Common Pool Resources with Endogenous Equity Shares*

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March 25, 2021

Abstract

We consider a common pool resource (CPR) where, in the first stage, every firm chooses an equity share on its rivals' profits (cross-ownership), in the second stage, firms compete for the resource, and in the third stage, firms compete again for the resource after it regenerated at a given rate. We identify equilibrium equity shares in this setting, and compare them against the socially optimal shares that maximize welfare. Our results show that equity shares are welfare improving under certain conditions, but can lead to a socially insufficient exploitation of the CPR if shares are large enough; as in a merger where firms equally share equity. We discuss how equity taxes can help firms approach socially optimal appropriation levels.

Keywords: Resource regeneration; Common pool resources; Endogenous equity shares; Social optimum; Equity share taxes.

JEL classification: L10, D43, D62, Q28, Q5.

*We thank the editor, Leo Simon, and an anonymous reviewer for their insightful suggestions, which improved the presentation of the paper. We also thank Ana Espinola-Arredondo, Jinhui Bai, Raymond Batina, and Alan Love for their discussions. We are also grateful to seminar participants at Washington State University, conference participants at AAEA 2017 in Chicago, at the World Congress of Environmental and Resource Economists in Gothenburg, Sweden, and at the Pacific Northwest Regional Economic Conference 2017 in Bend, Oregon, for their comments.

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1 Introduction

Firms hold equity shares on each other's profits in common pool resources (CPRs), especially in fisheries where cross-ownership is often maximal. In this context, firms equally share profits and their behavior coincides with that under a merger (or a multi-plant monopoly) where firms coordinate their appropriation levels.[1] In the case of U.S. "corporate-cooperative management" fisheries, for instance, companies exploiting the resource coordinate their appropriation decisions as a single entity. Examples include the Northeast Tilefish fishery, Kitts et al. (2007); the Alaskan Chignik Salmon fishery, Deacon et al. (2008); the Pacific Whiting fishery, Sullivan (2001); and the Bearing Sea Pollock fishery, Kitts and Edwards (2003).[2,3] The Whiting and Pollock conservation cooperatives, for instance, significantly reduced catches, as reported by the private fishery harvest monitoring service SeaState, Inc. and the U.S. National Marine Fisheries Service; see Sullivan (2001). In this paper, we study in which cases equity shares can help to protect the CPR by reducing appropriation and when, instead, they lead to an under-exploitation of the CPR, thus becoming socially damaging.

Our model considers that, in the first stage, every firm chooses its equity share on the other firm's profits; in the second stage, observing the profile of equity shares, every firm selects its appropriation level of the resource; and, in the third stage, allowing for a share of the resource to regenerate, every firms chooses its appropriation level again. First, we identify the equity shares that firms choose in equilibrium, anticipating how this equity acquisition softens subsequent competition for the resource.[4] Second, we find in which contexts the equilibrium appropriation of the resource

[1]Equity shares across rival firms are also common in other sectors, and are often referred to as "partial cross ownership." In the automobile industry, for instance, Renault currently holds 44.3% equity shares in Nissan, while Nissan holds 15% in Renault; see Bárcena-Ruiz and Campo (2012) and www.nissan-global.com. Cross-ownership is also common in the financial sector, where Allianz AG holds 5% of Deutsche Bank, while Deutsche Bank holds 10% in Allianz AG. Similarly, Allianz AG owns 22.5% of Dresdner Bank, who owns 10% of Allianz AG; see La Porta et al. (1999). In the U.S. mutual funds industry, State Street Corporation (STT) owns minority shares in other funds, such as 4.77% of T. Rowe Price and 3.05% in Black Rock. Similarly, T. Rowe Price Group owns 3.28% of STT, and Black Rock Inc. owns 2.68% of T. Rowe Price Group; see Levy and Szafarz (2017). Other examples include only one firm holding equity shares on their rival's profits, such as Gillette, which owns 22.9% of the non-voting stock of Wilkison Sword, Gilo et al. (2006); Ford, which purchased 25% of Mazda's shares in 1979; and General Motors, which acquired 20% of Subaru's stock in 1999; see Ono et al. (2004). Finally, this type of partial ownership is also common in the media sector, where Comcast owns multiple large media outlets including NBC, The Weather Channel, and CNBC; Time Warner owns CNN, HBO and Cartoon Network; and News Corp, which owns the Wall Street Journal and the New York Post.

[2]In "corporate management" systems, however, firms transfer their appropriation decisions to a separate corporation, which centrally determines the appropriation levels for each member. While these systems have not been fully implemented yet, some fisheries have adopted variants of this approach since 1995, such as New Zealand's Bluff Oyster and Challenger Scallop fisheries, Yang et al. (2014); and Australia's Exmouth Gulf Prawn fishery, Rogers (2009).

[3]While cooperatives of individuals or firms exploiting a CPR fit our model, "catch share" programs do not. In catch shares, such as those supported by NOAA, a portion of the catch for a species of fish is allocated to individual fishermen. Some programs allow every fisherman to purchase a larger catch share from other fishermen, which lets the fisherman increase his individual appropriation. The catch share program, however, does not provide the fisherman with a proportion of other fishermen profits.

[4]This result has been empirically confirmed in several industries where cross-ownership reduces output, such as telecommunications, Parker and Roller (1997); Italian banks, Trivieri (2007); and energy industry in Northern Europe, Amundsen and Bergman (2002).

lies above the socially optimal appropriation, leading to a socially excessive exploitation, and in which cases it lies below, entailing a socially insufficient appropriation. This comparison helps us explore taxes on equity acquisition, which induces firms to hold equity levels that yield a welfare improving exploitation of the resource in future stages.[5] This type of policy is commonly used in financial transactions taxes, as we describe below, but in this paper we identify an environmental benefit that should be considered by regulators in the design of this tax. Unlike common policies, such as quotas or emission fees, equity share taxes/subsidies do not require posterior supervision or monitoring of firms' appropriation (e.g., fish catches at ports and vessel inspections). If monitoring costs are large enough, equity subsidies can become a preferred policy tool.

Intuitively, when every firm shares a high proportion of its profits with other companies, its profit-maximization problem resembles that of a merged firm since the number of "effective independent actors" decreases, which helps firms coordinate on more efficient equilibria. Each firm reduces its appropriation since it now internalizes the cost externality that its exploitation imposes on its rival. If a reduction in output is relatively small, the presence of equity shares can help equilibrium appropriation approach its socially optimal level. In other words, equity shares can ameliorate the over-exploitation of the stock that arises in the commons. Specifically, the welfare benefits from the output reduction due to equity shares (larger profits and a lower environmental damage) exceed the welfare loss (reduction in consumer surplus), ultimately increasing overall welfare. Regulators can then intervene in a number of ways. While traditional approaches suggest the use of quotas or taxes on resource appropriation, these policy tools may be difficult as appropriation is costly to monitor. Policy makers could, instead, consider directly taxing (or subsidizing) equity acquisition, thus inducing firms to produce at the social optimum, as described by Kanjilal and Munoz-Garcia (2019). Specifically, subsidies lower firms' cost of equity acquisition, inducing them to increase their equity on rivals' profits, and ultimately increasing welfare. If equity shares are significant, however, the decrease in appropriation can be severe, leading firms to exploit the resource below what the social planner would recommend. In this case, firms' equilibrium behavior changes from an overexploitation of the stock (under no equity shares) to an underexploitation (when equity shares are significant). In extreme settings where firms equally share profits (such as in a merger like those discussed above in fishing grounds), our results find that underexploitation can be sustained under large parameter combinations. In this context, the welfare benefits from equity shares do not offset its welfare loss, thus providing regulators with incentives to tax equity shares in such a CPR.

Equity subsidies can be an alternative policy tool in some CPRs where common policies, such as quotas or emission fees, require costly ex-post monitoring (e.g., in the case of fisheries, supervising catches at port, searching vessels, and setting fines if necessary). In contrast, equity subsidies only require firms to report information about their equity acquisitions, which is often collected

[5] Ownership of fishing quotas by foreign fishing companies can be observed in various countries. In England, more than two-fifths of the quota is held by foreign-controlled fishing businesses. Similar examples can be seen in Denmark, Norway, The Faeroe Islands, Iceland, Greenland Scotland, Spain and Portugal, according to The Guardian (2014) and Atlantic Shipping Brokers (2020).

for accounting and tax purposes anyway, without the need to conduct ex-post monitoring. Equity subsidies are, thus, especially attractive when monitoring costs are relatively high; as empirically shown in fisheries by Grafton (1996) for Canada, Hatcher (1998) for the UK, Milazzo (1998) for the U.S., and Arnason et al. (2000) for Norway.[6] Equity subsidies can, nonetheless, be combined with other policies. Indeed, our setting also allows for a policy mix between equity subsidies (before firms choose their equity shares) and fees (after selecting their equity shares). In this case, equity subsidies are lower than in a setting with equity subsidies alone, while emission fees are less stringent than in a context with emission fees alone. As such, the policy mix can provide two benefits: subsidies would be less expensive to fund (an important point if tax collection produces large market distortions), and less stringent emission fees would probably face less political resistance.

Taxes/subsidies on equity acquisition are relatively minor in most countries, although 40 nations use different forms of taxes or subsidies on financial transactions.[7] However, our results show that the implementation of these taxes/subsidies could be attractive to firms in affected industries.

Related literature. Several studies have analyzed the overexploitation of the commons; for a detailed review of the literature see Ostrom (1990), Ostrom et al. (1994) and Faysse (2005).[8] We consider an alternate policy tool for the commons, and evaluate its effectiveness in helping avoid overexploitation. In particular, we allow firms to hold equity shares on each others' profits. Ellis (2001) presents a similar model, but he takes equity shares as exogenously given, and assumes that welfare coincides with the sum of firms' profits, thus ignoring the role of consumer surplus and environmental damage from the exploitation of the CPR.[9] We show that our model can reproduce Ellis' results in the special case in which all appropriation is sold overseas (no consumer surplus) and exploitation does not cause any environmental damage. In that setting, firms only behave optimally if they all hold maximal equities. However, when consumer surplus and/or environmental damage are considered, this finding no longer applies.

[6] In particular, monitoring costs in the UK for fisheries were estimated at 7.5% of the landings in 1996/97, Hatcher (1998); 15% of the landings in the U.S., Milazzo (1998); 8% in Norway and at least 15% in Newfoundland, Arnason et al. (2000). Similarly, a report by MRAG (2007) states that the monitoring of quotas in the Northern Prawn fishery is approximately 2 million AUD a year.

[7] Worldwide, financial transaction taxes raise more than $US 38 billion. For instance, the US Section 31 fee imposes $21.80 per million dollars for securities transactions; and the UK uses the Stamp Duty Reserve Tax at a rate of 0.5% on purchases of shares of companies headquartered in the UK, raising around $US4.4 billion per year. Similar equity taxes exist in France, Sweden, Taiwan, Singapore, Japan, and India. For a detailed review of this type of taxation across different countries, see Anthony et al. (2012).

