

Campaign Limits and Policy Convergence:
Asymmetric Agents and Donation Constraints
Appendices

April 3, 2019

Appendix

Appendix 1 - Further details on the numerical simulation

Downsian (1957) specification ($\gamma = 0$). Under the Downsian (1957) specification, candidates seek to solely maximize their probability of winning the election. By setting both $\gamma = 0$ and $\lambda = 0$, we obtain the original model and result proposed by Downs in that each candidate maximizes his probability of winning the election by positioning himself at exactly the median voter. For values of $\lambda > 0$, campaign contributions also determine the probability of winning the election, and our results are presented in figure A1 below.

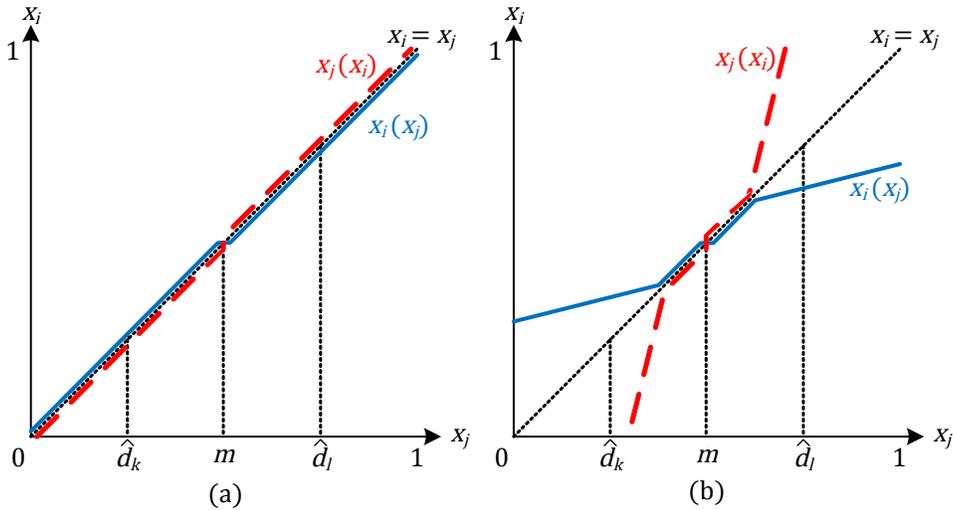


Figure A1. Downsian specification with $\lambda = 0$ and $\lambda > 0$, respectively.

Figure A1(a) plots the results in the original Downsian (1957) model since $\lambda = 0$. The best response for either candidate i is to position himself $\varepsilon > 0$ closer towards the median voter relative to his opponent. This behavior continues until both candidates converge at the median voter and have even odds of winning the election. In figure A1(b), we have the Downsian specification of our model where $\lambda > 0$. Similar to the Downsian model, our model has every consumer i positions himself closer to the median voter's ideal position than his opponent, where the best response is to position $\varepsilon > 0$ closer to the median voter for

positions near the median voter. As candidate i 's opponent deviates significantly from the median voter's ideal policy position, however, candidate i 's best response is now to increase the distance between his own position and that of his opponent (flatter best response function when x_j is either high or low). Intuitively, at the more extreme points of the best response function, the ability to increase his probability of winning by targeting voter preferences diminishes quickly due to the concavity of the probability function.¹

The equilibrium results of the original Downsian model and the Downsian specification of our model remain the same. Thus, the only Nash equilibrium we find is $x_i^* = x_j^* = m$, the location of the median voter, and $k_i^* = k_j^* = l_i^* = l_j^* = 0$, as no donor has any incentive to donate to either candidate. Intuitively, in the Downsian specification of our model, as candidates approach the median voter donations to each candidate effectively disappear. Every candidate has incentive to position at the median voter to maximize his vote share, as well as incentive to deny campaign contributions to his opponent, as described in corollary 4. Furthermore, lowering c , the marginal cost of donations only exacerbates this effect.

Wittman (1983) specification ($\gamma = 1$). In the Wittman (1983) specification, instead of maximizing the probability of winning the election, every candidate i maximizes his expected policy outcome. As a result, candidates position themselves closer to their ideal policy position rather than the median voter's ideal (as in the Downsian (1957) model). When we set $\gamma = 1$ and $\lambda = 0$, we can obtain both the original model and results as presented by Wittman. Allowing for donations to affect the probability of winning the election ($\lambda > 0$), we obtain the results in figure A2.

