimate the effect of campaign contribution limits. Jacobson (1978) and Coate (2004) focus on how contribution limits affected the margin of victory between candidates (namely, an incumbent against a challenger). Our model, however, examines the effect of contribution limits on equilibrium positioning, rather than the margin of victory.

[8] In 2012, Mitt Romney faced a long primary challenge, which required him to spend 87% of the $153 million raised up until June 2012, when he clinched the Republican nomination. In contrast, incumbent President Barack Obama spent 69% of the $303 million that he had raised over the same period. To raise additional funds, Romney courted donors who had either supported his rivals in the primary election or stayed out completely. These donors were able to observe Romney's policy positions long before ever offering him their aid.

in the second stage. Our setting, however, assumes that funding comes from donors. Anticipating the donors' ideal policies, we show that candidates can alter their announced policies in the first stage to capture a larger contribution than their rival. We also allow for public funding to play a role in candidates' policies, as in Ortuno and Schultz (2005). Unlike our model, they consider that candidates do not receive contributions from donors. Instead, they assume that candidates campaign is financed through two sources: public funding, based on the candidate's vote share; and non-public sources (a lump sum subsidy) which does not provide candidates with incentives to alter their policy positions to receive more funds. In contrast, our model allows for strategic effects to arise between candidates and donors.

Section 2 describes the model, while section 3 presents equilibrium results in the first and second stage. Section 4 provides a numerical simulation to illustrate our findings, and section 5 extends our model to asymmetric candidates, donors, and voter distributions; allows for public funding; and analyzes the effect of donation constraints in equilibrium policy positions. Section 6 concludes and discusses our results.

2 Model

Consider two candidates competing for office. Every candidate $i \in \{A, B\}$ chooses a policy position $x_i \in [0, 1]$. Every voter has his own preferred policy position distributed along $[0, 1]$. Voters choose the candidate that gives them the higher utility level and are more likely to vote for a candidate whose policy position is closer to their own. Every candidates seeks to maximize a combination of his probability of winning the election (as in Downs (1957)), and his expected utility they receive from the elected policy (as in Wittman (1983)).

In addition to candidates and voters, we also include two donors, where each donor k is endowed with an exogenous sum of money, $\bar{k}$, and with a preferred position, $\hat{d}_k$, where $k \in \{L, R\}$.[9] Donors face a marginal cost of contributions c, which represents the opportunity cost of money. As we describe in the following sections, every donor uses these donations to induce candidates to position closer to the donor's ideal policy. In turn, a campaign with more funding is able to better advertise their respective candidate, and will increase the probability that candidate wins the election.

[9] We assume that donors are solely interested in policy outcomes, as in Ball (1999a) and Austen-Smith (1987).

The time structure of the game is the following: In stage 1, candidates simultaneously and independently announce their policy positions, which are observed by both donors and voters. In stage 2, every donor responds simultaneously and independently allocating donations to one, both, or no candidates. In the last stage, votes are a function of the candidates' chosen positions as in Downs' and Wittman's models; where this probability is affected by donors' contributions to each candidate. Since this probability function is exogenous, its effect is only considered when analyzing candidates' probability of winning (subsection 3.1), and donors' expected utility from the implemented policy (subsection 3.2).

3 Equilibrium results

3.1 Stage 2 - *Donor's funding decisions*

In stage 2, every donor $k \in \{L, R\}$ seeks to maximize his expected utility based on the outcome of the election, which we can express with the following objective function:

$$\max_{k_i, k_j \geq 0} \quad p_i(x_i, x_j, D_i, D_j)u_k(x_i; \hat{d}_k) + [1 - p_i(x_i, x_j, D_i, D_j)]\, u_k(x_j; \hat{d}_k) - (k_i + k_j)\, c \tag{1}$$

$$\text{s.t. } k_i + k_j \leq \bar{k}$$

where k_i denotes donor k's monetary contribution to candidate i's campaign, where $i \in \{A, B\}$, $p_i(\cdot)$ represents the subjective probability that candidate i wins the election, x_i denotes candidate i's policy position, D_i captures the aggregate donations received by candidate i, i.e., $D_i \equiv k_i + l_i$; and $u_k(\cdot)$ represents the utility that donor k receives based on the outcome of the election. As in the standard Downsian (1967) and Wittman (1983) models, candidates do not have precise knowledge of voters' utility functions, which prevents candidates from knowing exactly how voters will vote. (All proofs are relegated to the Appendix)

For compactness, let $g_k(x_i, x_j) \equiv u_k(x_i; \hat{d}_k) - u_k(x_j; \hat{d}_k)$ be the utility gain that donor k obtains from candidate i relative to $j \neq i$. Solving for $g_k(x_i, x_j)$ in expression (2) yields $g_k(x_i, x_j) \leq \hat{p}_i$, where $\hat{p}_i \equiv \frac{c}{\frac{dp_i}{dD_i}}$. Figure 1 depicts the utility gains of donor k, $g_k(x_i, x_j)$ in the horizontal axis; and includes $\hat{p}_i$, $\hat{p}_j$ and the origin where $g_k(x_i, x_j) = 0$, which gives rise to three different regions, A-C. For instance, region A satisfies $g_k(x_i, x_j) \geq \hat{p}_i$; region B has $\hat{p}_j < g_k(x_i, x_j) < \hat{p}_i$; and similarly for region C.

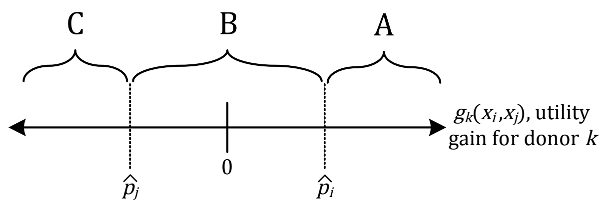

Figure 1. Donor behavior.