[8] Ostrom et al. (1994) suggested that CPRs can be managed by local governance structures. Kirkley et al. (2003) examined the importance of preserving CPRs for the long term, especially in developing countries where there is excess capacity, Hackett et al. (1994) analyzed equal appropriation rules in irrigation in India, and Coward (1979) discusses water assignments as a function of land held in the Phillipines. Other articles consider uncertainty in the resource's stock, and how such uncertainty affects individual appropriation levels in the commons, approaching them to socially optimal levels; see Suleiman and Rapoport (1988), Suleiman et al. (1996), and Apesteguia (2006).

[9] His model was extended in Ellis and Nouweland (2006) which considers that every individual firm exploits the resource in the first stage, and invests in equity shares during the second stage; earning profits only at the end of the game. Anticipating the equilibrium profile of shares at the second stage, every firm's appropriation during the first stage approaches the cooperative solution. While their model endogenizes equity acquisition, it assumes that it happens at the last stage of the game; as opposed to our setting where equity acquisition is chosen during the first stage.

Our paper connects with the literature analyzing the effect of equity shares in industrial organization. In particular, Reynolds and Snapp (1986) examines a standard Cournot model when firms hold exogenous shares in each other's profits, showing that equilibrium quantities decrease as equity shares increase, regardless of which company share increases.[10] While our paper considers a similar model, it extends their setting along several dimensions: it allows for cost externalities, thus helping understand how their results apply to CPRs; considers a polluting industry and its optimal environmental regulation; and endogenizes equity share acquisition. Dietzenbacher et al. (1999) use data from the Dutch financial sector, empirically confirming that output is lower when firms hold shares on each other than otherwise.[11]

Qin et al. (2017) consider a Cournot game which, like in our paper, allows for firms to select equity shares before their subsequent choice of output.[12] In particular, the authors characterize 'pairwise stability' wherein no two firms can make themselves better off by trading any further shares. They also find that equilibrium equity shares increase (approaching collusive outcomes) when only a few firms compete in the industry. Unlike our paper, their setting does not consider cost externalities, thus not allowing for the analysis of CPRs; and does not compare equilibrium equity shares against the social optimum. Kanjilal and Munoz Garcia (2019) analyze in which cases taxes (or subsidies) on equity shares can be used to induce socially optimal output levels in Cournot games with partial-cross ownership. However, their setting does not allow for cost externalities or resource regeneration, which we study in this paper.

Finally, Bárcena-Ruiz and Campo (2012) investigate a country's optimal emission fee on a polluting firm when this company holds equity shares on another polluting firm located in a different country. The article then compares emission fees in a non-cooperative setting (independent environmental policies across countries) and a cooperative context (coordination of environmental policies). However, it assumes exogenous and symmetric equity shares across firms, and that firms do not impose cost externalities on one another since they exploit different CPRs. We relax these assumptions. Similarly, Heintzelman et al. (2009) model exogenous "partnerships" while Kaffine and Costello (2011) consider "unitization" and how they can curb socially excessive extraction in CPRs or reduce output in a oligopoly.[13] Unlike our paper, however, they do not allow for partnership shares to be endogenously determined.

[10] Farrell and Shapiro (1990) consider a Cournot oligopoly in which firms buy new capital either by acquiring it from a rival, from a third party (not a rival), or buying shares from a rival. Fanti (2015) modifies this setting, by considering that only one firm holds an exogenous participation on its rival's profit. In addition, the paper allows for asymmetries in production costs, showing that the output reduction arising from cross-ownership can be welfare improving if the firm holding stock on its rival's profits is less efficient than the rival.

[11] Malueg (1992) considers a setting in which firms hold exogenous symmetric shares on each others profits, showing that collusive behavior becomes more difficult to sustain than when firms do not own equity shares. Gilo and Spiegel (2006) extend this model to a context in which firms are allowed to hold asymmetric equity shares, but still exogenously given shares, demonstrating that collusion can become easier to sustain under certain equity profiles.

[12] For empirical studies analyzing firms' motivations to hold equity on their rivals' profits, see Demsetz and Lehn (1985), which considers 511 U.S. firms; and Bishop et al. (2002), which examines the 162 largest Hungarian firms.

[13] In Kaffine and Costello (2011), owners (similar to firms in our model) can choose whether or not to participate in a unitization scheme. The mechanism is enforced by defectors (who do not share profits) being punished in future periods. Because there are no environmental externalities, full unitization is socially optimal, which can be sustained in equilibrium.

The following section describes the model. Section 3 identifies equilibrium appropriation, Section 4 finds socially optimal appropriation, and compares it against equilibrium values. Section 5 provides a discussion on our results.

2 Model

Consider two firms exploiting a CPR of size $\theta \in (0, 1]$. In the first period, every firm $i = \{1, 2\}$ simultaneously and independently chooses its equity share on its rival's profit, and in the second stage every firm selects its appropriation level x_i. For simplicity, we assume that firms sell their appropriation in a perfectly competitive market, facing a price normalized to 1.[14] In addition, firm i's second-period cost function is

$$C_i(x_i, x_j) = \frac{x_i(x_i + x_j)}{\theta},$$

entailing a marginal cost, $\frac{\partial C_i(x_i,x_j)}{\partial x_i} = \frac{2x_i+x_j}{\theta}$, which decreases in the available stock, θ, but increases in the firm's appropriation, x_i. Intuitively, firm i finds the resource easier to exploit as it becomes more abundant, but more difficult to capture as it increases its own appropriation (i.e., convex cost). In addition, total and marginal costs are also increasing in firm j's appropriation, q_j, indicating that the resource is more difficult to exploit as firm j increases its appropriation.[15]

In the second period, every firm i observes the aggregate second-period appropriation $X = x_i + x_j$, and responds with its third-period appropriation q_i, facing cost function

$$C_i(q_i, q_j) = \frac{q_i(q_i + q_j)}{\theta(1 + g) - X},$$

where $g \geq 0$ denotes the growth rate of the initial stock. Intuitively, when $g = 0$, the stock does not regenerate and the net stock available at the beginning of the third period is $\theta - X$. In contrast, when $X = g\theta$, the stock is fully recovered, so the initial stock θ is available again at the beginning of the third period. In this case, the second-period cost function is symmetric to that in the second period.

The time structure of the game is as follows:

1. In the first stage, every firm i simultaneously chooses its equity share on its rival's profit.

2. In the second stage, observing the equity shares selected during the first stage by all firms, every firm i simultaneously chooses its second-period appropriation x_i.

[14] For instance, firms sell all their fish captures in an international market for that fish variety, where they compete against many other fishermen, each of them representing a negligible proportion of aggregate sales.

[15] Firm i's marginal cost is $\frac{2q_i+q_j}{\theta}$, while a marginal increase in firm j's appropriation causes an increase of $\frac{q_i}{\theta}$, which is smaller than the marginal cost for all admissible parameter values. Hence, a marginal increase in its own appropriation produces a larger increase in firm i's costs than a marginal increase in its rival's appropriation, i.e., $\frac{\partial C_i(q_i,q_j)}{\partial q_i} > \frac{\partial C_i(q_i,q_j)}{\partial q_j}$.

3. In the third stage, observing equity shares (α_i, α_j) and aggregate second-period appropriation, X, every firm i responds selecting its third-period appropriation level q_i.

3 Equilibrium Analysis

3.1 Third stage - Equilibrium appropriation

In the third stage, every firm observes the equity share profile (α_i, α_j), and the aggregate second-period appropriation, X. Firm i then selects its third-period appropriation level q_i to solve

$$\max_{q_i \geq 0} \; V_i^{3rd} = (1-\alpha_j)\pi_i^{3rd} + \alpha_i \pi_j^{3rd} \tag{1}$$

where $\pi_i^{3rd} \equiv q_i - \frac{q_i(q_i+q_j)}{\theta(1+g)-X}$ denotes firm i's third-period profit, $j \neq i$, and prices are normalized to one.

Therefore, each firm i has two components in its objective function: (1) the share that firm i keeps in its own profit π_i^{3rd}, after subtracting the share that firm j holds, $\alpha_j \in [0, 1/2]$; and (2) a share $\alpha_i \in [0, 1/2]$ on the firm j's profits π_j^{3rd}.[16] Specifically, when firms hold no equity shares, $\alpha_i = \alpha_j = 0$, the above objective function collapses to π_i^{2nd}, indicating that every firm only considers its own profit when choosing its individual appropriation level q_i; as in standard CPR models. When $\alpha_i = \alpha_j = 1/2$, the above objective function simplifies to $\frac{\pi_i^{3rd}+\pi_j^{3rd}}{2}$, and firms equally share their profits (as in a merger of symmetric firms). In that setting, every firm i fully considers the profits of its rival in its individual appropriation decisions, and the number of "effective independent actors" collapses to just one. Finally, when $\alpha_i > 0$ but $\alpha_j = 0$, the objective function in (1) becomes $\pi_i^{2nd} + \alpha_i \pi_j^{2nd}$, indicating that firm i fully retains its profit and receives a share α_i of firm j's profits.[17]

We start our equilibrium analysis by describing firms' best response function in the second stage. (All proofs are relegated to the appendix.)

Lemma 1. *In the third period, every firm i's best response function is*

$$q_i(q_j) = \begin{cases} \frac{\theta(1+g)-X}{2} - \frac{1}{2}\left(1 + \frac{\alpha_i}{1-\alpha_j}\right) q_j & \text{if} \quad q_j \leq \frac{(1-\alpha_j)[\theta(1+g)-X]}{1+\alpha_i-\alpha_j} \\ 0 & \text{otherwise.} \end{cases}$$

As depicted in Figure 1, the vertical intercept of the best response function, $\frac{\theta(1+g)-X}{2}$, is only affected by the CPR's net stock at the beginning of the third period, $\theta(1+g) - X$; while its slope, $\frac{1}{2}\left(1 + \frac{\alpha_i}{1-\alpha_j}\right)$, depends on firms' equity shares.

[16] Allowing for equity shares above $1/2$ would entail that a firm holds more equity on its rival than the rival holds in its own company; as in an acquisition. For simplicity, we do not consider these cases in our analysis.

[17] Alternatively, parameter α_i could represent firms' altruistic concerns. In that case, $\alpha_i = 0$ reflects a selfish agent who only cares about its own payoff, whereas $\alpha_i = 1/2$ indicates a fully altruistic agent. See Velez et al. (2009) for controlled experiments in artisanal fisheries in Colombia, reporting that individuals who exploit a fishery display altruism and other socially favorable behaviors. Our subsequent analysis applies, nonetheless, to both interpretations.

For presentation purposes, we next examine the best response function in different settings.