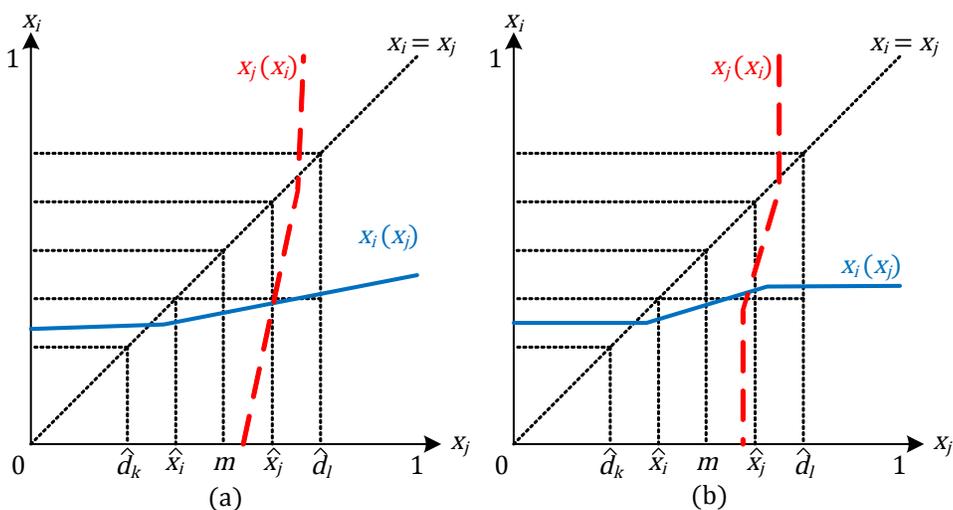


Figure A2. Wittman specification with $\lambda = 0$ and $\lambda > 0$, respectively.

In figure A2(a), we have the original Wittman (1983) model where $\lambda = 0$. Every candidate i positions himself close to his ideal policy position, i.e., the best response function $x_i(x_j)$ lies close to \hat{x}_i . As candidate j positions himself closer to the median voter, candidate i responds by increasing his own policy position, but at a much slower rate than seen in the Downsian (1957) specification (flatter best response function). This leads to an equilibrium where candidate policy positions diverge from the median voter and from their own ideal policies. Figure A2(b) contains the Wittman specification of our model where $\lambda > 0$. The key difference between the two models happens when x_j is relatively high. In this case, the best response of candidate i is to remain even closer to his own ideal policy position since candidate j positions himself significantly above the median voter; see flat segment in the right-hand side of $x_i(x_j)$. The intuition is similar to that of the Downsian specification, as candidate i receives a larger benefit from campaign contributions rather than targeting voter preferences when his opponent positions himself at extreme locations.

The location of our equilibrium under the Wittman (1983) specification with donations can vary relative to the equilibrium of Wittman's model, itself. Intuitively, by introducing donations we both lower voter

¹At these parts of the best response function, candidate i obtains a higher probability of winning the election by distancing himself from his opponent and enabling donors to contribute to his own campaign (as donors will contribute approximately zero when candidates position next to one another).

sensitivity, and introduce bias for whichever candidate receives more donations, as described in Wittman’s (1983) paper. We can reproduce Wittman’s results by setting an inverse relationship between Wittman’s voter sensitivity parameter s and our donation effectiveness parameter, λ ; and a proportional relationship between Wittman’s bias parameter B and a combination of our λ and $N(D_i^\eta - D_j^\eta)$ terms.² As λ increases, voter sensitivity decreases, which causes candidates to move closer to their own ideal policy positions; and the bias increases in magnitude, which causes candidates to move towards whichever candidate the bias favors.³

As the marginal cost of donations, c , decreases, donors contribute more to their preferred candidates. This influx of donations causes both candidates to position closer to each other, since each candidate is able to deny donations to his opponent; as explained in corollary 4. Due to the concavity of the donors’ utility functions, having a candidate that is positioned farther away from the donor move closer entails a much larger increase in utility than having a candidate that is close to the donor move away. Thus, candidate i can significantly reduce the donations that candidate j receives by moving slightly closer to candidate j ’s policy position while only experiencing a slight decrease in the amount of donations that he receives.⁴

In summary, when the marginal cost of donations, c , is sufficiently high $c > c_w$ (low $c < c_w$), our results predict less (more) convergence than in Wittman’s model.⁵ When $\lambda = 0$ (as in the original Wittman model), candidates position between their own ideal policy positions and the ideal policy position of the median voter. As λ increases, candidates put less weight on the location of the median voter’s ideal policy and more weight on the source of the bias, the donors’ ideal policy position. At the extreme case of $\lambda = 1$, candidates disregard the median voter’s ideal policy position entirely when they receive donations, and position themselves between their own ideal position and the midpoint of the donors’ ideal policies, $\frac{\hat{d}_k + \hat{d}_l}{2}$. Other effects are similar to those described in Wittman (1983). If the ideal policy position of either candidate increases (decreases), both candidates increase (decrease) their equilibrium policy position.⁶ Intuitively, if candidate j ’s ideal policy position increases, he positions closer to it, which induces candidate i to increase his own policy position to receive more votes and to deny candidate j additional donations. Likewise, if either donor increases (decreases) his most preferred policy position, both candidates increase (decrease) their equilibrium policy position.⁷

Mixed specification ($0 < \gamma < 1$). Using a mixed specification, we obtain results that fall between the Downsian (1957) and Wittman (1983) specifications. When γ is low ($\gamma < 0.655$ in our example), we find that policy convergence at the median voter occurs. For values of γ above this threshold, policy positions diverge until they reach those at the Wittman (1983) specification when $\gamma = 1$. Interestingly, the best response functions for both candidates show properties of both the Downsian and Wittman models as shown in figure

²Term $N(D_i^\eta - D_j^\eta)$ is our normally distributed contribution to the probability that candidate i wins the election based on their received donations.

³As described in propositions 3B and 4B in Wittman’s (1983) paper. The net effect of an increase in λ is ambiguous, as it strongly depends on the symmetry of ideal policy positions, but in general, as λ increases, candidates have stronger incentives to deny donations to their opponent by moving closer to one another; as described in Corollary 4.