Proposition 1. *Equilibrium donations* (k_i^*, k_j^*) *satisfy:*

1. Region A. $0 < k_i^* \leq \bar{k}$, $k_j^* = 0$ if $g_k(x_i, x_j) \geq \hat{p}_i$.

2. Region B. $k_i^* = k_j^* = 0$ if $\hat{p}_j < g_k(x_i, x_j) < \hat{p}_i$.

3. Region C. $k_i^* = 0$, $0 < k_j^* \leq \bar{k}$ if $g_k(x_i, x_j) \leq \hat{p}_j$.

In region A (C) of figure 1, donor k's marginal benefit of contributions to candidate i (j) is greater than or equal to his marginal cost. Intuitively, the policy position that candidate i (j) presents to donor k is favorable relative to candidate $j \neq i$'s $(i \neq j$'s$)$ and donor k makes a contribution to candidate i's (j's) campaign. When the marginal benefit of contributions is equal to the marginal cost, donor k contributes some positive amount, $k_i^* > 0$ $(k_j^* > 0)$, whereas when the marginal benefit strictly exceeds the marginal cost, donor k contributes as much as possible to candidate i (j), i.e., $k_i^* = \bar{k}$ $(k_j^* = \bar{k}$, respectively).

Lastly, in region B, the marginal benefit that donor k receives from contributions to either candidate i is strictly less than the marginal cost. This induces donor k to withhold all contributions from either candidate. Intuitively, candidate i and $j \neq i$'s policy positions do not differ enough for donor k to support either candidate monetarily.

With the equilibrium behavior of both donors defined as in proposition 1, we can sign the best response functions for both candidates as presented in corollaries 1 and 2.

Corollary 1. *When donors k and $l \neq k$ each contribute to the same (opposite) candidate, their contributions are decreasing (increasing) in the other donor's contribution, i.e.,* $\frac{dk_i}{dl_i} = -1$ $\left(\frac{dk_i}{dl_j} > 0\right)$.

When both donors support the same candidate, additional contributions from either candidate decreases the marginal benefit of further contributions.[10] On the contrary, when donors support opposite candidates, additional contributions from either candidate increases the marginal benefit of additional contributions to the other donor.[11]

Corollary 2. *Donor k's contribution to candidate i, k_i^*, weakly increases (decreases) in candidate i's position, x_i, if $x_i < \hat{d}_k$ ($x_i > \hat{d}_k$, respectively, where $\hat{d}_k$ is donor k's ideal policy position). Furthermore, k_i^* weakly decreases (increases) in candidate j's position, x_j if $x_j < \hat{d}_k$ ($x_j > \hat{d}_k$, respectively).*

When candidate i moves closer to k's ideal policy position, the utility gain that donor k receives from candidate i's position increases. On the other hand, if candidate j moves towards donor k's ideal policy position, the utility that donor k receives from candidate j's position increases, thus decreasing the marginal benefit of donor k contributing to candidate i.

3.2 Stage 1 - *Policy decisions*

In the first stage, every candidate $i \in \{A, B\}$ seeks to maximize his expected payoff, given what he anticipates how donors will respond in the second stage. For simplicity, let $p_i \equiv p_i(x_i, x_j, k_i^* + l_i^*, k_j^* + l_j^*)$ denote the probability that candidate i wins the election. Therefore, candidate i solves

$$\max_{x_i} \; \gamma\underbrace{[p_i(v_i(x_i) + w) + (1 - p_i)v_i(x_j)]}_{\text{Wittman}} + (1 - \gamma)\underbrace{p_i}_{\text{Downs}} \tag{2}$$

where $v_i(x_i)$ represents the payoff that candidate i receives when his chosen policy is elected (i.e., he wins the election); $v_i(x_j)$ denotes candidate i's payoff from losing the election (but having

[10] Intuitively, both donors benefit from the contribution made to the supported candidate, and a similar situation as a public good arises, where donors free ride upon each others' contributions. In particular, every dollar donated to candidate i by donor l decreases donor k's contribution by exactly one dollar.

[11] Intuitively, contributions from one donor are detrimental to the other donor's outcome in the election, and the other donor has incentive to further contribute to his own candidate to protect his interests in the election.

candidate j's policy position implemented); w represents the utility that candidate i receives from holding office; and γ reflects how candidate i weighs his expected payoff from policy implementation against his probability of winning the election. Hence, the first term in expression (2) coincides with the specification used in Wittman (1983), where candidates are expected payoff maximizers, while the second term in expression (2) corresponds with the original Downsian (1957) model where candidates maximize the probability to win the election. Thus, by setting $\gamma = 1$, our model becomes identical to Wittman's (1983), or if we set $\gamma = 0$, we obtain Downs' (1957) model. Function $v_i(\cdot)$ is strictly concave, reaching a maximum at candidate i's ideal policy position, $\hat{x}_i$, which we assume to be exogenously given.[12] The next lemma studies equilibrium conditions from candidate i's problem in (2).

Lemma 1. *Candidate i's equilibrium position, x_i, solves*

$$\underbrace{\frac{dp_i}{dx_i}\left[\gamma(v_i(x_i) + w - v_i(x_j)) + (1-\gamma)\right]}_{\text{Direct effect}} + \underbrace{p_i\gamma\frac{dv_i(x_i)}{dx_i}}_{\text{Indirect effect}} = 0 \tag{3}$$

where term $\frac{dp_i}{dx_i}$ can be expressed as

$$\frac{dp_i}{dx_i} = \frac{\partial p_i}{\partial x_i} + \frac{\partial p_i}{\partial D_i}\left[\frac{dk_i^*}{dx_i} + \frac{dl_i^*}{dx_i}\right] + \frac{\partial p_i}{\partial D_j}\left[\frac{dk_j^*}{dx_i} + \frac{dl_j^*}{dx_i}\right] \tag{4}$$

In words, an increase in x_i improves candidate i's probability of winning the election (making expression (4) positive) if: (1) the additional votes he receives offset the potential loss in contributions (relative to candidate j's donations); (2) if, instead, the loss in votes he experiences from increasing x_i is compensated by the more generous contributions he receives from donors; or a combination of both cases.[13]

Corollary 3. *In the Downsian specification ($\gamma = 0$), expression (3) simplifies to $\frac{dp_i}{dx_i} = 0$. In*

[12] Many studies consider that functions such as $v_i(x_i) = -A(x_i - \hat{x}_i)^2$, where $A > 0$, which is negative everywhere except at its max when $x_i = \hat{x}_i$, i.e., the implemented policy coincides with candidate i's ideal, where $v_i(\hat{x}_i) = 0$.