Case 1: CPR models without equity shares. When firms hold no shares on their rivals' profits, $\alpha_i = \alpha_j = 0$, every firm i's best response function collapses to

$$q_i(q_j) = \frac{\theta(1+g) - X}{2} - \frac{1}{2}q_j \tag{BRF$_1$}$$

which is positive for all $q_j \leq \theta(1+g) - X$; as in standard CPR models. In words, firm i appropriates $\frac{\theta(1+g)-X}{2}$ when its rival does not exploit the resource, $q_j = 0$, but decreases its appropriation as q_j increases, i.e., firms' exploitation levels are strategic substitutes.

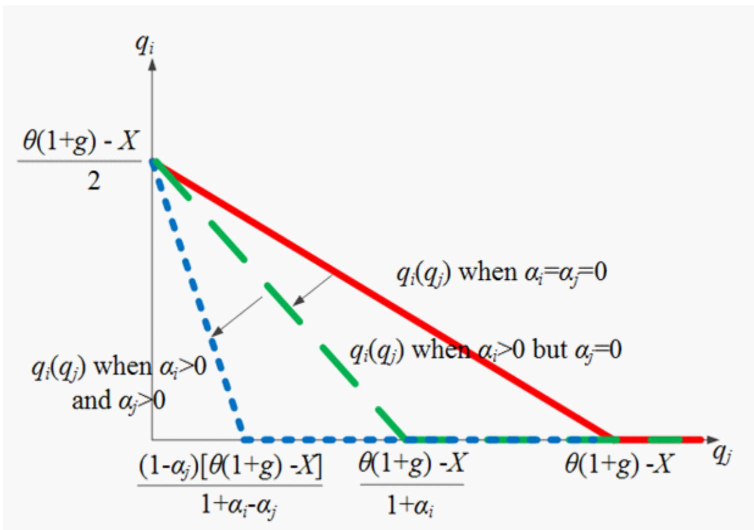

Figure 1. Firm i's best response function.

Case 2: CPR with firm i holding equity shares. When firm i is the only company holding a positive equity on its rival's profits, $\alpha_i > 0$ but $\alpha_j = 0$, its best response function becomes

$$q_i(q_j) = \frac{\theta(1+g) - X}{2} - \frac{1+\alpha_i}{2}q_j, \tag{BRF$_2$}$$

thus pivoting inwards relative to BRF$_1$, as depicted in Figure 1.[18] Intuitively, firm i internalizes a share of the external effect that its appropriation causes on its rival's profit, and thus reduces its own exploitation of the resource. Since the best response function is now steeper, firms' appropriation becomes strategic substitutes to a greater extent.

Case 3: CPR with both firms holding equity shares. If both companies hold equity shares, $\alpha_i, \alpha_j > 0$, we obtain the best response function in Lemma 1, which experiences a further pivoting

[18] In this setting, the horizontal intercept of the best response function is $q_j = \frac{\theta(1+g)-X}{1+\alpha_i}$, where $\frac{\theta(1+g)-X}{1+\alpha_i} \leq \theta(1+g) - X$ since $\alpha_i \geq 0$.

effect inwards relative to BRF_2 (where $\alpha_i > 0$ but $\alpha_j = 0$); as illustrated in Figure 1.[19] In this case, exploitation also decreases, which is now due to the fact that firm i retains a smaller share of its own profits when $\alpha_j > 0$.

The following Proposition analyzes equilibrium appropriation levels.

Proposition 1. *Every firm i's equilibrium third-stage appropriation is $q_i^* = \frac{(1-\alpha_i)[\theta(1+g)-X]}{3-\alpha_i-\alpha_j}$, which is strictly positive for all admissible parameter values. In addition, q_i^* is increasing in θ, in g, and in α_j, decreasing in X and α_i, and satisfies $q_i^* \geq q_j^*$ if and only if $\alpha_i \leq \alpha_j$.*

Confirming our discussion of firms' best response function in Lemma 1, equilibrium appropriation q_i^* is increasing in the net stock available at the beginning of the third period, $\theta(1+g)-X$, but decreasing in firm i's equity share on firm j, α_i, since firm i internalizes the cost externality that it imposes on its rival to a larger extent. In addition, firm i exploits the resource more intensively than its rival if it holds a smaller share of equity, $\alpha_i \leq \alpha_j$.

Finally, firm i's exploitation increases in the share that its rival holds in its profit, α_j. To understand this result, consider first the case in which only firm j holds a positive equity share on i's profit, $\alpha_i = 0$ and $\alpha_j > 0$. In this context, firm i's best response function becomes

$$q_i(q_j) = \frac{\theta(1+g)-X}{2} - \frac{1}{2}q_j$$

which coincides with that when both firms hold no equity on each other's profits, $\alpha_i = \alpha_j = 0$. In this setting, an increase in firm j's equity share, α_j, does not affect firm i's best response function. However, firm j's best response function in this context is

$$q_j(q_i) = \frac{\theta(1+g)-X}{2} - \frac{1+\alpha_j}{2}q_i.$$

A marginal increase in firm j's equity share, α_j, produces a pivoting effect in firm j's best response function: not affecting its slope, $\frac{\theta(1+g)-X}{2}$, but making it steeper. Intuitively, as firm j has a bigger stake on firm i's profits, it internalizes a larger share of the cost externality that its appropriation imposes on firm i. As a consequence, firm j decreases its appropriation to lower firm i's costs. Because best response function $q_i(q_j)$ is unaffected by a marginal increase in α_j but $q_j(q_i)$ becomes steeper, equilibrium appropriation increases for firm i but decreases for j, as illustrated in figure 2a. This explains why firm i's equilibrium third-period appropriation evaluated at $\alpha_i = 0$ and $\alpha_j > 0$, $q_i^* = \frac{\theta(1+g)-X}{3-\alpha_j}$, is increasing in α_j; while that of firm j, $q_j^* = \frac{(1-\alpha_j)[\theta(1+g)-X]}{3-\alpha_j}$, is

[19] Specifically, the difference in the horizontal intercepts yields $\frac{\theta(1+g)-X}{1+\alpha_i} - \frac{(1-\alpha_j)[\theta(1+g)-X]}{1+\alpha_i-\alpha_j} = \frac{\alpha_i\alpha_j[\theta(1+g)-X]}{(1+\alpha_i)(1+\alpha_i-\alpha_j)}$, which is positive for all admisible parameter values.

decreasing because $\frac{\partial q_j^*}{\partial \alpha_j} = -\frac{2[\theta(1+g)-X]}{(3-\alpha_j)^2}$.

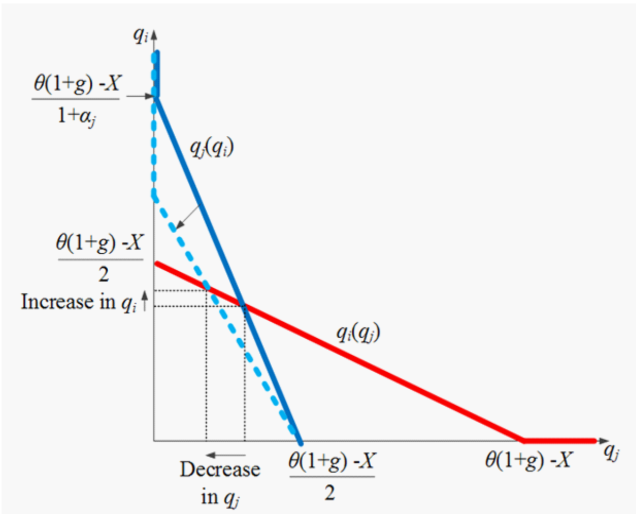

Figure 2a. An increase in α_j when $\alpha_i = 0$.

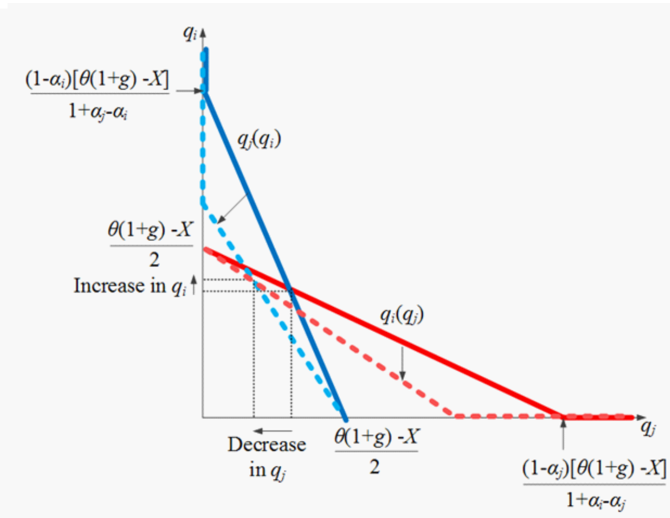

Figure 2b. An increase in α_j when $\alpha_i > 0$.

These effects are ameliorated when firm i also holds a positive equity share (i.e., $\alpha_i, \alpha_j > 0$), where its best response function becomes $q_i(q_j) = \frac{\theta(1+g)-X}{2} - \frac{1}{2}\left(1 + \frac{\alpha_i}{1-\alpha_j}\right) q_j$. As depicted in figure 2b, an increase in α_j produces a clockwise rotation in firm i's best response function. Intuitively, firm i has less motivation to appropriate when his rival holds a larger equity share in its profits. An increase in α_j, however, also yields a clockwise rotation in firm j's best response function, $q_j(q_i) = \frac{\theta(1+g)-X}{2} - \frac{1}{2}\left(1 + \frac{\alpha_j}{1-\alpha_i}\right) q_i$, which we found above. In sum, the rotation of $q_i(q_j)$ induces firm i to appropriate less, while that of $q_j(q_i)$ leads this firm to appropriate more. Overall, however, the effect on $q_j(q_i)$ dominates, entailing that, for a given equity α_i, a marginal increase in α_j induces firm i to increase its appropriation, as identified in Proposition 1.[20]

The following corollary evaluates equilibrium appropriation at special cases.

Corollary 1. *Third-period equilibrium appropriation q_i^* becomes*

1. *CPR model without equity shares:* $q_i^* = \frac{\theta(1+g)-X}{3}$, *i.e.,* $\alpha_i = 0$ *for every firm* i*;*

2. *CPR model with symmetric equity shares:* $q_i^* = \frac{(1-\alpha)[\theta(1+g)-X]}{3-2\alpha}$, *i.e.,* $\alpha_i = \alpha_j = \alpha$*;*

3. *CPR model with equally shared equity:* $q_i^* = \frac{\theta(1+g)-X}{4}$, *i.e.,* $\alpha_i = \alpha_j = \frac{1}{2}$.

