

# Nonprofit Product Differentiation

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## Abstract

In this note we analyze two-dimensional product differentiation in competition between nonprofit firms. Unlike for-profit settings, which finds maximal differentiation in the characteristic most salient to consumers and minimal differentiation in the other dimension, we show that the presence of at least one nonprofit firm leads to minimal differentiation in both dimensions. We extend the analysis to mixed competition between a nonprofit and for-profit firm.

KEYWORDS: Product differentiation, dominant dimension, nonprofit, for-profit.

JEL CLASSIFICATION: L1, L3, R3.

# 1 Introduction

Fifty years after Hotelling (1929) concluded that competing firms were insufficiently differentiated, in what came to be known as “the principle of minimum differentiation”, D’Aspermont, Gaszewicz and Thisse (1979) showed no pure strategy equilibrium existed in the Hotelling framework. Replacing Hotelling’s linear penalty for a firm’s output not meeting the consumer’s ideal with a quadratic penalty, they found, in contrast to Hotelling, firms will maximally differentiate. In an extension to that result, Irmen and Thisse (1998) showed that, in multi-characteristic spaces, two competing firms choose to maximize differentiation in a characteristic of dominant importance to consumers, and to minimize differentiation in all other characteristics.

In this note we analyze product differentiation in competition between nonprofit and for-profit firms. First, we show that when all firms are nonprofit, firms minimize differentiation both in dominant and dominated dimensions. As a consequence, firms’ products look alike and split the market equally. We extend the analysis to asymmetric firms, one nonprofit and another for-profit. We show that, when at least one firm is nonprofit, differentiation becomes nil in the dominant domain under all parameter values. Our results help explain, for instance, the competition between nonprofit and for-profit hospitals, where their prices, types and quality of care they offer, or length of stay, are extremely similar.<sup>1</sup>

## 2 Model

Our model is a two-dimensional variant of the Irmen and Thisse framework. Consumers are continuously distributed uniformly over a unit square  $[0, 1]^2$  in a configuration commonly known as a “Hotelling city.” Each point on the square, as defined by its location  $(z_1, z_2)$ , indicates the preferred product characteristics of the consumer there located. Two firms,  $A$  and  $B$ , produce a good, with the product characteristics defined by the firms locations, so the characteristics of the good produced by firm  $A$  are given by the pair  $a = (a_1, a_2)$  and likewise the product from firm  $B$  is defined by  $b = (b_1, b_2)$ . Firms are identical except for product location (characteristics). There is a constant marginal cost of production, which for simplicity is set equal to zero, and is independent of the product characteristics chosen by the firm. There are no fixed costs. As opposed to Irmen and Thisse (1998), where firms maximize profits, we consider that every firm  $j$  maximize a function of market share,  $D_j$ ,  $f(D_j)$  where  $j = \{A, B\}$ ,  $f' \geq 0$  and  $f'' \leq 0$ .<sup>2</sup>

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<sup>1</sup>Plante (2009) examined differences between the types of patients nonprofit and for-profit hospitals treat and the length of time it took to treat them. The results indicated no significant difference between the variables used as indicators of patient type, including Medicare percentage, Medicaid percentage, or case-mix index. Similarly, a 1999 study of 43 hospitals that converted to for-profit, found that, on average, there were not statistically significant differences in prices, the levels of uncompensated care provided or the provision of unprofitable services like trauma care, burn care and substance abuse treatment; see Becker (2014). Last, for-profit and nonprofit hospitals adopted more similar technologies, as shown by Robinson and Luft (1985).

<sup>2</sup>Thompson (1994) provides empirical evidence showing that nonprofit hospitals in the U.S. compete for market share. Newhouse (1970) and Lakdawalla and Philipson (2006) present settings where firms maximize a similar objective function. In Appendix 1 we show that maximizing consumer surplus is consistent with the the firm’s objective function we use.

Consumers buy one unit of the good, either from firm  $A$  or  $B$ . If consumer in location  $(z_1, z_2)$  buys from firm  $A$ , she derives a net utility of

$$u_A(z_1, z_2) = S - p_A - t_1(z_1 - a_1)^2 - t_2(z_2 - a_2)^2$$

where  $S > 0$  denotes surplus, which is assumed to be sufficiently large to ensure that consumers buy from one seller or the other,  $p_A$  is the price she pays for the good, and transportation costs  $t_1$  and  $t_2$  define how important each characteristic is to consumers. Following Irmen and Thisse, we assume that characteristic 2 dominates characteristic 1,  $t_2 > t_1$ .<sup>3</sup> An analogous expression holds if the consumer buys from firm  $B$  instead.

The time structure of the game is the following: first, firms simultaneously and independently choose location; second, firms select prices; and finally consumers choose which firm to buy from. We solve the model by applying backward induction.