[13] In the first term on the right-hand side, an increase in x_i corresponds with candidate i moving closer to (away from) the median voter, thus making his policy position more (less) attractive to voters and increasing (decreasing) the probability that he wins the election. The second term depicts how candidate i's policy position affects the contributions he receives from each donor. We know that $\frac{\partial p_i}{\partial D_i} > 0$ since an increase in donations to candidate i increases his chances of winning the election. The signs of $\frac{dk_i^*}{dx_i}$ and $\frac{dl_i^*}{dx_i}$ depend on candidate i's policy position relative to the ideal policies of donors k and l, respectively.

the Wittman's specification ($\gamma = 1$), expression (3) reduces to

$$\frac{dp_i}{dx_i}(v_i(x_i) + w - v_i(x_j)) + p_i \frac{dv_i(x_i)}{dx_i} = 0$$

whereas expression (4) remains unchanged in both specifications.

In the Downsian specification, $\frac{dp_i}{dx_i} = 0$ indicates that candidate i changes his policy position until his probability of winning no longer increases, disregarding the specific policy he implements. In the Wittman specification, candidates focus instead on maximizing their expected policy payoff.

Lemma 2: *If* $x_i \leq \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ *then it is strictly dominated by* $\min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\} + \varepsilon$ *where* $\varepsilon > 0$ *and* m *represents the median voter's ideal policy. If* $x_i \geq \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$*, then it is strictly dominated by* $\max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\} - \varepsilon$.

Intuitively, Lemma 2 implies that both candidates, when choosing their positions, balance the effects of both their own policy positions and the donations that those policy positions yield in order to maximize their expected payoff.

From the above lemma, we restrict our attention to undominated strategies $\min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\} < x_i < \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$. Due to the presence of corner solutions outlined in the second stage, an analytical solution is not feasible. Furthermore, our model does not satisfy Ball's (1999) assumption 4, which requires that $p(x_i, x_j) = \Pi(x_i + x_j)$, where $\Pi(x_i + x_j)$ is some continuous function; implying that it does not necessarily converge to a fixed point donation.

Several of the below results are driven by every candidate's incentive to reduce his rival's donations, as shown in the following corollary. Let us define symmetric donors as those who, when having the same ideal policy $\hat{d}_k$, experience the same utility from every policy x_i, i.e., $u_k(x_i; \hat{d}_k) = u_l(x_i; \hat{d}_k)$ for every x_i.

Corollary 4. *When donors are symmetric and support separate candidates, and if* $\left|x_i - \hat{d}_k\right| < \left|x_i - \hat{d}_l\right|$*, then* $\frac{dk_i}{dx_i} > \frac{dl_j}{dx_i}$.

In words, this corollary says that when candidate i positions himself closer to donor k's ideal policy than donor l's ideal policy, i.e., $\left|x_i - \hat{d}_k\right| < \left|x_i - \hat{d}_l\right|$, an increase in policy position from candidate i causes donor l to reduce his contribution to candidate j by more than donor k's reduction to candidate i.[14] Both donors reduce their donations to their respective candidates, but candidate j's reductions decrease by a greater amount. Subsequent sections show that candidates have incentives to lower their rival's donations because of the above result.

4 Numerical Analysis

To simulate best response functions for candidate $i \in \{A, B\}$, our goal was to make the probability function as general as possible. Starting with the voters, we assume that their ideal policy positions are distributed according to the Beta distribution. This functional form was chosen for its general flexibility and because its support aligns with the range of our voter ideologies. Let $B(x; \alpha, \beta)$ represent the cumulative distribution function for the Beta Distribution at point x, given shape parameters α and β.[15] If $x_i < x_j$ ($x_i > x_j$), the contribution to the probability that candidate i wins the election based on their policy position relative to the voters' ideal policy positions is $B\left(\frac{x_i+x_j}{2}; \alpha, \beta\right)$ $\left(1 - B\left(\frac{x_i+x_j}{2}; \alpha, \beta\right)\right)$ where $\frac{x_i+x_j}{2}$ represents the midpoint between the candidate policy positions. Intuitively, point $\frac{x_i+x_j}{2}$ is where the indifferent voter is located along the policy spectrum. If $x_i < x_j$ ($x_i > x_j$), all voters below $\frac{x_i+x_j}{2}$ prefer the policy of candidate i (j), while all those above prefer the policy of candidate j (i).

With regard to donations, the contribution to the probability that candidate i wins the election from donations is assumed to follow a standard normal distribution. Let $N(x)$ represent the cumulative distribution function of the standard normal distribution. The contribution to the probability that candidate i wins the election based on their received donations is $N(D_i^\eta - D_j^\eta)$ where $\eta \in (0, 1)$ is an additional concavity parameter that assists with differentiation. Intuitively, a candidate that already receives more in donations than their rival will not benefit as much from additional donations. For simplicity, the effectiveness of donations at increasing candidate i's probability of winning

[14] Since candidate i was relatively close to donor k's ideal policy position, and donors' utility is concave by definition, candidate i's increase in x_i causes only a small reduction in donor k's expected utility, but gives donor l a much larger gain in his expected utility.

[15] Utilizing α and β, we can simulate several different population distributions. When $\alpha = \beta > 1$, we obtain a symmetric population of voters whose mean is 0.5 and are more concentrated towards the center of the distribution. Alternatively, when $\alpha = \beta < 1$, we obtain a symmetric population of voters whose mean is 0.5 but are more concentrated on the tails of the distribution. With $\alpha > \beta > 1$, we obtain a distribution that has a mean above 0.5 and has a negative skew. Lastly, with $\beta > \alpha > 1$, we obtain a distribution that has a mean below 0.5 and has a negative skew.

is not affected by the candidate's political platform. However, richer environments could consider that, besides the donation advantage that candidate i receives relative to his rival, candidates in certain political platforms are more likely to benefit from this donation advantage.