[20] Our above discussion can be alternatively presented by noting that the derivative $\frac{\partial q_i^*}{\partial \alpha_j} = \frac{(1-\alpha_i)[\theta(1+g)-X]}{(3-\alpha_i-\alpha_j)^2}$ is monotonically decreasing in α_i, originating at $\left.\frac{\partial q_i^*}{\partial \alpha_j}\right|_{\alpha_i=0} = \frac{\theta(1+g)-X}{(3-\alpha_j)^2}$ when $\alpha_i = 0$ and decreasing to $\left.\frac{\partial q_i^*}{\partial \alpha_j}\right|_{\alpha_i=1/2} = \frac{2[\theta(1+g)-X]}{(5-2\alpha_j)^2}$ when $\alpha_i = 1/2$. More formally, the cross-partial $\frac{\partial^2 q_i^*}{\partial \alpha_j \partial \alpha_i} = -\frac{(1+\alpha_i-\alpha_j)[\theta(1+g)-X]}{(3-\alpha_i-\alpha_j)^3}$ is negative since $\alpha_i, \alpha_j \in [0, 1/2]$ by definition.

In the standard CPR model without equity shares, third-period appropriation is $q_i^* = \frac{\theta(1+g)-X}{3}$; which decreases to $q_i^* = \frac{(1-\alpha)[\theta(1+g)-X]}{3-2\alpha}$ when firms hold symmetric equity shares, $\alpha_i = \alpha_j = \alpha$. In addition, $q_i^* = \frac{(1-\alpha)[\theta(1+g)-X]}{3-2\alpha}$ decreases in α, reaching its lowest level when firms equally share profits, $\alpha = 1/2$, as in a merger, where their equilibrium appropriation becomes $q_i^* = \frac{\theta(1+g)-X}{4}$. Finally, if in this setting the resource fully regenerates across periods, $g = \frac{X}{\theta}$, appropriation is $q_i^* = \frac{\theta}{4}$.

3.2 Second stage - Equilibrium appropriation

In the second stage, every firm i anticipates the equilibrium appropriation pair $\left(q_i^*, q_j^*\right)$ in the subsequent stage of the game. Evaluating its equilibrium profits at the third stage, we obtain

$$\pi_i^{3rd}\left(q_i^*, q_j^*\right) = \frac{(1-\alpha_i)[\theta(1+g)-X]}{(3-\alpha_i-\alpha_j)^2},$$

while those of firm j are symmetric, $\pi_j^{3rd}\left(q_i^*, q_j^*\right)$. Therefore, firm i's third-period payoff is

$$\begin{aligned} V_i^{3rd}\left(q_i^*, q_j^*\right) &= (1-\alpha_j)\pi_i^{3rd}\left(q_i^*, q_j^*\right) + \alpha_i \pi_j^{3rd}\left(q_i^*, q_j^*\right) \\ &= \frac{(1-\alpha_j)[\theta-(1-r)X]}{(3-\alpha_i-\alpha_j)^2} \end{aligned}$$

Anticipating this profit, every firm i simultaneously and independently chooses the second-period appropriation level x_i that solves

$$V_i^{2nd} = \max_{x_i \geq 0} \underbrace{\left[(1-\alpha_j)\pi_i^{2nd} + \alpha_i \pi_j^{2nd}\right]}_{\text{Second period}} + \underbrace{V_i^{3rd}\left(q_i^*, q_j^*\right)}_{\text{Third period}} \tag{2}$$

where the term in brackets reflects the firm's first-period payoff, and $\pi_i^{2nd} = x_i - \frac{x_i(x_i+x_j)}{\theta}$. For simplicity, we assume no payoff discounting. The following lemma identifies the second-period best response function.

Lemma 2. *In the second period, firm i's best response function is*

$$x_i(x_j) = \begin{cases} \frac{\theta(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{2(3-\alpha_i-\alpha_j)^2} - \frac{1}{2}\left(1+\frac{\alpha_i}{1-\alpha_j}\right)x_j & \text{if } \; x_j \leq \frac{\theta(1-\alpha_j)(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{(1+\alpha_i-\alpha_j)(3-\alpha_i-\alpha_j)^2} \\ 0 & \text{otherwise.} \end{cases}$$

As in our analysis of firms' best response function during the third period (section 3.1), we first evaluate $x_i(x_j)$ in the case that firms hold no equity shares, $\alpha_i = \alpha_j = 0$. In this setting, the above best response function collapses to

$$x_i(x_j) = \frac{4\theta}{9} - \frac{1}{2}x_j.$$

When firm i holds equity shares on its rival's profits, $\alpha_i > 0$, we obtain

$$x_i(x_j) = \frac{\theta(4-\alpha_i)(2-\alpha_i)}{2(3-\alpha_i)^2} - (1-\alpha_i)x_j.$$

Therefore, an increase in α_i shifts firm i's best response function downward, and makes it steeper, thus indicating that firm i unambiguously decreases its first-period appropriation, for a given x_j from its rival. Finally, when both firms sustain positive equity shares, $\alpha_i, \alpha_j > 0$, their best response function coincides with that in Lemma 2, entailing that it originates below than the best response function when only firm i holds equity shares, and becomes steeper. Intuitively, with α_j now also becoming positive, the appropriation of the resource for a given level of the rival's appropriation decreases even further.

We next evaluate equilibrium appropriation in this period.

Proposition 2. *Every firm i's second-period equilibrium appropriation is*

$$x_i^* = \frac{\theta(1-\alpha_i)(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3},$$

which is strictly positive for all admissible parameter values. In addition, x_i^* is increasing in θ, and in α_j, decreasing in α_i, and satisfies $x_i^* \geq x_j^*$ if and only if $\alpha_i \leq \alpha_j$.

Firm i's appropriation of the resource is, then, increasing in its initial stock, θ, in α_j, but decreasing in α_i, and satisfies $x_i^* \geq x_j^*$ if and only if $\alpha_i \leq \alpha_j$; thus following a similar pattern as third-period appropriation in Proposition 1.

The following corollary evaluates equilibrium appropriation at special cases.

Corollary 2. *Second-period equilibrium appropriation x_i^* becomes*

1. *CPR model without equity shares: $x_i^* = \frac{8\theta}{27}$, i.e., $\alpha_i = 0$ for every firm i;*
2. *CPR model with symmetric equity shares: $x_i^* = \frac{4\theta(2-\alpha)(1-\alpha)^2}{(3-2\alpha)^3}$, i.e., $\alpha_i = \alpha_j = \alpha$;*
3. *CPR model with equally shared equity: $x_i^* = \frac{3\theta}{16}$, i.e., $\alpha_i = \alpha_j = \frac{1}{2}$.*

As in Corollary 1, second-period appropriation is the highest in the standard CPR model without equity shares, $q_i^* = \frac{8\theta}{27}$; and decreases to $q_i^* = \frac{3\theta}{16}$ when firms hold symmetric equity shares, $\alpha_i = \alpha_j = \alpha$, which decreases in α, reaching its lowest level when firms equally share profits, $\alpha = 1/2$, as in a merger.

3.3 First period - Equilibrium equity shares

Anticipating the above equilibrium results in the second and third stages, in the first stage every firm i chooses its equity share on its rival's profits, α_i, to solve

$$\begin{aligned}\max_{\alpha_i \geq 0}\ \Pi_i &= (1-\alpha_j)V_i^{2nd} + \alpha_i V_j^{2nd} + NW_i\alpha_j^{\beta} - C(\alpha_i) \\ &= \frac{\theta(1-\alpha_j)\left[(11+\alpha_i^2-(7-\alpha_j)\alpha_j-\alpha_i(7-2\alpha_j))^2+(3-\alpha_i-\alpha_j)^4 g\right]}{(3-\alpha_i-\alpha_j)^6} + NW_i\alpha_j^{\beta} - C(\alpha_i)\end{aligned} \tag{3}$$

where V_i^{2nd} was defined in problem (2), and $NW_i\alpha_j^{\beta}$ denotes the monetary amount that firm i receives when firm j acquires α_j equity on firm i's profits. The last term, $C(\alpha_i) = F + NW_j\alpha_i^{\beta}$, represents firm i's cost of acquiring an equity α_i from firm j. For generality, this term includes a fixed cost $F \geq 0$ (e.g., broker fees), and firm j's net worth, NW_j, where $\infty > NW_j \geq 0$.[21] When $\beta > 1$, firm i's marginal cost of acquiring equity from firm j, $NW_j\beta\alpha_i^{\beta-1}$, is increasing in α_i. Intuitively, as firm i purchases more shares from firm j, the cost of additional equity increases, i.e., firm j's equity becomes more scarce, and firm i's opportunity cost of capital goes up.[22] Most studies analyzing endogenous equity assume that equity acquisition is costless, which our model allows as a special case when $F = NW_j = 0$; but also allows for more general cost structures. (For completeness, Appendix 2 considers endogenous NW_i, finding equilibrium equity shares in that context.)

Differentiating with respect to α_i, yields

$$(1-\alpha_j)\frac{\partial V_i^{2nd}}{\partial \alpha_i} + V_j^{2nd} + \alpha_i\frac{\partial V_j^{2nd}}{\partial \alpha_i} = \beta NW_j\alpha_i^{\beta-1}$$

In a symmetric equilibrium where $\alpha_i = \alpha_j$, firms anticipate that their appropriation levels in all subsequent stages of the game will coincide, entailing that profits also satisfy $V_i^{2nd} = V_j^{2nd}$. Therefore, the above first-order condition can be rewritten as

$$MB_i \equiv V_j^{2nd} + \frac{\partial V_i^{2nd}}{\partial \alpha_i} = \beta NW_j\alpha_i^{\beta-1} \equiv MC_i \tag{4}$$

Intuitively, an increase in equity shares produces a marginal benefit (left side of the above equation) that can be decomposed into a direct effect, since firm i now keeps a larger future profit of its rival, and an indirect effect, as a larger equity share softens the competition for appropriation in all subsequent stages. The right-hand side indicates, in contrast, the marginal cost of acquiring equity.

[21] The share price of a firm includes both its current profits but also other factors, which we consider exogenous. Examples of firms with net worths significantly above current profits include Amazon, Netflix, Broadcom Limited, and Ionis Pharmaceuticals Verten; see Forbes (2017)

[22] Several companies pay a premium for acquiring a substantial stake in their rival's profits, above the current market price of the shares they acquire; suggesting that the cost of acquiring equity may be convex. A common example is that of Renault, buying 36.4% of Nissan for $5.4 billion, while Nissan's market capitalization at the time was worth approximately $3.6 billion; see New York Times (March 17, 1999) and CNN Money (March 27, 1999).