⁴We also find that for every cost $c < \bar{c}$, no Nash equilibrium exists. Intuitively, as donations become extremely cheap, candidate behavior becomes erratic. Candidates receive large donations for even small deviations from their current positions, and constantly vie for the most donations from their respective donors. This causes no equilibrium to emerge. As a note, for large donations, the concavity property of our normal distribution also breaks down, which could be driving this result.

⁵In our numerical analysis, $\gamma = 1$, $\lambda = 0.5$, $w = 0$, $\alpha = \beta = 2$, $\hat{d}_k = 0.2$, $\hat{d}_l = 0.8$, $\hat{x}_i = 0.3$, $\hat{x}_j = 0.7$, $\eta = 0.5$, we obtain that for the value of $c_w = 0.053$, the Wittman model and the Wittman specification of our model yield the same equilibrium policy positions for both candidates.

⁶This includes extreme cases where candidates’ ideal policies are at the endpoints of the policy line, $\hat{x}_i = 0$ and $\hat{x}_j = 1$.

⁷Once again, this also holds for extreme ideal policies, $\hat{x}_k = 0$ and $\hat{x}_l = 1$.

A3. Their behavior depends on each candidate's location relative to their ideal position.

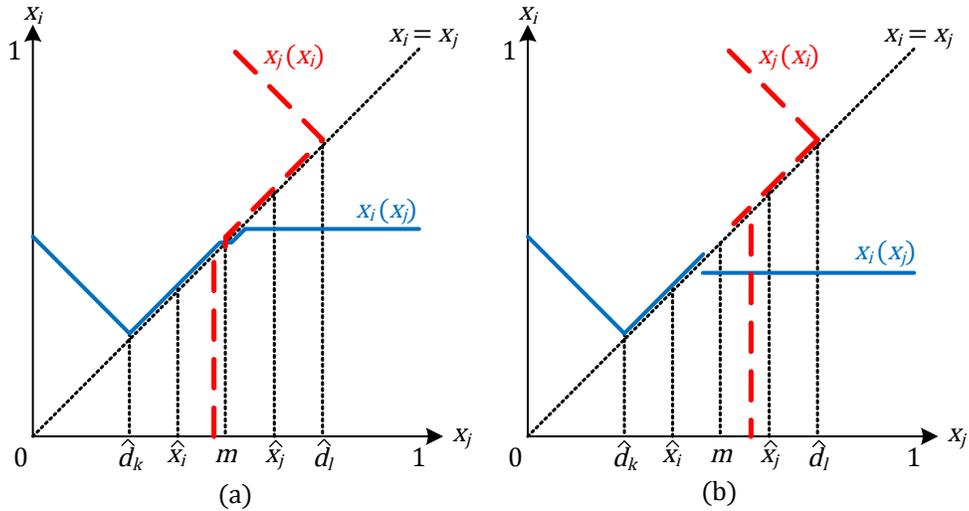


Figure A3. Mixed specification with $\hat{x}_i < m < \hat{x}_j$ for low and high values of γ .

Without loss of generality, when candidate i prefers a lower policy position than the median voter and candidate j prefers a higher policy position than the median voter, i.e., $\hat{x}_i < m < \hat{x}_j$, for low values of x_j , candidate i 's best response is to target the voters consistent with the Downsian model as seen in figure A3(a), and he positions himself $\varepsilon > 0$ closer to the median voter than candidate j . This occurs up until a policy point above the median voter, where candidate i no longer increases his position in response to an increase in candidate j 's position. At this point, candidate i 's expected policy payoff dominates his preference to maximize his probability of winning the election, and he behaves more in line with the Wittman model. An analogous argument applies for candidate j 's best response to candidate i 's position. In this case, both candidates position themselves at exactly the median voter in equilibrium, and policy convergence occurs. In equilibrium, neither donor contributes to either candidate's campaign, and the election is decided by a coin flip.

In figure A3(b), we have the case where γ is large enough to induce policy diversion. The major difference in this case is that both candidates shift from maximizing their probability of winning the election to maximizing their expected policy payoff at a position below (above for candidate j) the median voter. This leads to behavior more in line with the Wittman specification rather than the Downsian, where a single Nash equilibrium in pure strategies exists where candidates select different policy positions, ones that are closer to their most ideal position.

Appendix 2 - Comparing candidate positions against donor's ideals

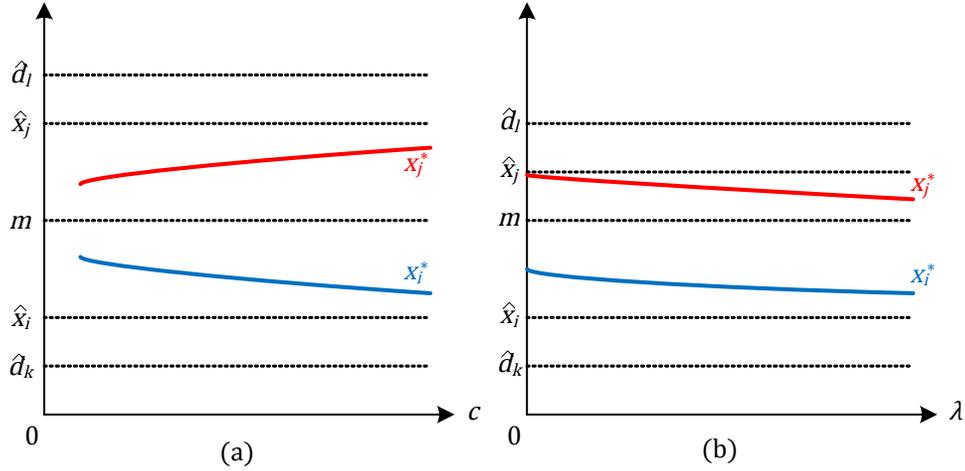


Figure A4. Equilibrium candidate positions as a function of c and λ when $\gamma = 1$.