Combining both of these contributions, the probability that candidate i wins the election is the linear combination of both contributions, i.e.,

$$p(x_i, x_j, D_i, D_j) = \begin{cases} (1-\lambda)B\left(\frac{x_i+x_j}{2};\alpha,\beta\right) + \lambda N(D_i^\eta - D_j^\eta) & \text{if } x_i < x_j \\ (1-\lambda)\left[1 - B\left(\frac{x_i+x_j}{2};\alpha,\beta\right)\right] + \lambda N(D_i^\eta - D_j^\eta) & \text{if } x_i > x_j \end{cases} \quad (5)$$

where $\lambda \in (0,1)$ denotes the weight of donations received relative to candidate policy position on the probability that candidate i wins the election.[16]

For utility functions, donors face the following utility functions,

$$u_k(x_i;\hat{d}_k) = -(x_i - \hat{d}_k)^2$$

while candidates face similar utility functions,

$$v_i(x_j) = -(x_i - \hat{x}_j)^2$$

These functions were chosen since they allow for a bliss point, where donors and candidates maximize their utility at exactly their most preferred policy position, and lose utility as they move away from that position.[17]

The analysis was performed as follows:

1. We first divided the $[0, 1]$ interval into 1,001 equally spaced points (e.g., 0, 0.001, 0.002,

[16] This is a significant departure from Ball's (1999a) original model, which did not consider the effectiveness of donations as a linear combination, but rather as a parameter (Ball refers to it as γ). We choose this method as it guarantees that the probability that candidate i wins the election falls between 0 and 1, inclusive.

[17] This specification can lead to convexity issues. While we are unable to prove that the set of all Downsian and Wittman utilities are convex analytically, we examined several instances of this set numerically. For this analysis, all donation levels and the position of candidate j, x_j, were fixed, and only the values of candidate i's policy position, x_i and the linear combination term between the Downsian and Wittman specifications, γ, were varied. We then calculated the resulting payoffs for candidate i. Generally, we find that candidate i's best response function only exhibits discontinuities when γ is extremely low, as described later in this section, or in the special case in which both candidate i and j have the same ideal policy position. In these cases, we can numerically show that our model predicts policy convergence. Otherwise, candidate i's best response function is continuous and the set of Downsian and Wittman utilities is convex around candidate i's best response.

etc.).[18]

2. We pick one value of x_j from the above range at a time. For each value of x_j, we consider all 1,001 values of x_i, calculating for each policy pair the donations made to each candidate i and j, and their corresponding expected payoffs.[19]

3. We identify the value of x_i that maximizes candidate i's expected payoff. This identifies candidate i's best response to the chosen value of x_j.

4. We then repeat the process for all values of x_j, to characterize candidate i's best responses.

5. We then repeat steps (1)-(4) picking all values of x_i at a time, to identify candidate j's best response.

6. We finally find where the two best responses intersect to identify the Nash equilibrium of the donation game.

This discretization of the best responses converges to a continuous function as the number of equally spaced points approaches infinity, as described in Lemma 3.

Lemma 3: *For increasingly smaller intervals between equally spaced points, the approximate equilibrium described in step (6) converges to the exact equilibrium of the underlying continuous function.*

Intuitively, Lemma 3 implies that as we add more equally spaced points to our numerical simulation, our equilibrium result approaches the analytical solution to candidate i's political position solving equation (3).

[18] To approximate continuity, the numerical analysis was also performed using 2,001 and 5,001 equally spaced points, confirming that the results are unaffected. We then reproduced our simulations by randomly drawning 1,001 points on the $[0, 1]$ interval, showing that the results were essentially identical to those using equally spaced points.

[19] For instance, if parameters are $\gamma = 1$, $\lambda = 0.5$, $w = 0$, $\alpha = \beta = 2$, $\hat{d}_k = 0.2$, $\hat{d}_l = 0.8$, $\hat{x}_i = 0.3$, $\hat{x}_j = 0.7$, $\bar{k} = \bar{l} = 1$, $\eta = 0.5$, and $c = 0.03$; if we start with $x_j = 0.1$, candidate i's highest payoff occurs at $x_i = 0.3$ where his payoff becomes -0.0053.

Figure 4 summarizes the results of this numerical simulation.

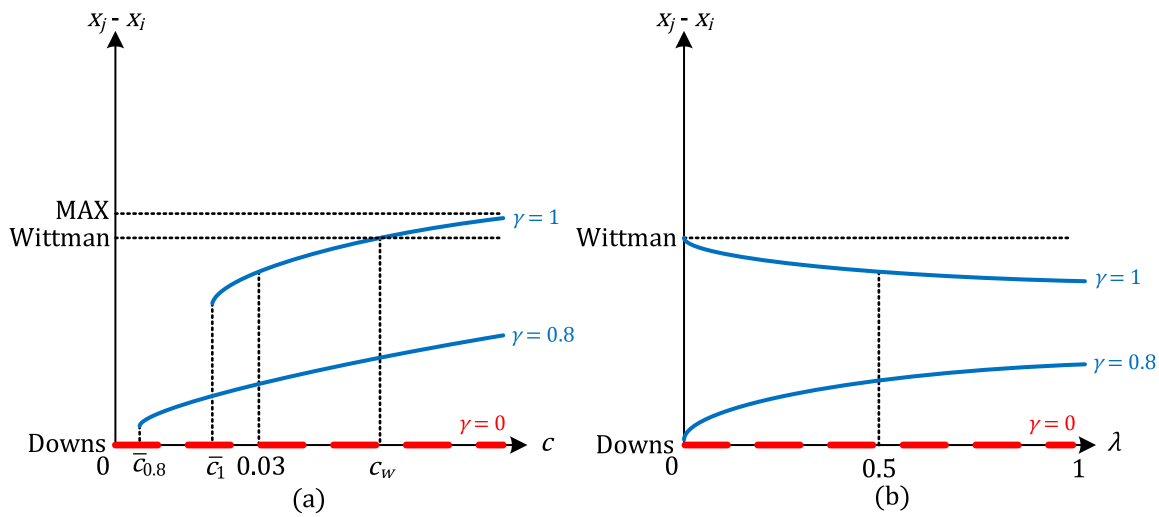

Figure 4. Policy divergence as a function of c and λ for different values of γ.