### 3 Equilibrium analysis

#### 3.1 Third stage

Following Irmen and Thisse (assuming  $b \geq a$ ) the demand for firm  $A$  is given by

$$D_A = \frac{p_B - p_A + t_1(b_1^2 - a_1^2) + t_2(b_2^2 - a_2^2) - t_1(b_1 - a_1)}{2t_2(b_2 - a_2)}$$

and that of firm  $B$  is  $D_B = 1 - D_A$ .

#### 3.2 Second stage

Anticipating the above demand functions  $D_A$  and  $D_B$ , every firm  $j$  chooses its price  $p_j$  to maximize  $f(D_j)$  subject to  $p_j D_j \geq 0$ . It is easy to show that, for any given location pair,  $\partial f(D_j)/\partial p_j \leq 0$ , which implies that  $j$ 's best response function collapses to  $p_j(p_k) = 0$  for all  $p_k$ . That is, when a firm is non-profit, setting a price equal to average cost (zero) becomes a weakly dominant strategy, as it is unaffected by its rival's price.<sup>4</sup> As a consequence, equilibrium prices are  $p_A^* = p_B^* = 0$ .

#### 3.3 First stage

Using the demands found above, and noting that with a firm trying to maximize  $f(D_j)$  will do so by maximizing the demand for its product, we find that every firm  $j$  chooses its location pair  $(j_1, j_2)$  to maximize  $D_j(p_j^*, p_k^*)$ . It is straightforward to show that equilibrium locations are  $a_1^* = b_1^* = 1/2$  in the dominated dimension. This result extends Irmen and Thisse's finding to contexts in which

<sup>3</sup>With only two characteristics, we are restricting ourselves to what Irmen and Thisse call "strong dominance".

<sup>4</sup>A nonprofit firm constrained to not making losses maximizes sales by setting its price equal to average cost. Since in our set-up average and marginal costs are zero, the firm sets its price at zero.

firms are nonprofit. (As a reference, Appendix 2 includes the derivation of Irmen and Thisse's result where both firms are for-profit.)

We next examine optimal locations in the dominant dimension  $j_2$ . Inserting equilibrium prices in the second stage, firm  $A$ 's program during the first stage becomes

$$\max_{a_2} D_A(p_A^*, p_B^*) = \frac{a_2 + b_2}{2}$$

which, differentiating with respect to  $a_2$ , yields  $1/2$ , i.e., increasing location  $a_2$  is a weakly dominant strategy for firm  $A$ . Similarly, firm  $B$ 's program simplifies to

$$\max_{b_2} D_B(p_A^*, p_B^*) = \frac{2 - a_2 - b_2}{2}$$

After differentiating with respect to  $b_2$ , we obtain  $-1/2$ , i.e., decreasing location  $b_2$  is a weakly dominant strategy for firm  $B$ . Intuitively, firm  $A$  ( $B$ ) has monotonic incentives to increase (decrease) its location, which is only compatible with the initial assumption  $b \geq a$  if  $a_2$  converges to  $b_2$  from below, ultimately entailing that

$$a_2^* = b_2^* = 1/2.$$

In words, at the Nash equilibrium both firms locate at the center, not differentiating their product in any dimension.

## 4 Extension - Only one nonprofit

In this section, we examine the case where firm  $A$  is nonprofit while its rival  $B$  is for-profit. Starting again with the third-stage result, we have

$$D_A = \frac{p_B - p_A + t_1(b_1^2 - a_1^2) + t_2(b_2^2 - a_2^2) - t_1(b_1 - a_1)}{2t_2(b_2 - a_2)} \quad \text{and} \quad D_B = 1 - D_A.$$

Differentiating  $D_A$  with respect to  $p_A$  again leads to  $p_A = 0$ . The optimal price for the for-profit firm  $B$  is found from

$$\frac{\partial \pi_B}{\partial p_B} = p_B \left( 1 - \frac{p_B - p_A + t_1(b_1^2 - a_1^2) + t_2(b_2^2 - a_2^2) - t_1(b_1 - a_1)}{2t_2(b_2 - a_2)} \right) = 0$$

Anticipating that  $p_A = 0$  and that  $a_1^* = b_1^* = 1/2$ , and solving for price  $p_B$ , yields

$$p_B = \frac{2t_2(b_2 - a_2) - t_2(b_2^2 - a_2^2)}{2}.$$

In this case the nonprofit firm  $A$ 's program becomes

$$\max_{a_2} D_A(p_A^*, p_B^*) = \frac{(b_2 + a_2 + 2)}{4}$$

which, differentiating with respect to  $a_2$ , yields  $1/4$ , i.e., increasing location  $a_2$  up to where firm  $B$  is located is a weakly dominant strategy for firm  $A$ . In contrast, the for-profit firm  $B$ 's program simplifies to

$$\max_{b_2} \pi_B(p_A^*, p_B^*) = \frac{t_2(b_2 - a_2)(b_2 + a_2 - 2)^2}{8}$$

with best response function  $b_2(a_2) = \frac{2}{3} + \frac{1}{3}a_2$ . Inserting  $a_2^* = 1$  into this function, yields  $b_2^* = 1$  entailing that, when only one of the firms is nonprofit, nil product differentiation emerges and the for-profit firm, to compete for any customers, would also have to set its price equal to zero.<sup>5</sup>

This conclusion has important implications for mixed competition. Lakdawalla and Philipson (2006) argue that for-profit firms cannot compete with nonprofit providers if there is sufficient nonprofit preferences among the suppliers of a good, and mixed competition is possible only when there is “insufficient” nonprofit goals. Our results concur with this conclusion.