Figure A4(a) plots candidate equilibrium position as a function of the marginal cost of donations, c . Starting from the right side of the figure, when c is large, every candidate receives fewer donations and has little incentive to position closer to his opponent in order to deny him of those donations. As a result, policy divergence is higher with large values of c . As c decreases, each candidate positions closer to his rival, as he has strong incentives to deny his rival of the additional donations that are available due to the reduced marginal cost.

Figure A4(b) depicts candidate equilibrium position as a function of the effectiveness of donations, λ . In this figure, candidate i has an ideal policy position closer to the midpoint between the donors' ideal policy position (donors are slightly asymmetric towards candidate i), and thus, candidate i responds quickly to an increase in λ by moving closer to his own ideal policy position. Candidate j follows at a slower rate, increasing policy divergence for low values of λ . As λ increases further, candidate i responds less to further increases (his line becomes flatter), and candidate j is able to deny more donations to his rival. For high values of λ , we observe decreased policy divergence.

Appendix 3 - Candidate positions with donation constraints

For parameter values of $\hat{d}_k = 0.2$, $\hat{d}_l = 0.8$, $\hat{x}_i = 0.3$, $\hat{x}_j = 0.7$, $\eta = 0.5$, $\lambda = 0.5$, and $a = \beta = 2$, the following results were obtained:

$\bar{k} = \bar{l}$	Equilibrium Values (x_i^*, x_j^*)		
	$c = 0.01$	$c = 0.02$	$c = 0.03$
0.8	(0.346, 0.654)	(0.346, 0.654)	(0.346, 0.654)
0.9	(0.346, 0.654)	(0.346, 0.654)	—
1.2	(0.346, 0.654)	(0.346, 0.654)	—
1.3	(0.346, 0.654)	(0.346, 0.654)	(0.387, 0.613)
1.5	(0.346, 0.654)	(0.346, 0.654)	(0.387, 0.613)
1.6	(0.346, 0.654)	—	(0.387, 0.613)
2.1	(0.346, 0.654)	—	(0.387, 0.613)
2.2	—	—	(0.387, 0.613)
2.5	—	—	(0.387, 0.613)
2.6	—	(0.398, 0.602)	(0.387, 0.613)
4.1	—	(0.398, 0.602)	(0.387, 0.613)
4.2	(0.422, 0.578)	(0.398, 0.602)	(0.387, 0.613)
10	(0.422, 0.578)	(0.398, 0.602)	(0.387, 0.613)

For $c = 0.01$, the low marginal cost of donation allows donors to contribute large donations to their respective candidates. When donation constraints are set low, $\bar{k} < k_1 = 2.1$ in this case, candidates behave as if no

donations are received and position themselves at $(x_i^*, x_j^*) = (0.346, 0.654)$. For values of $k_1 < \bar{k} < k_2 = 4.2$, no equilibrium exists, as candidates leverage constraints on their opponents to position closer to their own ideal policy positions. Lastly, when $\bar{k} > k_2$, candidates act as if they were unconstrained and position at $(x_i^*, x_j^*) = (0.422, 0.578)$.

As we increase c to 0.02 or 0.03, we find that the values of k_1 and k_2 decrease. For example, when $c = 0.02$, the higher marginal cost of donations causes donors to reduce their contribution levels, and thus the breakpoints for each scenario must also decrease. We find that $k_1 = 1.5$ and $k_2 = 2.6$ when $c = 0.02$; and $k_1 = 0.9$ and $k_2 = 1.3$ when $c = 0.03$. Of note, the fully constrained equilibrium does not change when $\bar{k} < k_1$ regardless of the value of c since candidates behave as if they receive no contributions, but as c decreases, the unconstrained equilibria show reduced policy divergence.

Appendix 4 - Public Funding Numerical Results

The presence of public funding in our model behaves qualitatively similar to adding an asymmetry. For example, when γ is low (as in the Downsian Specification), equal public funding levels among candidates retains our equilibrium at the median voter. For even small public funding donation advantages, however, we arrive at situations where no equilibrium in pure strategies exists, much like the cases described in the asymmetry section.

In contrast, when γ is high (as in the Wittman Specification), a public funding advantage does not prevent the emergence of an equilibrium in pure strategies. Under these conditions, both candidates again behave as if an asymmetry were present, positioning closer to the candidate with the public funding advantage, the results of which are shown below in figure A2.