Figure 4a depicts policy divergence as a function of c when λ is held at 0.5. When $\gamma = 0$, as in the Downsian (1957) specification, we obtain zero policy divergence and both candidates position at the median voter's ideal policy. This result holds regardless of the costs of donations, c, and their effectiveness, λ. Intuitively, since candidates reduce their rivals donations by moving closer to one another, positioning at the median voter maximizes the probability of winning when no donations are made When $\gamma = 1$, as in the Wittman (1983) specification, policy divergence is a function of the marginal cost of donations, c, as well as their effectiveness, λ, as depicted in figure 4b. Beginning from the right side of figure 4a, when c approaches infinity, candidates receive no donations, and respond positioning at maximal policy divergence.[20] This result is more divergent than that in the original Wittman model due to the reduced voter sensitivity, as explained in Wittman (1983). As c decreases, candidates converge in order to deny donations to their rival. For intermediate values of c, $\bar{c} < c < c_w$, our model produces less policy divergence than in the original Wittman model as candidates continue to position closer to one another. Lastly, for values of $c < \bar{c}$, no equilibrium emerges.

[20] In this situation, candidate i positions between his own ideal policy, $\hat{x}_i$ and the location of the median voter, but prefers positioning closer to $\hat{x}_i$.

Figure 4b depicts policy divergence as a function of λ, where c is held at 0.03. When γ is sufficiently low ($\gamma < 0.655$), policy convergence arises for low values of λ; which includes the case of $\gamma = 0$ where policy convergence occurs for all values of λ, as illustrated in the figure. In contrast, when γ is relatively high, positive policy divergence emerges as depicted in the figure. Graphically, this is represented in the vertical axis of figure 4b, where donations are ineffective ($\lambda = 0$), and where policy divergence increases in γ. When donations become more effective ($\lambda > 0$), this policy divergence is attenuated, since candidates seek to position closer to their donors.[21]

On the other hand, as the effectiveness of donations increases, we see candidates shift from targeting the median voters' ideal preferences to the donors' ideal preferences. When donors' ideal policies are not symmetric around the median voters, this leads to a scenario where candidates' chosen policies are skewed towards donors relative to those preferred by the voters. Intuitively, if voters have a high value of λ, they easily influenced by campaign contributions (in the form of advertisements, etc.). In this setting, they can be convinced to vote for a candidate that receives large contributions whose policy is farther away from their own ideal policy rather than a candidate whose policy is closer to their own ideal, but receives fewer contributions. This leads candidates to align themselves with donor preferences in order to maximize his own contributions and minimize those received by his opponent.

4.1 Asymmetries

As we introduce asymmetries into the model, they take three distinct forms: asymmetries in voter distribution, candidates' ideal policies, and donors' ideal policies. This allows us to have voters that favor one end of the policy spectrum over the other, or candidates and donors whose ideal policy position are no longer equidistant to the median voter. Introducing any form of asymmetry into our model (voter distribution, candidates' ideal policies, and donors' ideal policies), causes significant changes to models with low values of γ, but minimal changes to models with high values of γ.

High Value of γ. For models with high values of γ (which contains the Wittman (1983) model where $\gamma = 1$), asymmetries simply change the location of the equilibrium, but do not

[21] In Appendix 2, we show how equilibrium candidate policy position shifts as the parameters c and λ vary. Consistent with the results in Corollary 4, as the marginal cost of donations decreases, candidates position themselves closer to the median voter to deny donations to their opponent.

prevent equilibria from emerging. With an asymmetric voter distribution that results in the median voter having a higher (lower) ideal policy position, candidates respond by increasing (decreasing, respectively) their equilibrium location to be closer to the median voter. Likewise, for asymmetric ideal policy positions among candidates or donors that result in the midpoint of their ideal policies being above (below) the midpoint of the policy spectrum, both candidates respond by increasing (decreasing, respectively) their equilibrium location. Intuitively, candidates have strong incentives to move in the same direction as changes to the ideal policies of the voters, donors, and themselves.

These asymmetries can lead to policy misalignment between voters and candidates. Without donations, the equilibrium policy positions of both candidates reflect their own preferences, as well as those of the voters, and this relationship is maintained with the introduction of donors as long as symmetry exists in the candidate ideal policy positions, donor ideal policy position, and voter distribution.[22] As λ increases, candidates put more weight on donor ideal policy positions, and less on those of their own voters, leading them to choose equilibrium policy positions that are not reflective of their voting constituencies. We depict this relationship below in figure 5, which depicts the skewness of candidate policy position on the vertical axis away from their voters, measured as $\left|x_i^* + x_j^* - 2m\right|$, and the degree of asymmetry of donors on the horizontal axis, which we measure

[22] Intuitively, under symmetry, donor preferences also align with voter preferences.

as $|x_k + x_l - 2m|$.[23]

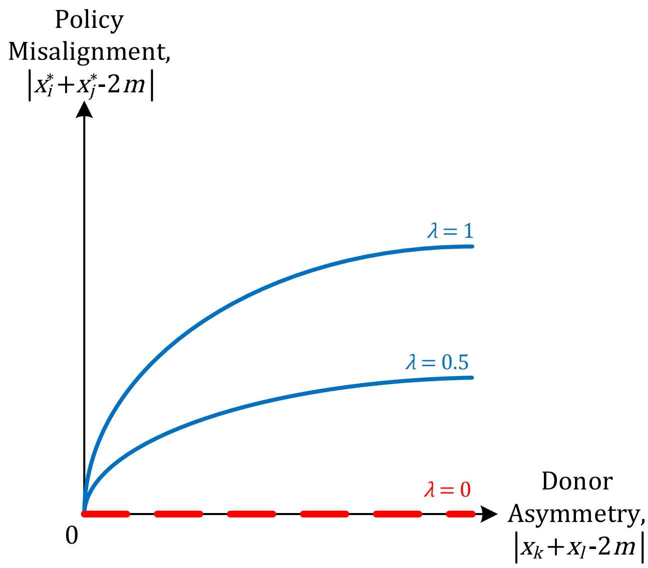

Figure 5. Policy misalignment.

In figure 5, as the asymmetry between donors increases, we find that candidates shift further away from their voters' preferred policies towards their donors. In fact, when $\lambda = 1$, candidates completely disregard voter preferences in favor of those of the donors, as voters become perfectly influenced by campaign contributions, and the election is effectively bought by the donors. The effect is not as large when $\lambda < 1$, as voters cannot be influenced as strongly by campaign contributions, requiring candidates to take their ideal policies into account as well. Lastly, when $\lambda = 0$ and we return to the case where voters cannot be influenced by campaign contributions, we observe no policy misalignment.