Inserting the expressions of V_i^{2nd} and V_j^{2nd} into (4), we find

$$MB_i \equiv \frac{2\theta(1-\alpha_i)[4(3-\alpha_i)(1-\alpha_i)[11-2\alpha(7-2\alpha_i)]+(3-2\alpha_i)^4 g]}{(3-2\alpha_i)^7} = \beta NW_j \alpha_i^{\beta-1} \equiv MC_i.$$

It is easy to show that MB_i originates at $\frac{2\theta(44+27g)}{729}$, when $\alpha_i = 0$, and increases to $\frac{\theta(25+16g)}{128}$ when $\alpha_i = \frac{1}{2}$. In addition, MB_i decreases in g since $\frac{\partial MB_i}{\partial g} = -\frac{2\theta(1-\alpha_i)}{(3-2\alpha_i)^3} < 0$. Graphically, an increase in the regeneration rate of the resource, g, produces a downward shift in MB_i, indicating that firm i has less incentives to acquire equity on its rival's profits since it anticipates more stock being available in the last period, ultimately decreasing its equilibrium equity α_i^*.

Because MB_i is highly nonlinear in α_i, the comparative statics with respect to θ are, however, more involved, so figure 3a numerically evaluate the expressions of MB_i and MC_i at parameter values $\theta = 1/2$, $\beta = 2$, $g = 0.9$, and $NW_j = 1/2$. The figure illustrates that $MC_i = \alpha_i$ in this context, thus being linearly increasing in α_i, while MB_i becomes

$$MB_i = \frac{(1-\alpha_i)\left[2049 - 8\alpha_i(673 - 2\alpha_i(319 - \alpha_i(129 - 19\alpha_i)))\right]}{10(3-2\alpha_i)^7}$$

which is also increasing, but concave, in α_i, producing an equilibrium equity share is $\alpha_i^* = 0.105$. Intuitively, the marginal benefit is increasing as firm i acquires additional equity on its rival's profit, while the marginal cost of acquiring equity is, in this setting, constant. Finally, an increase in parameter β, entailing more convex costs from acquiring equity, produces a counterclockwise rotation in MC_i, which decreases the equity share that firm i acquires in equilibrium, α_i^*.

For comparison purposes, the figure also evaluates MB_i at a more abundant stock, $\theta = 0.7$, showing that firms have more incentives to acquire equity shares on each other's profits, producing an increase in α_i^* to $\alpha_i^* = 0.154$.

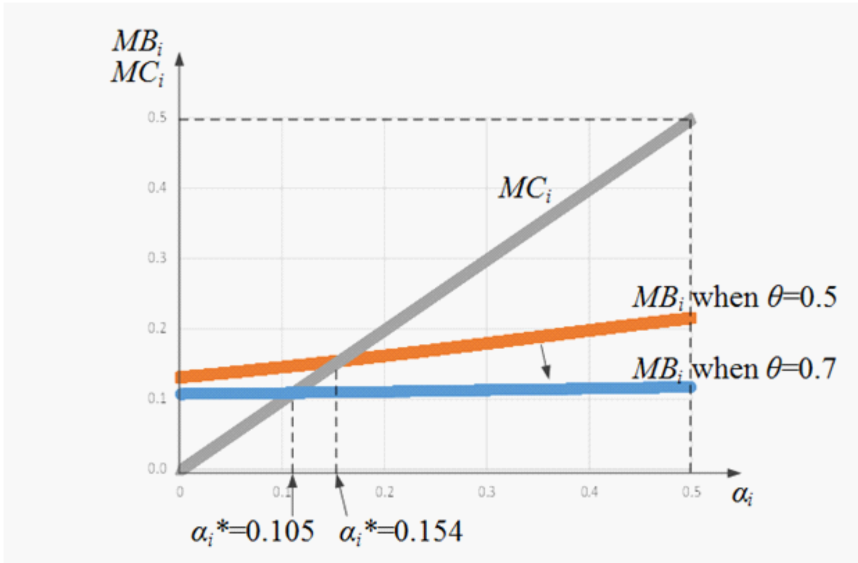

Figure 3. MB_i and MC_i - Numerical example.

Table I finds the equilibrium equity α_i^* for other parameter values. We first analyze the symmetric case, assuming that both firms face the same net worth, $NW_i = NW_j = NW$ and $\beta = 2$. We consider different parameter combinations of θ and NW.[23]

Net Worth NW / Stock θ	$\theta = 1/10$	$\theta = 3/10$	$\theta = 1/2$	$\theta = 7/10$	$\theta = 1$
$NW = 1/2$	0.019	0.060	0.105	0.154	0.241
$NW = 1$	0.009	0.028	0.050	0.071	0.105
$NW = 2$	0.005	0.014	0.024	0.034	0.049
$NW = 3$	0.003	0.009	0.016	0.022	0.032

Table I. Equilibrium equity share α_i^*.

As the net worth of firm j increases, the cost of acquiring a given percentage for firm i increases, leading firm i to reduce its equity acquisition on j. The net worth of firm j affects firm i's equity acquisition decision in stage 1, which ultimately impacts the exploitation of the resource during stages 2 and 3. For example, when $\theta = 1/10$ and $NW = 1/2$, firms acquire 0.135 share in each other's profit. However, as the net worth increases to 3, optimal equity share acquired becomes much lower at 0.003. We can also see that, as the size of the resource (θ) increases, so does the share of equity acquisition across firms.

[23] Table I restricts the values of α_i^* to its admissible range $\alpha_i^* \in [0, 1/2]$. In addition, in case of multiple roots, we report the only root that lies in the admissible range.

4 Welfare analysis

We next compare the equilibrium results against the social optimum. The social planner's problem is to maximize welfare across each stage of the game.

Third stage. In the third stage, the social planner seeks to find the optimal appropriation levels for each firm, q_i^{SO} and q_j^{SO}, solving

$$\max_{q_i,q_j\geq 0} \quad SW^{3rd} = CS^{3rd} + PS^{3rd} - d(q_i+q_j)^2 \tag{5}$$

where consumer surplus considers an inverse demand function $p(q_i,q_j) = a - b(q_i+q_j)$ with parameters $a > 1$ and $b > 0$. This domestic demand function is then evaluated at the internationally given price $p = 1$ to find consumer surplus $CS^{3rd} = \frac{(a-1)(q_i+q_j)}{2}$.[24] Producer surplus $PS^{3rd}(q_i,q_j) = V_i^{3rd} + V_j^{3rd}$ sums the objective functions of both firms in problem (1), and $Env(q_i,q_j) = d(q_i+q_j)^2$ denotes the environmental damage, which is convex in aggregate appropriation (q_i+q_j), and $d \in [0,1]$. Intuitively, exploiting the resource can affect the food chain connected to a fishing ground, damaging biodiversity in the area, or be due to the pollution from fishing vessels. Therefore, environmental damage is interpreted as a by-product of appropriation. Of course, when $d = 0$, appropriation does not entail environmental damages and social welfare only includes consumer and producer surplus, as in standard models.

Consumer surplus and environmental damage are not a function of equity shares and, in addition, producer surplus $PS^{3rd}(q_i,q_j)$ collapses to $\pi_i + \pi_j - 2F$. Therefore, after differentiating with respect to output q_i, the social planner's problem in (5) does not contain equity shares. Solving for q_i, yields

$$q_i^{SO}(q_j,x_i,x_j) = \frac{(1+a)[\theta(1+g)-(x_i+x_j)]}{4+4d[\theta(1+g)-(x_i+x_j)]} - q_j$$

In a symmetric equilibrium, $q_i = q_j$, we find that the socially optimal appropriation level in the third stage is

$$q_i^{SO}(x_i,x_j) = \frac{(1+a)[\theta(1+g)-(x_i+x_j)]}{8+8d[\theta(1+g)-(x_i+x_j)]},$$

which simplifies to $q_i^{SO}(x_i,x_j) = \frac{(1+a)[\theta(1+g)-X]}{8}$ when the resource extraction does not entail environmental externalities, as in most fishery models.

Second stage. After finding $q_i^{SO}(x_i,x_j)$, we can now substitute this expression into problem (5) again to obtain $SW^{3rd}(x_i,x_j)$, as a function of second-period appropriation levels x_i and x_j. In

[24] For simplicity, we assume firms are price takers, meaning that each of them produces a negligible share of the aggregate output (e.g., each firm is small relative to the international market of the product). However, when the regulator considers social welfare in a given country, he considers the domestic demand for the good when evaluating consumer surplus.

this period, the social planner chooses x_i and x_j to solve

$$\max_{x_i, x_j} \quad \underbrace{CS^{2nd} + PS^{2nd} - d(x_i + x_j)^2}_{\text{Second period}} + \underbrace{SW^{3rd}(x_i, x_j)}_{\text{Third period}} \tag{6}$$

where second-period welfare is analogous to that in the third period, as defined in expression (5). Differentiating with respect to x_i and invoking symmetry yields a first-order condition

$$\frac{1+a}{2} - \frac{(1+a)^2}{16(1+d[(1+g)\theta - 2x_i])^2} - \frac{4(1+d\theta)x_i}{\theta} = 0.$$

This expression is highly nonlinear in x_i, and thus we cannot obtain an explicit function of the socially optimal second-period appropriation, x_i^{SO}. Table II, however, numerically approximates x_i^{SO} considering the same parameter values as in Table I and $NW = 1/2$, $a = 2$, and $d = 0.7$. The second (third) column reports the equilibrium (socially optimal) second-period appropriation, x_i^* (x_i^{SO}), and the fourth (fifth) column lists the equilibrium (socially optimal) third-period appropriation, q_i^* (q_i^{SO}). Finally, the sixth (seventh) column reports the equilibrium (socially optimal) aggregate appropriation across both periods.

	Second period		Third period		Aggregate appropriation		
Stock, θ	x_i^*	x_i^{SO}	q_i^*	q_i^{SO}	$x_i^* + q_i^*$	$x_i^{SO} + q_i^{SO}$	$(x_i^* + q_i^*) - (x_i^{SO} + q_i^{SO})$
$\theta = 1/10$	0.029	0.024	0.043	0.048	0.072	0.072	0.000
$\theta = 3/10$	0.086	0.072	0.130	0.123	0.216	0.195	0.021
$\theta = 5/10$	0.140	0.116	0.214	0.18	0.354	0.296	0.058
$\theta = 7/10$	0.190	0.154	0.300	0.223	0.490	0.377	0.113
$\theta = 1$	0.254	0.201	0.420	0.274	0.674	0.475	0.199

Table II. Equilibrium and optimal appropriations – Benchmark.

Intuitively, Table II analyzes how second- and third-period appropriation levels are affected by a more abundant stock, θ. As anticipated, an increase in θ induces a higher equilibrium and optimal exploitation of the resource, in both the second and third periods. Appendix 1 examines how the above results are affected when g, NW, or a increase.