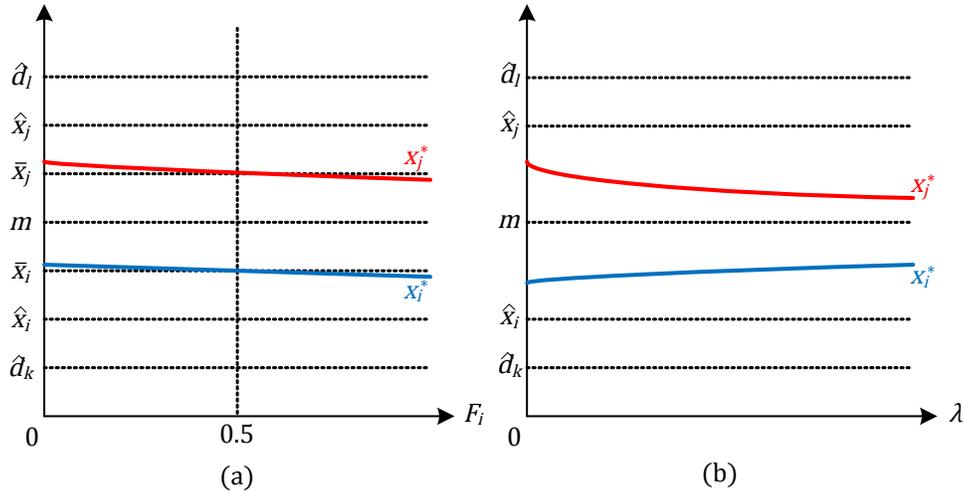


Figure A5. Equilibrium candidate positions as a function of F_i and λ when $\gamma = 1$.

Figure A5(a) depicts candidate equilibrium positions as a function of F_i while F_j is held constant at 0.5 and λ is held constant at 0.5. For comparison purposes, we denote point \bar{x}_i as candidate i 's equilibrium policy position without public funding. As seen in the figure, for values of $F_i < 0.5$, equilibrium positions are skewed upward, towards candidate j 's ideal. As F_i increases, however, the skewness at first disappears at $F_i = F_j = 0.5$, and then becomes skewed downward as F_i increases further above 0.5. Figure A5(b) depicts candidate equilibrium positions as a function of λ with $F_1 = 0.8$ and $F_2 = 0.5$. In this situation, we again observe the increased policy convergence as λ increases, as seen in figure 7b (as an increase in λ increases the effect of private, as well as public donations). However, we do observe an asymmetry in favor of candidate i , due to their advantage in public funding. As λ increases, candidates shift their priorities from the voters to the donors, but candidate i 's public donation advantage also shifts both candidates more towards candidate i 's ideal policy.

Proof of Proposition 1

Lemma A. *Donor k 's equilibrium donation to candidate i , k_i , solves*

$$\frac{dp_i}{dD_i} \left[u_k(x_i; \hat{d}_k) - u_k(x_j; \hat{d}_k) \right] \leq c \quad (A1)$$

Intuitively, donor k increases his contribution k_i until his marginal benefit from further donations (left-hand side of equation (A1)) coincides with his marginal cost (right-hand side of (A1)). Note that the marginal benefit captures the additional probability that candidate i wins the election thanks to larger donation, $\frac{dp_i}{dD_i} \geq 0$, and the utility gain that donor k obtains when candidate i wins the election to candidate j , $u_k(x_i; \hat{d}_k) - u_k(x_j; \hat{d}_k)$. Needless to say, if donor k prefers candidate j winning the election then $u_k(x_i; \hat{d}_k) < u_k(x_j; \hat{d}_k)$, and the left-hand side becomes unambiguously negative, ultimately yielding a corner solution where donor k does not contribute to candidate i 's campaign in equilibrium. This result suggests that ever donor k will only contribute to the candidate yielding the highest utility; as we prove in the next lemma.

Lemma B. *In equilibrium, every donor k contributes to one candidate at most.*

Proof: Assume that donor k contributes to both candidates A and B , i.e., $k_i, k_j > 0$ and thus equation (A1) binds with equality for candidates i and j , i.e.,

$$\frac{dp}{dD_i} [u_k(x_i) - u_k(x_j)] = 1 \quad (A2)$$

$$\frac{dp}{dD_j} [u_k(x_i) - u_k(x_j)] = 1 \quad (A3)$$

Setting equations (A2) and (A3) equal to one another and simplifying yields

$$\frac{dp}{dD_i} = \frac{dp}{dD_j} \quad (A4)$$

which cannot hold, since the left side of equation (A4) is positive, while the right side is negative. Therefore, donor k 's contribution to at least one of the candidates must equal zero. ■

Intuitively, if donor k contributes to candidate i , he does so to increase the probability that candidate i wins the election. On the contrary, any contribution that donor k makes to the other candidate $j \neq i$ lowers the probability that candidate i wins the election. Thus, contributions to both candidates are counterproductive, and every donor k only donates to the candidate whose policy position yields him the highest utility level.⁸ As a remark, note that if both policy positions yield the same utility for donor k , $u_k(x_i; \hat{d}_k) = u_k(x_j; \hat{d}_k)$ then his marginal benefit of contributing to candidate i (left-hand side of (2)) becomes nil, inducing no donations to either candidate, i.e., $k_i^* = 0$ for all i .

Lemmas A and B allow us to characterize the solution of the second stage of the game into several cases, as detailed in Proposition 1. ■

Proof of Corollary 1

First, we show that $\frac{dk_i^*}{dl_i} = -1$. Differentiating equation (2) with respect to l_i yields

$$\overbrace{\frac{d^2p}{dD_i^2} \left(\frac{dk_i^*}{dl_i} + 1 \right)}^{(-)} \overbrace{[u_k(x_i) - u_k(x_j)]}^{(+)} = 0$$

where the only value that can satisfy the above equation is $\frac{dk_i^*}{dl_i} = -1$.