Low Value of γ. When we allow for an asymmetric voter distribution, we observe situations in settings with low values of γ (including the Downsian (1957) model where $\gamma = 0$) where such distributions give rise to profitable deviations from the previous policy convergence at the median voter. Since the median voter is no longer located at the midpoint between the donors' ideal policies, deviations from his position induce large contributions from the donor located farther

[23] The asymmetry measurment is derived from the difference between each position and that of the median voter, m, $|x_k - m - (x_l - m)|$ which simplifies to our expression. Notably, when policy positions are equidistant from the median voter, the expression evaluates to zero.

away from the median voter, while the donor located nearer to the median voter contributes little in comparison. When the marginal cost of donations, c, is low or their effectiveness, λ, is high, it becomes profitable for candidate i to deviate from the median voter and target the larger of the two donations available. In response to this deviation, candidate j positions himself ε closer to the median voter than candidate i, which leads to no equilibrium emerging. In summary, no stable policy positions arise in equilibrium when contributions are relatively cheap (low c) and/or they are effective (high λ, indicating that voters' decisions are highly affected by large campaigns and advertising).

For asymmetries in donors' or candidates' ideal policies, we experience a similar problem in models with low values of γ. In the case of donors' ideal policies, again we find that the ability to obtain a large amount from the donor located farther away from the median voter induces no equilibrium to exist when either c is low or λ is high. Regarding asymmetries in candidates' ideal policy positions, the effect is more subtle.[24] For candidates with extreme, yet similar ideal policy positions, a profitable deviation from the median voter exists which is closer to the candidates' own ideal policy. These models also produce no equilibrium.

Extreme cases. For presentation purposes, we next consider three extreme cases about candidate preferences, donor preferences, or the voter distribution.

Candidate preferences. If candidates have identical policy preferences, i.e., $\hat{x}_i = \hat{x}_j$, we find that both candidates converge towards their (common) ideal policy.[25] Intuitively, since candidates are identical, they choose the same policy in equilibrium, leading donors to not contribute to either candidate. Therefore, both candidates maximize their expected utility by positioning at their common ideal policy. This includes the case where both candidates' ideals lie at extreme positions, such as $\hat{x}_i = \hat{x}_j = 0$.

*Donor preferences***.** When donors have identical policy positions, $\hat{d}_k = \hat{d}_l$, we still experience policy divergence among the candidates, where the candidate whose ideal policy is closest to the donors' (common) ideal policy locates closer to the donors and receives all donations, while the other candidate positions closer to the median voter to remain competitive by targeting voter

[24] When $\gamma = 0$, we return to policy convergence, as candidate policy preferences do not impact the Downsian equilibrium.

[25] This is the only case of complete policy convergence in the Wittman specification we found.

preferences.[26]

Voter distribution. For extreme voter distributions, we examine two cases. First, when the voter distribution has a single peak at the very end of the policy spectrum (by setting $\alpha = 2$ and $\beta = 0.5$, or vice versa), both candidates position below (above, in the case of $\alpha = 0.5$ and $\beta = 2$) the median voter, as locating above (below, respectively) such a position would be strictly dominated, as explained in Lemma 4. Second, we examine a voter distribution with dual peaks, such as one where voters are concentrated at the tails of the beta distribution (by setting $\alpha = \beta = 0.5$). In this context, policy divergence increases as candidates seek to accommodate voters at the ends of the distribution. Intuitively, since so few voters are located at the center of the policy line, candidates do not compete as fiercely for them, instead favoring their own ideal policies and those of the donors.

5 Donation Constraints

Of interest to policy makers is the effect of constraints that limit donors' ability to contribute to their respective candidates. A binding donation constraint can significantly alter the equilibrium of our model. Under certain circumstances, a donation constraint can prevent an equilibrium from emerging at all; as shown in this section. Under specifications with high γ (which includes the Downsian (1957) specification), a binding donation constraint has no effect on the equilibrium behavior of either candidate, since, as shown above, no donations are made in equilibrium to either candidate.

In contrast, in specifications with low γ (which captures the Wittman (1983) specification as a special case), when donors are constrained to a maximum donation, candidate behavior changes relative to the size of those constraints. For simplicity, we categorize the equilibrium behavior when donors are constrained into three cases, with cutoff values of the donation constraint, $\bar{k}$ at k_1 and k_2, where $k_1 < k_2$.[27]

[26] The candidate receiving donations has a higher probability of winning the election under these circumstances. The candidate who positions closer to the median voter (and receives no donations), however, still has a positive probability of winning the election. This is reminiscent of the 1896 United States presidential election won by William McKinley, an industrialist with strong backing by business interests. His opponent, William Jennings Bryan, adopted policy positions that were popular among the general population, but he was unable to raise money from potential donors. McKinley raised \$3.5 million to Bryan's \$0.5 milliion, which led to McKinley winning the election.

[27] For calculated values of k_1 and k_2, refer to the selected simulation results table in Appendix 3.

Case 1: $\bar{k} > k_2$. For high donation constraints, the constraint does not bind and we obtain the same equilibrium as described in the previous section. Of interest is the fact that k_2 is significantly higher than the unconstrained amount that donors contribute to their respective candidates in the unconstrained equilibrium, which we denote as k_U. In other words, if $\bar{k}$ is set to the unconstrained donation level, candidates do not position at the unconstrained equilibrium, as explained in the next case.

Case 2: $\bar{k} \in [k_1, k_2]$. For intermediate constraints, we find that every candidate i deviates from the unconstrained equilibrium. Figure 6 below demonstrates the effect of a binding constraint on the unconstrained equilibrium.