No environmental damages. Table III considers a setting where appropriation generates no environmental damages ($d = 0$ rather than $d = 0.7$ in Table II). In this setting, the above first-order condition simplifies to

$$\frac{1+a}{2} - \frac{(1+a)^2}{16} - \frac{4x_i}{\theta} = 0$$

yielding a socially optimal second-period appropriation of

$$x_i^{SO} = \frac{\theta(1+a)(7-a)}{64}.$$

As expected, a less severe externality has no effect on firm's appropriation levels in the second and third period, but increases the socially optimal appropriation in both periods. As a result, aggregate appropriation is socially insufficient; that is, $x_i^* + q_i^* < x_i^{SO} + q_i^{SO}$ for all values of stock θ. This result suggests that, because of the reduction in exploitation levels that occurs after firms acquire a positive equity share in each other's profits, their equilibrium appropriation may be suboptimal, especially if the exploitation of the resource does not generate environmental damages. We discuss below policy tools to address the socially insufficient (or excessive) appropriation that arises in equilibrium.

	Second period		Third period		Aggregate appropriation		
Stock, θ	x_i^*	x_i^{SO}	q_i^*	q_i^{SO}	$x_i^* + q_i^*$	$x_i^{SO} + q_i^{SO}$	$(x_i^* + q_i^*) - (x_i^{SO} + q_i^{SO})$
$\theta = 1/10$	0.029	0.023	0.043	0.054	0.072	0.077	-0.003
$\theta = 3/10$	0.086	0.070	0.130	0.161	0.216	0.231	-0.015
$\theta = 5/10$	0.140	0.117	0.214	0.269	0.354	0.386	-0.032
$\theta = 7/10$	0.190	0.164	0.300	0.376	0.490	0.540	-0.050
$\theta = 1$	0.254	0.234	0.420	0.537	0.674	0.771	-0.097

Table III. Equilibrium and optimal appropriations – No environmental damage.

4.1 Socially excessive equity acquisition?

From our above results, we showed that aggregate equilibrium appropriation is socially excessive, $x_i^* + q_i^* > x_i^{SO} + q_i^{SO}$, under large parameter values. In other words, while equity acquisition induces both firms to reduce their exploitation of the resource in equilibrium (relative to a setting where firm do not acquire equity), their appropriation is still excessive. Firms' equity decisions then ameliorate inefficiencies, but are insufficient to fully correct them. In short, firms acquire a socially insufficient amount of equity on each other's profits. This holds in particular when the exploitation of the resource generates environmental damages (as in Table II), the resource has a strong rate of regeneration (see Table AI in Appendix 1), or when firms' net worth is substantial (see Table AII in Appendix 1).

However, when environmental damages are absent (as in Table III) and demand is relatively strong (Table AIII in Appendix 1), firms may exploit the resource below the socially optimal levels, suggesting that they acquire a socially excessive equity on their rival's profits. In this case, equity shares induced firms to reduce too much their appropriations, ultimately leading to a suboptimal exploitation of the resource.

Finally, it is straightforward to check that, when environmental externalities are absent ($d = 0$) and consumer surplus is absent from the social planner's welfare function (e.g., firms do not sell products domestically), the objective function in problem (6) coincides with that of firm i's problem in (3) if and only if equity shares satisfy $\alpha_i = \alpha_j = 1/2$; as in a merger between firms i and j. Intuitively, equilibrium appropriation is only socially optimal when firms equally share their profits;

as shown in Ellis (2001, Proposition 1). Otherwise, the resource is overexploited.

5 Policy Tools

As shown above, aggregate equilibrium appropriation may not coincide with the social optimum. Common policy tools to correct this inefficiency include quotas and Pigouvian taxes. However, monitoring the actual exploitation of the resource can be costly and is often inaccurate. In this section, we propose an alternate policy tool: taxes on equity acquisition inducing firms to choose the socially optimal equity share. While this policy tool was examined in Kanjilal and Munoz-Garcia (2019) for the case of Cournot duopolists, it cannot be readily applied to our CPR problem where the resource regenerates across periods. In particular, the socially optimal appropriation in each period is different when the stock grows at a rate g, as shown in the expressions for x^{SO} and q^{SO} found above. In such a setting, it is not possible to find a socially optimal equity share α^{SO} that ensures both $x^* = x^{SO}$ and $q^* = q^{SO}$, i.e., a unique equity share α^{SO} cannot guarantee that equilibrium appropriation is socially optimal in both time periods.

First best. To achieve a first-best, equity shares would have to change across periods, but this would be lead to a different game structure allowing firms to choose their equity shares at the beginning of each period —as opposed to the setting we considered, where firms commit to one equity share at the beginning of the first period. We then examine a policy tool that takes the strategic setting as given and seeks to introduce a tax (or subsidy) on equity acquisition to ensure that aggregate equilibrium appropriation coincides with the aggregate social optimum, that ensures that $x^* + q^* = x^{SO} + q^{SO}$. This policy does not guarantee that, at a given period, equilibrium appropriation is socially optimal, and thus can be understood as a second best.

Second best. To find the (second-best) equity share α^{SO2} that solves $x^* + q^* = x^{SO} + q^{SO}$, let us first consider the two equilibrium appropriations that we obtained from the firm's problem, namely,

$$q_i^*(x_i, x_j) = \frac{(1-\alpha_i)[\theta(1+g) - (x_i + x_j)}{3 - \alpha_i - \alpha_j} \quad \text{and} \tag{7}$$

$$x_i^* = \frac{\theta(1-\alpha_i)(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} \tag{8}$$

From simulations in previous section, we found the aggregate socially optimal appropriation, $x_i^{SO} + q_i^{SO}$, for specific parameter values.

We know that when $NW_i = NW_j$, equilibrium appropriation levels satisfy $x_i^* = x_j^* = x^*$ and $q_i^* = q_j^* = q^*$. Thus, from the conditions of symmetry and inserting equation (2) into (1), we get

$$q^* = \frac{\theta(1-\alpha)\left[11 - 14\alpha + 4\alpha^2 + (3-2\alpha)^3 g\right]}{3-2\alpha} \quad \text{and} \tag{9}$$

$$x^* = \frac{\theta(1-\alpha)(4-2\alpha)(2-2\alpha)}{(3-2\alpha)^3} \tag{10}$$

From Table II, we can numerically solve the value for $x_i^{SO} + q_i^{SO}$. Thus, from the equations in (5.3), we can find the value of α^{SO2} that induces firms to choose an aggregate socially optimal appropriation, $x_i^{SO} + q_i^{SO} \equiv \mathbf{Q}^{SO}$ by solving:

$$q^*(\alpha^{SO2}; \theta, g) + x^*(\alpha^{SO2}; \theta, g) = \mathbf{Q}^{SO} \tag{11}$$

Assuming values for θ, a and g, we can numerically solve for $\mathbf{Q}^{SO}$ as in Table II. This leaves us with one unknown in equation (5.4), α^{SO2}. Table IV solves for the value of α^{SO2} consider similar parameter values as in previous tables, namely, $NW_i = NW_j = \frac{1}{2}$, $g = \frac{9}{10}$, $a = 2$ and $d = \frac{7}{10}$.

	Equilibrium outcomes		Soc. optimal outcomes		Tax/Subsidy
Stock, θ	$x_i^* + q_i^*$	α_i^*	$x_i^{SO} + q_i^{SO}$	α^{SO2}	t
$\theta = 1/10$	0.073	0.019	0.072	0.059	-0.662
$\theta = 3/10$	0.218	0.060	0.195	0.313	-0.751
$\theta = 5/10$	0.361	0.105	0.296	0.454	-0.673
$\theta = 7/10$	0.502	0.154	0.377	0.553	-0.569
$\theta = 1$	0.710	0.241	0.475	0.644	-0.384

Table IV. Aggregate equilibrium and optimal appropriations – Benchmark.

This table shows that aggregate appropriation is socially excessive, $q^* + x^* > q^{SO} + x^{SO}$, entailing that firms' equity share acquisition in equilibrium is socially insufficient, $\alpha^{SO} \geq \alpha^*$. Therefore, policies helping firms increase their cross-ownership would lead to a decrease in the exploitation of the resource, thus reducing the aggregate level of resource appropriation.[25]

Table V considers the same parameter values as Table IV, but a lower environmental damage, $d = \frac{2}{10}$. Like in Table IV, when the aggregate appropriation is socially excessive, i.e., $q^* + x^* > q^{SO} + x^{SO}$, we have $\alpha^{SO2} > \alpha^*$ and $t < 0$. This is the case when $\theta \geq \frac{7}{10}$. However, when $q^* + x^* < q^{SO} + x^{SO}$, we can see that $\alpha^{SO2} < \alpha^*$. This is because reducing equity sharing will help increase appropriation to a socially optimal level. This is especially clear when $\theta = \frac{3}{10}$. Here, a positive tax prevents a socially excessive equity acquisition. When $\theta = \frac{1}{10}$, we have a unique case where $\alpha^{SO_2} < 0$, which is not possible since equity shares are bounded in $\alpha^{SO2} \in [0, 0.5]$. Hence, in this case, the social planner can set an indefinitely high tax to ensure that no equity is acquired, or ban equity acquisition.

[25] To find the tax/subsidy that induces firms to acquire the socially optimal equity share, α^{SO2}, one only needs to change the marginal cost of equity acquisition in expression (4) to $\beta NW_j(1+t)\left(\alpha_i^{SO2}\right)^{\beta-1}$. Setting it equal to the marginal benefit of equity acquisition in (4), and solving for t, yields the tax/subsidy reported in Tables IV and V. This policy tool is analogous to that in Kanjilal and Garcia (2019) for firms competing a la Cournot.

	Equilibrium outcomes		Soc. optimal outcomes		Tax/Subsidy
Stock, θ	$x_i^* + q_i^*$	α_i^*	$x_i^{SO} + q_i^{SO}$	α^{SO2}	t
$\theta = 1/10$	0.073	0.019	0.076	-0.163	$+\infty$
$\theta = 3/10$	0.216	0.060	0.219	0.011	4.16
$\theta = 5/10$	0.354	0.105	0.354	0.112	-0.059
$\theta = 7/10$	0.490	0.154	0.480	0.199	-0.188
$\theta = 1$	0.674	0.241	0.655	0.299	-0.143

Table V. Aggregate equilibrium and optimal appropriations – Lower environmental damage.

6 Discussion

Our results identify the equity share that firms exploiting a CPR hold in equilibrium, how this equity acquisition leads a decrease in resource exploitation, and ultimately whether this reduction still entails a socially excessive appropriation of the resource or, instead, it yields a socially insufficient exploitation. We then show how equity taxes can be implemented to induce welfare improving appropriation levels.

Taxes on equity acquisition. When resource appropriation does not generate large externalities (d is small), when the rate of regeneration of the resource g is high, and when demand becomes strong (high a), the planner seeks a larger appropriation level than that emerging in equilibrium. In this setting, it may become socially optimal for firms to not sustain equity shares on each other's profits; as otherwise their output would be socially insufficient. In such a case, the social planner can either: set a lower bound on equity shares that firms can hold, or subsidize the exploitation of the resource to ameliorate underexploitation. However, these policy tools require costly monitoring and supervision.