⁸From disclosure website opensecrets.org, data obtained suggests that no major super PAC supports multiple candidates in the same election. This holds true for several elections, dating back to before 2008.

Next, we show that $\frac{dk_i}{dl_j} > 0$. Using equation (2) with respect to k_i^* and l_j^* , we have

$$\begin{aligned}\frac{dp}{dD_i} [u_k(x_i) - u_k(x_j)] &= 1 \\ \frac{dp}{dD_j} [u_l(x_i) - u_l(x_j)] &= 1\end{aligned}$$

Setting these two equations equal to one another and rearranging terms yields

$$\frac{dp}{dD_i} [u_k(x_i) - u_k(x_j)] - \frac{dp}{dD_j} [u_l(x_i) - u_l(x_j)] = 0$$

Using the implicit function theorem,

$$\frac{dk_i}{dl_j} = \frac{\overbrace{\frac{d^2p}{dD_j^2} [u_l(x_i) - u_l(x_j)]}^{(-)}}{\underbrace{\frac{d^2p}{dD_i^2} [u_k(x_i) - u_k(x_j)]}_{(-)}} > 0$$

where the signs of $u_k(x_i) - u_k(x_j)$ and $u_l(x_i) - u_l(x_j)$ are by definition and the signs of $\frac{d^2p}{dD_i^2}$ and $\frac{d^2p}{dD_j^2}$ are due to the concavity and convexity, respectively of D_i and D_j on p . ■

Proof of Corollary 2

Differentiating equation (2) with respect to x_i and x_j yields

$$\underbrace{\frac{d^2p}{dD_i^2}}^{(-)} \frac{dk_i^*}{dx_i} \overbrace{[u_k(x_i) - u_k(x_j)]}^{(+)} + \overbrace{\frac{dp}{dD_i}}^{(+)} \frac{du_k(x_i)}{dx_i} = 0 \quad (A4)$$

$$\underbrace{\frac{d^2p}{dD_j^2}}^{(-)} \frac{dk_j^*}{dx_j} \underbrace{[u_k(x_i) - u_k(x_j)]}_{(-)} - \underbrace{\frac{dp}{dD_j}}_{(+)} \frac{du_k(x_j)}{dx_j} = 0 \quad (A5)$$

Since the probability that candidate i wins the election is increasing and concave, we know that $\frac{dp}{dD_i} > 0$ and $\frac{d^2p}{dD_i^2} < 0$. Likewise, if donor k is contributing to candidate i , $u_k(x_i) - u_k(x_j) > 0$. The sign of $\frac{du_k(x_i)}{dx_i}$ ($\frac{du_k(x_i)}{dx_j}$) can be determined by candidate i 's (j 's) position relative to donor k 's ideal position. If $x_i < x_k$ ($x_j < x_k$), an increase in candidate i 's (j 's) policy position will entail an increase in the utility that donor k receives from that position, and thus $\frac{du_k(x_i)}{dx_i} > 0$ ($\frac{du_k(x_i)}{dx_j} > 0$). On the contrary, if $x_i > x_k$ ($x_j > x_k$), an increase in candidate i 's (j 's) policy position will entail a decrease in the utility that donor k receives from that position, and thus $\frac{du_k(x_i)}{dx_i} < 0$ ($\frac{du_k(x_i)}{dx_j} < 0$). This leaves $\frac{dk_i^*}{dx_i}$ ($\frac{dk_i^*}{dx_j}$) as the only unknown sign in the above equation. In order for the equation to hold with equality, it is necessary that $\frac{dk_i^*}{dx_i}$ ($\frac{dk_i^*}{dx_j}$) have the same (opposite) sign as $\frac{du_k(x_i)}{dx_i}$ ($\frac{du_k(x_i)}{dx_j}$), i.e., $\frac{dk_i^*}{dx_i} > 0$ ($\frac{dk_i^*}{dx_j} < 0$) if $x_i < x_k$ ($x_j < x_k$) and $\frac{dk_i^*}{dx_i} < 0$ ($\frac{dk_i^*}{dx_j} > 0$) if $x_i > x_k$ ($x_j > x_k$, respectively). ■

Proof of Lemma 2

Equilibrium location pairs (x_i^*, x_j^*) must satisfy equation (4) for both candidates, which will depend on the signs of each term in those equations. Unambiguously, we know that $\frac{dp_i}{dD_i} > 0$, since by assumption, if candidate i receives more donations from either candidate, their subjective probability of winning the election

will increase. Likewise, we have $\frac{dp_i}{dD_j} < 0$, as an increase in the amount of donations received by candidate $j \neq i$ causes candidate i 's subjective probability to decrease. As shown in Corollary 1, when the subjective probability that candidate i wins the election is concave and $x_i < \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$, we have that $\frac{dk_i^*}{dx_i} > 0$, $\frac{dl_i^*}{dx_j} < 0$, $\frac{dl_i^*}{dx_i} > 0$, and $\frac{dl_i^*}{dx_j} < 0$. In addition, since $x_i < \hat{x}_i$, an increase in candidate i 's position increases the utility he receives if he wins the election, thus $\frac{dv_i(x_i)}{dx_i} > 0$. Due to these relationships, the left-hand side of equation (4) is positive, and thus, it cannot hold with equality. Thus, any $x_i < \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ cannot be a solution to stage 1. A similar approach when $x_i > \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ shows that the left-hand side of equation (3) is unambiguously negative and also cannot solve equation (4).