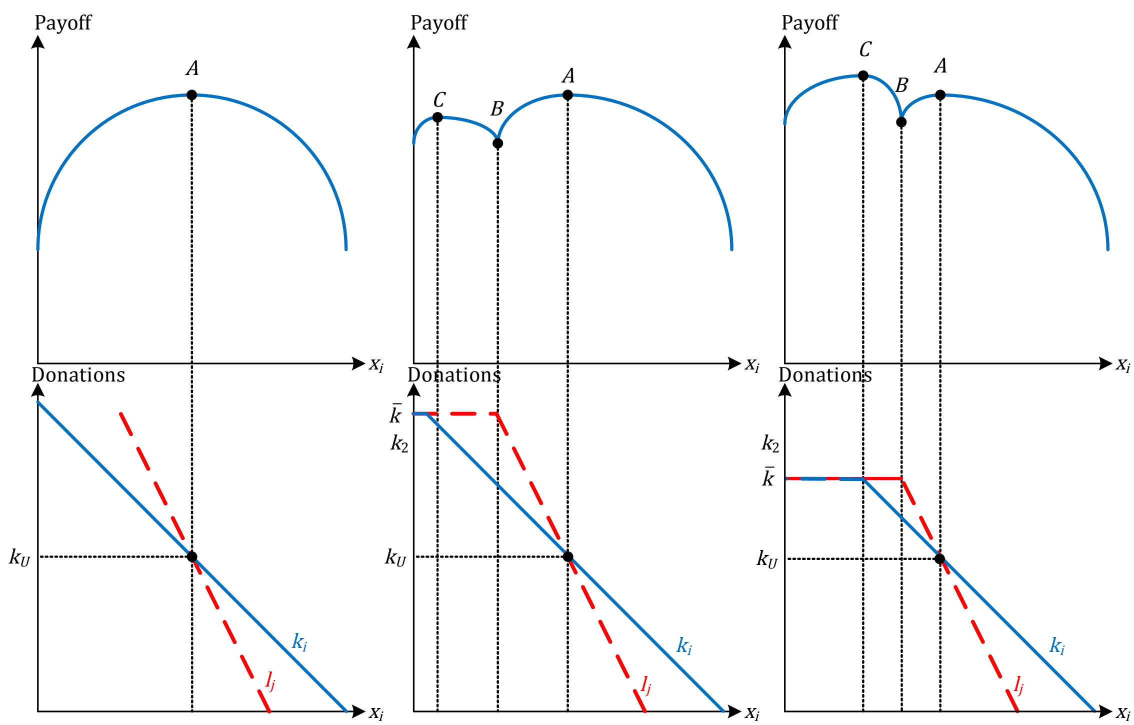

Figure 6. Candidate i's best response at different donation constraints.

The left panel of figure 6 depicts candidate i's payoff function and both candidates' donation levels when donations are unconstrained. In this case, candidate i maximizes his expected payoff by also positioning at the unconstrained equilibrium (located at point A), and receives the same donation level as candidate j.

The center panel of figure 6 plots a constraint $\bar{k} > k_2$, as described in case 1. From Corollary 4, the donations that candidate j receives from donor l are more sensitive to candidate i's position than the donations that candidate i receives from donor k. Thus, as candidate i decreases his position from the unconstrained equilibrium, candidate j receives more donations than candidate i up until point B, where donor l reaches the donation constraint. At this point, candidate i can continue to decrease his position further without candidate j receiving any additional donations. This causes candidate i's payoff to increase since he is able to move closer to his own ideal position as well as receive more donations relative to candidate j (since he is constrained). In this situation, $\bar{k}$ is sufficiently high to allow candidate i to maximize his expected policy by positioning himself at the unconstrained equilibrium (since point A yields a higher payoff than point C).

The right panel of figure 6 represents the case which $\bar{k}$ is above the unconstrained donation level k_U, but below k_2. In this situation, since $\bar{k}$ is sufficiently low, candidate i reaches a higher utility at point C than he does at point A, and thus deviates from the unconstrained equilibrium. The best response functions for both candidate i and j are depicted below in figure 7.

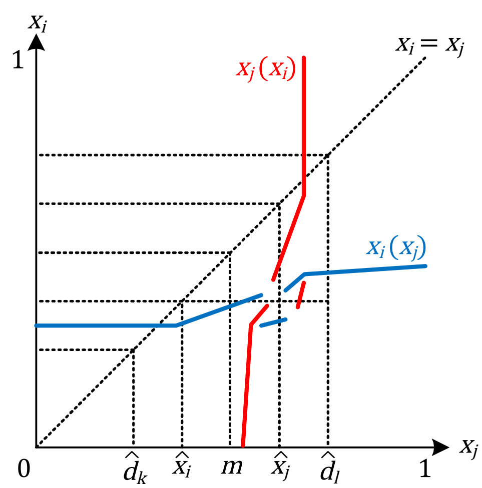

Figure 7. Best response functions at intermediate constraints.

As shown in figure 7, the profitable deviations from the unconstrained equilibria cause significant discontinuities to appear in the best response function, leading to no Nash equilibrium.

Case 3: $\bar{k} < k_1$. For low donation constraints, donors contribute $\bar{k}$ to their respective candidates, but the candidates behave as if they receive no donations at all. When every candidate i is constrained, he cannot reduce the amount of donations his opponent receives by increasing his own policy position (as with low constraints, both candidates remain constrained). Thus, the incentive to position close to one another as described in Corollary 4 does not exists, and candidates position themselves at the location which maximizes their expected policy payoff had they not received any donations.[28]

6 Public Funding

Several countries provide publicly funded lump-sum contributions to political parties based on votes received in the previous election; as in Australia and most European nations.[29] This leads to potential donation advantages that are entirely exogenous. If a candidate knows he has an initial advantage over his rival in donations, he may choose to position closer to his own ideal policy position.

We can adjust our model to include public funding by adding two terms into our probability of winning the election function. Let F_i and F_j denote the amount of public funding candidates i and j receive from the government, which we incorporate into the probability that candidate i wins the election, equation 6, a follows., $N(D_i^\eta + F_i^\eta - (D_j^\eta + F_j^\eta))$.