In this paper, we explore an alternative policy, taxes on equity acquisition, which can approach equilibrium appropriation to the social optimum and only needs information about firms' equity shares, which is generally requested by government authorities anyway. Yet another policy approach could allow for the ecosystem services market to strategically purchase equity shares on different firms (fishing companies), which may be attractive for their owners when appropriation generates large environmental damages, ultimately helping to ameliorate the CPR's overexploitation problem.

Subsidies on equity acquisition. When appropriation generates more severe environmental damages (higher d), socially optimal appropriation decreases, leading to the possibility that resources are overexploited in equilibrium. In this setting, policy makers would want firms to hold a larger equity share on their rival's profits, which would reduce their equilibrium appropriation, ultimately curbing their associated environmental damage (e.g., pollution and biodiversity loss). A similar argument applies when the stock becomes more abundant (higher θ), which makes socially excessive exploitation more likely. In extreme settings where the stock is abundant, but its exploitation generates substantial environmental damages, firms would only have incentives to produce socially optimal levels if they equally share profits.

When overexploitation emerges in equilibrium, the social planner could set an upper bound on equity shares that firms can hold, which may not be politically feasible. Alternatively, regulators can set an emission fee on the exploitation of the resource itself (approaching equilibrium appropriation to its socially optimal level), or subsidize equity acquisition. As emission fees are often costly to monitor and implement, taxes or subsidies on equity acquisition may be more attractive in overexploited CPRs. However, this outcome can also be induced by taxes on equity acquisition. To increase firms' equity acquisition in equilibrium, these taxes would have to be negative, and act as subsidies. This would be both relatively costless to monitor, and subsidies are generally politically more feasible than taxes.

Further research. Our model can be extended to consider settings in which each firm exploits a different CPR, rather than both appropriating from the same commons; and to industries in which firms do not perfectly observe the extent to which one firm places a cost externality on another. Additionally, our results can be empirically estimated, and tested in field experiments, to evaluate if firms' observed exploitation of the CPR approaches our theoretical predictions. Finally, our model assumes no separation between ownership and control of every firm. In CPRs where managers do not own the firm (e.g., fishermen hired by the boat owner to operate the boat), the manager's objective functions would differ from (1) and (2), while the owner's would coincide with (3), giving rise to principal-agent problems. Intuitively, the manager would could have less incentives to appropriate than the manager, especially when firms hold a large equity share in each other's profits.

7 Appendix

7.1 Appendix 1 - Numerical simulations

Table AI examines how appropriation levels change when the growth rate of the resource, g, increases from $g = 0.9$ (in the benchmark Table II) to $g = 1$. Intuitively, as the commons regenerate faster, firms decrease their second-period exploitation to allow the resource to grow more, thus increasing their third-period appropriation. However, the social planner seeks more exploitation in both periods to increase consumer surplus. In this setting, as in Table II, aggregate appropriation is socially excessive, i.e., $x_i^* + q_i^* > x_i^{SO} + q_i^{SO}$ for all values of θ.

	Second period		Third period		Aggregate appropriation		
Stock, θ	x_i^*	x_i^{SO}	q_i^*	q_i^{SO}	$x_i^* + q_i^*$	$x_i^{SO} + q_i^{SO}$	$(x_i^* + q_i^*) - (x_i^{SO} + q_i^{SO})$
$\theta = 1/10$	0.029	0.024	0.047	0.052	0.076	0.076	0.000
$\theta = 3/10$	0.086	0.073	0.140	0.129	0.226	0.202	0.024
$\theta = 5/10$	0.139	0.117	0.231	0.187	0.370	0.304	0.066
$\theta = 7/10$	0.189	0.155	0.320	0.232	0.509	0.387	0.122
$\theta = 1$	0.251	0.202	0.448	0.283	0.699	0.485	0.214

Table AI. Equilibrium and optimal appropriations – More regeneration.

Table AII studies how our equilibrium results are affected when the firms' net worth increases (from $NW = 1/2$ in Table II to $NW = 1$ in Table AII). When NW increases, it becomes costlier for every firm i to acquire equity on its rival's profit, reducing equilibrium α_i^*. As a consequence, privately optimal appropriation increases in every period, also increasing aggregate exploitation, $x_i^* + q_i^*$. However, the (higher) cost of acquiring equity shares cancels out in the social planner problem, thus not affecting socially optimal appropriation (i.e., x_i^{SO}, q_i^{SO}, and their sum $x_i^{SO} + q_i^{SO}$ coincide with those in Table II).

	Second period		Third period		Aggregate appropriation		
Stock, θ	x_i^*	x_i^{SO}	q_i^*	q_i^{SO}	$x_i^* + q_i^*$	$x_i^{SO} + q_i^{SO}$	$(x_i^* + q_i^*) - (x_i^{SO} + q_i^{SO})$
$\theta = 1/10$	0.029	0.024	0.044	0.048	0.073	0.072	0.001
$\theta = 3/10$	0.088	0.072	0.130	0.123	0.218	0.195	0.023
$\theta = 5/10$	0.144	0.116	0.217	0.180	0.361	0.296	0.065
$\theta = 7/10$	0.200	0.154	0.302	0.223	0.502	0.377	0.125
$\theta = 1$	0.280	0.202	0.430	0.274	0.710	0.475	0.235

Table AII. Equilibrium and optimal appropriations – Higher net worth.

Finally, table AIII examines how the equilibrium results change when demand increases (from $a = 2$ in Table II to $a = 3$ in Table AIII). When a increases, equilibrium appropriation is unaffected (x_i^* and q_i^* in Table AIII coincide with those in Table II), but consumer surplus increases, thus inducing a higher socially optimal exploitation. As a result, the parameter values for which equilibrium exploitation is socially excessive decreases, giving rise to more settings where such exploitation is socially insufficient.

	Second period		Third period		Aggregate appropriation		
Stock, θ	x_i^*	x_i^{SO}	q_i^*	q_i^{SO}	$x_i^* + q_i^*$	$x_i^{SO} + q_i^{SO}$	$(x_i^* + q_i^*) - (x_i^{SO} + q_i^{SO})$
$\theta = 1/10$	0.029	0.027	0.043	0.062	0.072	0.089	-0.015
$\theta = 3/10$	0.086	0.086	0.130	0.156	0.216	0.242	-0.026
$\theta = 5/10$	0.140	0.142	0.214	0.227	0.354	0.369	-0.015
$\theta = 7/10$	0.190	0.192	0.300	0.285	0.490	0.477	0.013
$\theta = 1$	0.254	0.256	0.420	0.352	0.674	0.608	0.066

Table AIII. Equilibrium and optimal appropriations – Stronger demand.

7.2 Appendix 2 - Endogenous NW_i

In this appendix, we allow for endogenous NW_i to understand how results are affected. NW_i being endogenous implies that $NW_i = (1-\alpha_j)V_i^{2nd} + \alpha_i V_j^{2nd}$, so firm i's net worth coincides with its future profits in the subsequent stages. Equilibrium results in the third and second periods are, of course, unchanged. Firm i's problem in the first period, however, now becomes

$$\begin{aligned} \max_{\alpha_i \geq 0} \ \Pi_i &= (1-\alpha_j)V_i^{2nd} + \alpha_i V_j^{2nd} + \underbrace{\left[(1-\alpha_j)V_i^{2nd} + \alpha_i V_j^{2nd}\right]}_{NW_i} \alpha_j^2 - C(\alpha_i) \qquad (3') \\ &= (1-\alpha_j^2)\left[(1-\alpha_j)V_i^{2nd} + \alpha_i V_j^{2nd}\right] - C(\alpha_i). \end{aligned}$$

Differentiating with respect to α_i, we obtain

$$(1-\alpha_j^2)\left[(1-\alpha_j)\frac{\partial V_i^{2nd}}{\partial \alpha_i} + V_j^{2nd} + \alpha_i \frac{\partial V_j^{2nd}}{\partial \alpha_i}\right] = \beta \alpha_i^{\beta-1}\left[V_j^{2nd} + \alpha_i \frac{\partial V_j^{2nd}}{\partial \alpha_i} - \alpha_j \frac{\partial V_i^{2nd}}{\partial \alpha_i}\right]$$

where the left side denotes the marginal benefit of increasing α_i, while the right-hand side represents the marginal cost since $C(\alpha_i) = F + NW_j \alpha_i^\beta$ and $NW_j = (1-\alpha_i)V_j^{2nd} + \alpha_j V_i^{2nd}$ by definition. In a symmetric equilibrium where $\alpha_i = \alpha_j = \alpha$, firms anticipate that their appropriation levels in all subsequent stages of the game will coincide, entailing that profits also satisfy $V_i^{2nd} = V_j^{2nd}$. Therefore, the above first-order condition becomes

$$(1-\alpha^2)\left[(1-\alpha)\frac{\partial V_i^{2nd}}{\partial \alpha} + V_i^{2nd} + \alpha \frac{\partial V_i^{2nd}}{\partial \alpha}\right] = \beta \alpha^{\beta-1}\left[V_i^{2nd} + \alpha \frac{\partial V_i^{2nd}}{\partial \alpha} - \alpha \frac{\partial V_i^{2nd}}{\partial \alpha}\right]$$

which, after rearranging, simplifies to

$$(1-\alpha^2)\left(V_i^{2nd} + \frac{\partial V_i^{2nd}}{\partial \alpha}\right) = \beta \alpha^{\beta-1} V_i^{2nd}.$$

This first-order condition is similar to that in expression (4), which assumed exogenous NW_i, but has the marginal benefit (on the left side of the equation) decreased by a factor $(1-\alpha^2)$, where $(1-\alpha^2) < 1$ given that $\alpha \in [0, 1/2]$ by definition. Intuitively, this decrease is due to the fact that a marginal increase in α_i now increases firm i's net worth NW_i, with firm i only keeping $1-\alpha^2$ of it. In addition, the marginal cost of acquiring equity (on the right side of the equation) also changes, relative to (4), considering an endogenous NW_i.

Inserting V_i^{2nd} in the above first-order condition, we obtain

$$MB_i = \frac{(1-\alpha^2)\left[528 - 2267\alpha + 3806\alpha^2 - 3212\alpha^3 + 1448\alpha^4 - 336\alpha^5 + 32\alpha^6 + (3-2\alpha)^4\left[4-\alpha(7-2\alpha)\right]g\right]\theta}{(3-2\alpha)^7}$$

and

$$MC_i = \beta\alpha^{\beta-1}\frac{\theta(1-\alpha)\left[\left[11-2\alpha(7-2\alpha)\right]^2 + (3-2\alpha)^4 g\right]}{(3-2\alpha)^6}.$$

It is easy to show that MB_i originates at $\frac{4(44+27g)\theta}{729}$, when $\alpha_i = 0$, and reaches $\frac{3(25+16g)}{1024}$ when $\alpha_i = \frac{1}{2}$. In addition, MB_i decreases in g since $\frac{\partial MB_i}{\partial g} = \frac{(1-\alpha^2)[4-\alpha(7-2\alpha)]\theta}{(3-2\alpha)^3} > 0$. Setting $MB_i = MC_i$, we find an expression that does not depends on parameters θ.