## 5 Discussion

D’Aspermont, Gaszewicz and Thisse (1979) identified two countervailing incentives from firm location. First, to maximize market share, firms seek to locate in the center of the demand field; and, second, to create market power and essentially become a local monopoly, firms seek to be apart. Irmen and Thisse show, in a setting with multiple dimensions, that the second incentive dominates for profit-maximizing firms, driving them to maximal differentiation in the dominant domain. We show that the first incentive dominates for nonprofits, inducing firms to compete for market share yielding nil product differentiation. Importantly, this result holds even if only one of the two firms is nonprofit.

Overall, our findings suggest that Irmen and Thisse’s minimal differentiation result in the dominated dimension extends to settings in which one or both firms are nonprofit. However, their maximal differentiation outcome in the dominant dimension breaks down if at least one of the firms is nonprofit. As suggested in our discussion of equilibrium prices, the presence of nil differentiation induces even for-profit firms to practice average cost pricing under large parameter conditions. Our results, hence, suggest that the presence of at least one nonprofit firms leads to minimal differentiation in products and services, subsequently strengthening price competition.

## 6 Appendix 1 - Maximizing consumer surplus

It is easy to show that inverting the direct demand  $D_A(p_A)$  for firm  $A$  we obtain the inverse demand curve  $p_A(D_A)$ , which is linear in  $D_A$ . Hence consumer surplus can be found by computing the area of the triangle  $CS_A = \frac{1}{2}p_A(0)D_A$ , where  $p_A(0)$  denotes the vertical intercept of the inverse demand

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<sup>5</sup>There is only one equilibrium when one firm is for-profit and the other nonprofit, occurring at  $a_2^* = b_2^* = 1$  when firm  $A$  is the only nonprofit, and at  $a_2^* = b_2^* = 0$  when firm  $B$  is the only nonprofit. If we consider the case in the main text, with firm  $A$  as the nonprofit firm, if both firms are at  $a_2^* = b_2^* = 1$ , firm  $B$  may try to separate itself from  $A$  by moving to  $b_2^* = 0$ . Since  $a_2 \geq b_2$  and we have a situation analogous starting with firm  $B$  as the nonprofit firm. Hence, in the mixed competition case this strategy profile is not a Nash equilibrium.

function  $p_A(D_A)$ . This consumer surplus yields  $CS_A = t_2(b_2 - a_2)D_A^2$ . Differentiating with respect to  $D_A$  produces  $\frac{\partial CS_A}{\partial D_A} = 2t_2(b_2 - a_2)D_A$ , which is positive for all  $b_2 > a_2$ . An analogous argument holds for firm  $B$ . Hence, our firm objective is consistent with a goal of maximizing consumer surplus, as long as the firms do not collude.

## 7 Appendix 2 - Two for-profits

The setting in which both firms are for-profit coincides with that Irmen and Thisse (1998). Using equilibrium location  $a_1^* = b_1^* = 1/2$  in the dominated dimension,<sup>6</sup> firm  $A$  solves

$$\max_{a_2} \pi_A(p_A^*, p_B^*) = \frac{t_2(b_2 - a_2)(b_2 + a_2 + 2)^2}{18}$$

The above program yields best response function  $a_2(b_2) = -\frac{2}{3} + \frac{1}{3}b_2$ , which is negative for all admissible values of  $b_2 \in [0, 1]$ . Hence, firm  $A$ 's best response function collapses to a flat line  $a_2(b_2) = 0$  for all  $b_2$ . Operating similarly for firm  $B$ , we obtain

$$\max_{b_2} \pi_B(p_A^*, p_B^*) = \frac{t_2(b_2 - a_2)(b_2 + a_2 - 4)^2}{18}$$

with best response function  $b_2(a_2) = \frac{4}{3} + \frac{1}{3}a_2$ , which is positive for all values of  $a_2$ . Simultaneously solving for  $a_2$  and  $b_2$ , and using the constraint that  $b_2, a_2 \in [0, 1]$ , we obtain equilibrium locations  $a_2^* = 0$  and  $b_2^* = 1$ . Therefore, when both firms are for-profit, product differentiation in the dominant dimension is maximal; as found by Irmen and Thisse (1998).

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<sup>6</sup>Inserting  $a_1^* = b_1^* = 1/2$  in the firm  $A$ 's objective function is equivalent to inserting it after taking first-order conditions with respect to choice variable  $a_2$  since  $a_1^* = b_1^* = 1/2$  is not a function of  $a_2$ . Otherwise, the optimal location  $a_2^*$  would not coincide using the first or second approach.

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