For compactness, we relegate the numerical results to Appendix 4, and discuss here the main difference with respect to the model without public funding. Specifically, we find that both candidates position closer to the ideal of the candidate with the public funding advantage, where the latter capitalizes on his funding advantage by moving toward his own ideal more than his opponent. This leads to an increase in policy divergence that remains approximately constant in the cost of donations, c, relative to the scenario with no public funding; thus producing a similar result as in figure 7a. Intuitively, if candidate i has a public funding advantage over his opponent, he can use the donation advantage to offset a less appealing position to the voters, moving closer to his own ideal. Candidate j must respond to this by also moving toward candidate i's ideal in order to

[28] Which, as described in the numerical analysis section, has more policy divergence than in the original Wittman (1983) model, but the candidates' do not experience the skewness towards donors' ideal policies due to $\lambda > 0$.

[29] Other countries with limits on campaign contributions are Uruguay, Belgium, Finland, France, Greece, Ireland, Poland, Japan and South Korea; with most limiting individual donations below $8,000. Countries limiting political parties spending include Canada, Austria, Belgium, Czech Republic, France, Greece, Hungary, Ireland, Israel, Italy, Poland, Japan, New Zealand and South Korea.

mitigate his own disadvantage by choosing a position more favorable to voters.

Candidates represent better voter preferences when public funding is available only if the candidate with the public funding advantage has an ideal close to the median voter. Otherwise, candidate's policy positions and voter preferences are misaligned, yet public funding provides less incentives for candidates to position closer to the median voter. Since public funding is distributed according to votes in previous elections, this may lead to the candidate who won previous electoral contests having a substantial funding advantage. If his ideal policies changed (e.g., becoming more radicalized), he could position at a more extreme policy, driving his rival to similar positions, and yet win the election again.

7 Conclusion and Discussion

Our results suggest that as donations become either cheaper or more effective at influencing the outcome of an election, candidates position themselves closer to one another to deny donations to one another. In addition, as asymmetries are included in the model, equilibrium policy positions shift from those that better represent voter preferences to those that better represent donor preferences as the effectiveness of donations increases. This problem is mitigated through strict donation constraints, but this comes at a cost of increased policy divergence among candidates. We summarize five main points further below.

Cheaper donations. Our results suggest that, policies lowering donors' cost of contributions to political campaigns (such as making a larger portion of them tax deductible) induces donors to increase their ability to contribute to either candidate. Anticipating the availability of more donations, we demonstrated that candidates converge in their policy positions, seeking to reduce each other's donations as much as possible; see Figure 4a. However, we also showed that, when donations become extremely cheap, candidates become so concerned about monetary contributions that no stable profile of political platforms from emerging. Hence, our findings indicate that societies that make contributions to political campaigns sufficiently cheap may not experience more policy convergence among candidates, but instead unstable political platforms (e.g., candidates who change their position after his rival alters his own, without ever reaching an equilibrium).

More effective donations. When voters become more influenced by large political campaigns featuring TV ads (higher λ), our results suggest that candidates shift their policy position, from targeting the median voter to targeting the midpoint of the donors. Essentially, money distorts the incentives in the Wittman's model, where candidates care about voter's preferences and their own ideal policy position, since now candidates must also care about the donors' ideal policies. In addition, we show that as donations become more effective at winning votes, both candidates position themselves closer to the midpoint of donors' ideal policies, yielding more policy convergence. Therefore, electorates highly influenceable by campaigns yield more policy convergence than otherwise. This holds, however, when candidates assign a sufficiently high weight to the policy that wins the election. Otherwise, policy convergence emerges around the median voter for all values of λ.[30]

Cynical candidates. Following Wittman's results, our paper confirms that, as candidates assign a larger weight to the utility they obtain from the policy that wins the election (higher γ), political platforms become more divergent, as candidates put less emphasis on maximizing their probability of winning the election and, as a result, seek to position themselves closer to their ideal policies. This occurs even in the absence of political contributions. When donations are present, this effect is attenuated, as candidates balance their own ideal policies with those of the donors.

Incumbent advantage. For public campaign funding systems that allocate based on the results of the previous election, an incumbent candidate starts with a funding advantage, leveraging it to position closer to his own ideal. If previous elections that are closely contested, this effect is minimal. In the case of a previous landslide electoral victory, however, the public funding advantage granted to the incumbent leads to a significant skewing of both candidate positions closer to the incumbent's ideal.

Limiting campaign budgets. Last, our findings help shed light on the effect of setting limits on the amount of money donors can contribute to political campaigns. We show that these constraints yield different results, depending on the severity of the constraint. When campaign contributions are substantially constrained, our findings entail more policy divergence between candidates. However, the midpoint of their policies is now closer to the median voter, so policies can be interpreted as becoming more aligned with voter preferences. In contrast, when constraints are relatively lax,

[30] Of note, there are intermediate values of γ that yield policy convergence only for low values of λ. For example, when $\gamma = 0.5$ with our standard parameters, policy convergence occurs for $\lambda < 0.74$, but policies diverge slightly for values above this.

our results indicate that candidates' positions do not reach an equilibrium. Therefore, if the social planner seeks to minimize policy divergence among candidates, donors must remain unconstrained. On the contrary, if the social planner's objective is to have candidates' policies in line with those of the voter distribution, a low donation constraint is optimal. In summary, aligning candidate policies with those of the median voter comes at a cost of increased policy divergence.

Further Research. Our work leads to several venues for future research. Many democratic countries elect their government from more than two candidates, and adding additional candidates to the model would allow for more robust results. In addition, we consider a one dimensional policy spectrum, where candidates typically chose among many dimensions (different policies) when forming their campaign policy. Following Ball (1999a, 1999b) and other papers in the literature, we consider a probabilistic voting model, which assumes a probability function for voters and, in addition we do not consider a specific utility function for these agents, which prevents welfare comparisons in equilibrium. Considering a utility function for these agents, however, could help us evaluate equilibrium utility, and welfare, which ultimately can lead to a study of the welfare effects of donation constraints to understand whether these constraints are welfare improving under some conditions. Furthermore, our paper assumes that candidates can commit to comply with the policy announcement made in the first stage. However, an alternative model could allow for commitment problems, so candidates can revise their political announcements after donors submit their contributions to each candidate. In that setting, donors would submit contributions menus, as in Grossman and Helpman (2001), and every candidate i would respond with policy $x_i(D_i, D_j)$. Lastly, we consider the effectiveness of campaign contributions, λ, to be uniform among all voters and candidates. For asymmetric values of λ, we could find situations where one candidate favors donations much more than the other.