As in section 3.3, the expression of MB_i is highly nonlinear in α_i, so we evaluate MB_i and MC_i at our ongoing parameter values, $\theta = 1/2$, $\beta = 2$, and $g = 0.9$, finding that

$$MB_i = \frac{(1-\alpha^2)[8196 - \alpha(35549 - 2\alpha(30451 - 8\alpha(3317 - \alpha[1562 - \alpha(381 - 38\alpha))))])]}{20(3-2\alpha)^7}$$

and

$$MC_i = \frac{\alpha(1-\alpha)\left[\frac{9}{10}(3-2\alpha)^4 + (11 - 2\alpha(7-2\alpha))^2\right]}{(3-2\alpha)^6}$$

as depicted in figure AI. Relative to the setting where NW_i is exogenous (figure 3), MC_i is still increasing in α_i but at a lower rate, thus indicating that the marginal cost of acquiring equity is lower. In addition, MB_i is generally above that where NW_i is exogenous, ultimately yielding an equilibrium equity share of $\alpha^* = 0.446$, which is higher than that when NW_i is exogenous. A similar result holds when parameter g increases, and can be provided by the authors upon request.

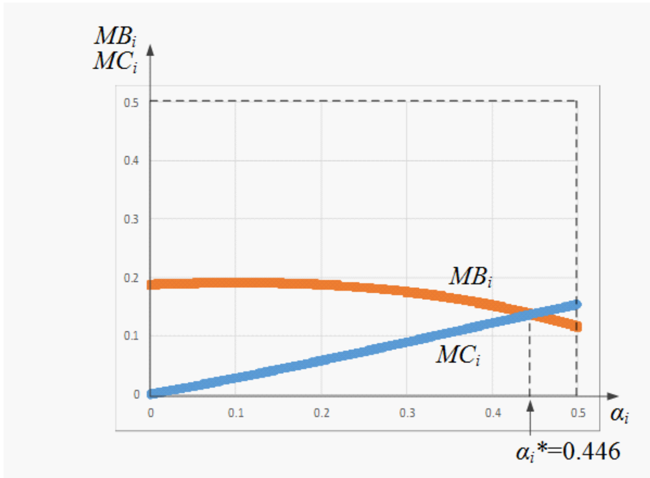

Figure AI. Marginal benefit and cost when NW_i is endogenous.

7.3 *Proof of Lemma 1*

Firm i solves problem (1), which we can more explicitly write as follows

$$\max_{q_i \geq 0} \; (1-\alpha_j)\left(q_i - \frac{q_i(q_i+q_j)}{\theta(1+g)-(x_i+x_j)}\right) + \alpha_i \left(q_j - \frac{q_j(q_i+q_j)}{\theta(1+g)-(x_i+x_j)}\right)$$

Differentiating with respect to q_i yields,

$$1-\alpha_j - \frac{2(1-\alpha_j)q_i + (1+\alpha_i-\alpha_j)}{\theta(1-g)-(x_i+x_j)} q_j.$$

Solving for q_i to get the best response function, we obtain

$$q_i(q_j) = \frac{\theta(1+g)-(x_i+x_j)}{2} - \frac{1}{2}\left(1+\frac{\alpha_i}{1-\alpha_j}\right).$$

Since firms appropriate weakly positive amounts, we can set the above expression greater or equal to zero, and solve for q_j, finding $q_j \leq \frac{(1-\alpha_j)[\theta(1+g)-X]}{1+\alpha_i-\alpha_j}$. Therefore, firm i's best response function is

$$q_i(q_j) = \begin{cases} \frac{\theta(1+g)-X}{2} - \frac{1}{2}\left(1+\frac{\alpha_i}{1-\alpha_j}\right) q_j & \text{if } q_j \leq \frac{(1-\alpha_j)[\theta(1+g)-X]}{1+\alpha_i-\alpha_j} \\ 0 & \text{otherwise.} \end{cases}$$

7.4 *Proof for Proposition 1*

From Lemma 1, we found the best response function for firms i and j. Simultaneously solving for q_i and q_j in $q_i(q_j)$ and $q_j(q_i)$, we obtain that the optimal appropriation for every firm i is

$$q_i^* = \frac{(1-\alpha_i)[\theta(1+g)-(x_i+x_j)]}{3-\alpha_i-\alpha_j}$$

which is positive for all admissible parameter values. We can now differentiate q_i^* with respect to parameters. First,

$$\frac{\partial q_i^*}{\partial \theta} = \frac{(1-\alpha_i)(1+g)}{3-\alpha_i-\alpha_j} > 0$$

thus indicating that q_i^* is increasing in θ. Second,

$$\frac{\partial q_i^*}{\partial g} = \frac{\theta(1-\alpha_i)}{3-\alpha_i-\alpha_j} > 0$$

meaning that q_i^* is increasing in the regeneration rate, g. Third,

$$\frac{\partial q_i^*}{\partial X} = -\frac{1-\alpha_i}{3-\alpha_i-\alpha_j} < 0$$

which says that q_i^* is decreasing in the aggregate second-period appropriation, X. Fourth,

$$\frac{\partial q_i^*}{\partial \alpha_i} = -\frac{(2-\alpha_j)[\theta(1+g)-(x_i+x_j)]}{(3-\alpha_i-\alpha_j)^2} < 0$$

which reflects that q_i^* is decreasing in α_i. Fifth,

$$\frac{\partial q_i^*}{\partial \alpha_j} = \frac{(1-\alpha_i)[\theta(1+g)-(x_i+x_j)]}{(3-\alpha_i-\alpha_j)^2} > 0$$

which implies that q_i^* is increasing in α_j. Finally, the difference

$$q_i^* - q_j^* = \frac{(\alpha_j-\alpha_i)[\theta(1+g)-(x_i+x_j)]}{3-\alpha_i-\alpha_j}$$

is weakly positive if and only if $\alpha_i \leq \alpha_j$.

Substituting q_i^* and q_j^* in the firm's third-period objective function yields

$$\pi_i^{2nd}(q_i^*, q_j^*) = \frac{(1-\alpha_j)[\theta(1+g)-(x_i+x_j)]}{(3-\alpha_i-\alpha_j)^2}.$$

7.5 *Proof of Lemma 2*

Firm i solves problem (2), which we can more explicitly written as follows

$$\max_{x_i \geq 0} \ (1-\alpha_j)\left(x_i - \frac{x_i(x_i+x_j)}{\theta}\right) + \alpha_i\left(x_j - \frac{x_j(x_i+x_j)}{\theta}\right) + \frac{(1-\alpha_j)[\theta(1+g)-(x_i+x_j)]}{(3-\alpha_i-\alpha_j)^2}$$

Differentiating with respect to x_i yields,

$$\frac{(1-\alpha_j)(\theta-2x_i-x_j)}{\theta}-\frac{1-\alpha_j}{(3-\alpha_i-\alpha_j)^2}-\frac{\alpha_i x_j}{\theta}=0$$

Solving for x_i, we obtain the best response function

$$x_i=\frac{\theta(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{2(3-\alpha_i-\alpha_j)^2}-\frac{1}{2}\left(1+\frac{\alpha_i}{1-\alpha_j}\right)x_j$$

Since firms appropriate weakly positive amounts, we can set the above expression greater or equal to zero, and solve for x_j, finding

$$x_j\leq\frac{\theta(1-\alpha_j)(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{(1+\alpha_i-\alpha_j)(3-\alpha_i-\alpha_j)^2}.$$

Therefore, firm i's best response function is

$$x_i(x_j)=\begin{cases}\frac{\theta(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{2(3-\alpha_i-\alpha_j)^2}-\frac{1}{2}\left(1+\frac{\alpha_i}{1-\alpha_j}\right)x_j & \text{if } x_j\leq\frac{\theta(1-\alpha_j)(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{(1+\alpha_i-\alpha_j)(3-\alpha_i-\alpha_j)^2}\\ 0 & \text{otherwise.}\end{cases}$$

7.6 *Proof of Proposition 2*

From Lemma 2, we found the best response function for firms i and j in second-period appropriation. Simultaneously solving for x_i and x_j in $x_i(x_j)$ and $x_j(x_i)$, we obtain that the optimal appropriation for every firm i is

$$x_i^*=x_i^*=\frac{\theta(1-\alpha_i)(1+\alpha_i-\alpha_j)(4-\alpha_i-\alpha_j)(2-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^2[3-2(\alpha_i+\alpha_j)-(\alpha_i-\alpha_j)^2]}$$

which is positive for all admissible parameter values. We can now differentiate x_i^* with respect to parameters. First,

$$\frac{\partial x_i^*}{\partial\theta}=\frac{(1-\alpha_i)(2-\alpha_i-\alpha_j)(4-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3}>0$$

thus indicating that q_i^* is increasing in θ. Second,

$$\frac{\partial x_i^*}{\partial\alpha_i}=-\frac{\theta\left[18+\alpha_i^2(2-\alpha_j)-\alpha_j(20-8\alpha_j+\alpha_j^2)-2\alpha_i(7-5\alpha_j+\alpha_j^2)\right]}{(3-\alpha_i-\alpha_j)^4}<0$$

which reflects that x_i^* is decreasing in α_i. Note that this can be easily found by separating the terms in the brackets. Fifth,

$$\frac{\partial x_i^*}{\partial\alpha_j}=\frac{\theta(1-\alpha_i)[6(1-\alpha_i-\alpha_j)(\alpha_i+\alpha_j)^2]}{(3-\alpha_i-\alpha_j)^4}>0$$

which implies that x_i^* is increasing in α_j. Finally, the difference in firm i's and j's equilibrium first-period appropriation is

$$x_i^* - x_j^* = \frac{\theta(\alpha_j - \alpha_i)(2 - \alpha_i - \alpha_j)(4 - \alpha_i - \alpha_j)}{(3 - \alpha_i - \alpha_j)^3}$$

which is weakly positive if and only if $\alpha_i \leq \alpha_j$.

Substituting x_i^* and x_j^* into the firm's second-period objective function, we obtain

$$\frac{\theta(1 - \alpha_j)[(11 + \alpha_i^2 - (7 - \alpha_j)\alpha_j - (7 - 2\alpha_j)\alpha_i)^2 + (3 - \alpha_i - \alpha_j)^4 g]}{(3 - \alpha_i - \alpha_j)^6}.$$

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