When $x_i = \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ ($x_i = \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$), a similar situation occurs. All signs described in the previous paragraph are identical except for the sign that corresponds with $\min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ ($\max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$). This term is equal to zero, as candidate i is either receiving the most possible subjective probability contribution by positioning himself at the median voter, the most possible utility by positioning at his own ideal policy, or the most possible donations from the donor with the lower (higher) ideal policy position by positioning at his most preferred policy position. The outcome is the same where the left side of equation (4) is unambiguously positive (negative), and cannot be satisfied when $x_i = \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ ($x_i = \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$, respectively). ■

Proof of Corollary 4

We can show the effect of an increase of candidate i 's policy position, x_i , graphically below in figure A3,

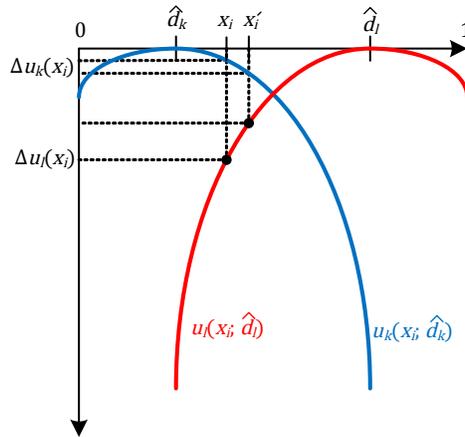


Figure A3. Donor utility functions.

As seen in the above figure, when candidate i moves from position x_i to position x_i' the decrease in utility to donor k , $\Delta u_k(x_i)$ is much less than the gain in utility to donor l , $\Delta u_l(x_i)$. Holding all other values of equation (2) constant, the resulting increase in candidate i 's policy position requires both equilibrium donation levels to decrease, but that for donor l to decrease by a larger amount. ■

Proof of Lemma 3

Let $x_1 = f_1(x_2)$ and $x_2 = f_2(x_1)$ be two continuous functions of the argument x_1 and x_2 each having domain $[a, b]$ where $a, b \in [0, 1]$ and range $[0, 1]$. An equilibrium is defined by solving the two equations simultaneously for the solution x_1^* and x_2^* . The solution can be characterized via substitution as $x_1^* = f_1 \circ f_2(x_1^*) = f(x_1^*)$ and $x_2^* = f_2(x_1^*)$, where the composite function $f = f_1 \circ f_2$ is also continuous.

Define an equally-spaced grid on the domain consisting of $n + 1$ points, as

$$\mathcal{G} = \left\{ a, a + \frac{b-a}{n}, a + \frac{2(b-a)}{n}, \dots, b \right\} \quad \text{for all } i = 1, 2.$$

Define piecewise linear approximations to the composite function $f = f_1 \circ f_2$ as

$$\hat{f}(x) = \sum_{k=1}^n \left[f(x_k) + (x - x_k) \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k} \right] I_{[x_k, x_{k+1})}(x) + f(x_{n+1}) I_{\{n+1\}}(x)$$

where $I_A(x)$ is an indicator function taking values $I_A(x) = 1$ if $x \in A$ and $I_A(x) = 0$ if $x \notin A$.

For compactness, define $\Delta_{k,k+1} \equiv \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k}$. Let \hat{x}_1^* represent the linear piecewise-approximated equilibrium value that solves $\hat{x}_1^* = \hat{f}(\hat{x}_1^*)$, i.e., a fixed point of the above piecewise linear approximation $\hat{f}(x)$. Then, either $\hat{x}_1^* = 1$ if $\hat{f}(1) = f(1) = 1$, or else $\hat{x}_1^* = \hat{f}(\hat{x}_1^*)$ for all $\hat{x}_1^* \in [x_k, x_{k+1})$. We can rewrite the last expression as $\hat{x}_1^* - \hat{f}(\hat{x}_1^*) = 0$ which, by the definition of $\hat{f}(x)$, expands as follows

$$\hat{x}_1^* - \hat{f}(\hat{x}_1^*) = \hat{x}_1^* - \underbrace{(f(x_k) + (\hat{x}_1^* - x_k)\Delta_{k,k+1})}_{\hat{f}(\hat{x}_1^*)} = 0 \quad \text{for all } \hat{x}_1^* \in [x_k, x_{k+1}). \quad (\text{A6})$$

We now focus on term $(\hat{x}_1^* - x_k)\Delta_{k,k+1}$ of expression (A6). Note that, upon letting the number of equally spaced grid points in the $[a, b]$ interval increase without bound, i.e., $n \rightarrow \infty$, it follows that $x_{k+1} \rightarrow x_k^+$, yielding

$$\begin{aligned} & \lim_{x_{k+1} \rightarrow x_k^+} (\hat{x}_1^* - x_k)\Delta_{k,k+1} \\ &= \lim_{x_{k+1} \rightarrow x_k^+} (\hat{x}_1^* - x_k) \times \lim_{x_{k+1} \rightarrow x_k^+} \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k} \\ &= 0 \times \left. \frac{df}{dx} \right|_{x_k^+} = 0 \end{aligned}$$

where $\left. \frac{df}{dx} \right|_{x_k^+}$ is the derivative of f or the subgradient of f from the right. Using this result in expression (A6), it follows that, in the limit as $n \rightarrow \infty$,

$$\hat{x}_1^* - \hat{f}(\hat{x}_1^*) = \hat{x}_1^* - [f(x_k) + 0] = 0$$

and the approximate and exact equilibriums converge. ■

References

- [1] Adamany, D. (1977) "Money, Politics, and Democracy: A Review Essay." *The American Political Science Review* (1927), 71(1), 289-304.
- [2] Alesina, A. (1988) "Credibility and policy convergence in a two-party system with rational voters." *The American Economic Review*, 78(4), 796-805.
- [3] Aragonès, E., Postlewaite, A., and Palfrey, T. (2007) "Political reputations and campaign promises." *Journal of the European Economic Association*, 5(4), 846-884.
- [4] Austen-Smith, D. (1987) "Interest groups, campaign contributions, and probabilistic voting." *Public Choice*, 54(2), 123-139.
- [5] Ball, R. (1999a) "Opposition backlash and platform convergence in a spatial voting model with campaign contributions." *Public Choice*, 98(3), 269-286.
- [6] Ball, R. (1999b) "Discontinuity and non-existence of equilibrium in the probabilistic spatial voting model." *Social Choice and Welfare*, 16(4), 533-555.
- [7] Barro, R. (1973) "The control of politicians: An economic model." *Public Choice*, 14(1), 19-42.
- [8] Bowen, L. (1994) "Time of voting decision and use of political advertising: The Slate Gorton-Brock Adams senatorial campaign." *Journalism Quarterly*, 71(3), 665-675.

- [9] Calvert, R. (1985) "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science*, 29(1), 69-95.
- [10] Coate, S. (2004) "Political Competition with Campaign Contributions and Informative Advertising." *Journal of the European Economic Association*, 2(5), 772-804.
- [11] d'Aspremont, C., and Gabszewicz, J. (1979) "On Hotelling's 'Stability in Competition'". *Econometrica*, 47(5), 1145-1150.
- [12] Downs, A. (1957) *An economic theory of democracy*. New York: Harper.
- [13] Evrenk, H., and Kha, D. (2011) "Three-candidate spatial competition when candidates have valence: Stochastic voting." *Public Choice*, 147(3-4), 421-438.
- [14] Goldstein, K., and Freedman, P. (2000) "New evidence for new arguments: Money and advertising in the 1996 Senate elections." *Journal of Politics*, 62, 1087-1108.
- [15] Gordon, B. R., and Hartmann, W. R. (2013) "Advertising effects in presidential elections." *Marketing Science*, 32(1), 19-35.
- [16] Grossman, G., and Helpman, E. (2001) *Special interest politics*. Cambridge, Mass.: MIT.
- [17] Hare, C., and Poole, K. (2014) "The Polarization of Contemporary American Politics." *Polity*, 46(3), 411-429.
- [18] Herrera, H., Levine, D. and Martinelli, C. (2008) "Policy platforms, campaign spending and voter participation." *Journal of Public Economics*, 92(3), 501-513.
- [19] Hinich, M. and Munger, M. (1994) *Ideology and the theory of political choice*. Ann Arbor: University of Michigan Press.
- [20] Hotelling, H. (1929) "Stability in Competition." *The Economic Journal*, 39(153), 41-57.
- [21] Jacobson, G. (1978) "The Effects of Campaign Spending in Congressional Elections." *The American Political Science Review* (1927), 72(2), 469-491.
- [22] Kaid, L. L. (1982) "Paid television advertising and candidate name identification." *Campaigns and Elections*, 3, 34-36.
- [23] McKelvey, R. (1975) "Policy Related Voting and Electoral Equilibrium." *Econometrica*, 43(5), 815-843.
- [24] Morton, R. (1993). "Incomplete information and ideological explanations of platform divergence." *American Political Science Review* 87: 382-392.
- [25] Ortuno-Ortín, I., and Schultz, C. (2005) "Public Funding of Political Parties." *Journal of Public Economic Theory*, 7(5), 781-791.
- [26] Poole, K., and Rosenthal, H. (1984) "The Polarization of American Politics." *The Journal of Politics*, 46(4), 1061-1079.
- [27] Shaw, D. R. (1999) "The effect of TV ads and candidate appearance on statewide presidential votes." 1988-1996. *American Political Science Review*, 93, 345-362.
- [28] Stratmann, T. (2009) "How prices matter in politics: The returns to campaign advertising." *Public Choice*, 140(3-4), 357-377.
- [29] Welch, W. (1980) "The allocation of political monies: Economic interest groups." *Public Choice*, 35(1), 97-120.
- [30] Welch, W. (1974) "The economics of campaign funds." *Public Choice*, 20(1), 83-97.
- [31] West, D. (2005) *Air wars: television advertising in election campaigns, 1952-2004* (4th edn.). Washington: CQ Press.

- [32] Wittman, D. (1983) "Candidate Motivation: A Synthesis of Alternative Theories." *The American Political Science Review*, 77(1), 142-157.
- [33] Zakharov, A., and Sorokin, V. (2014) "Policy convergence in a two-candidate probabilistic voting model." *Social Choice and Welfare*, 43(2), 429-446.