Endogenous Equity Shares in Cournot Competition: Welfare Analysis and Policy*

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Abstract

We consider a duopoly in which firms can strategically choose equity shares on their rival's profits before competing in quantities. We identify equilibrium equity shares, and subsequently compare them against the optimal equity shares that maximize social welfare. Most previous studies assume that equity shares are exogenous, and those allowing for endogenous shares do not evaluate if equilibrium shares are socially excessive or insufficient. Our results also help us identify taxes on equity acquisition that induce firms to produce a socially optimal output without the need to directly tax output levels.

KEYWORDS: Cournot competition; Endogenous equity shares; Social optimum; Equity share taxes.

JEL CLASSIFICATION: L10, L2, D43, D6.

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1 Introduction

Partial cross ownership (PCO) across firms implies that two or more firms hold equity shares in each others' profits, while firms continue to operate independently. PCOs are common in several industries, such as automobiles, banks, energy, media, and financial institutions.[1] The literature has extensively analyzed the role that equity shares play in limiting firms' competition[2]; but mostly assuming that equity shares are exogenously given. For instance, Reynolds and Snapp (1986) study how equilibrium quantities in a Cournot oligopoly with symmetric firms decrease as equity shares increase; and Fanti (2015) allows for asymmetric production costs, considering that only one firm holds an exogenous participation on its rival's profit.[3]

A few papers allow for endogenous equity shares. In a symmetric Cournot duopoly, Reitman (1994) shows that both firms have incentives to hold positive equity on each other's profits. His article, however, does not identify the specific equity shares that firms hold in equilibrium. Instead, Reitman (1994) checks if, for a given equity profile, firm i has incentives to unilaterally deviate by modifying its equity stake. In addition, the paper shows that, in oligopolies involving more than two firms, at least one firm finds it optimal to not hold equity on its rivals' profits.

Qin et al. (2017) finds the equilibrium equity shares that firms choose before competing a la Cournot. We also allow for endogenous equity acquisition, and show that our equilibrium equity shares reproduce those in Qin et al. (2017). However, we also identify the optimal equity share that maximizes social welfare, which we then compare against the equity share firms choose in equilibrium. This allows us to determine whether equilibrium equity shares are excessive or insufficient, relative to the social optimum. Social welfare considers consumer and producer surplus and, for completeness, it also includes the environmental damage that the production process generates (which embodies settings of no pollution as a special case).

We show that equilibrium equity shares are socially *insufficient* when: (1) a small proportion of output is sold domestically; and (2) the production process generates a significant amount of pollution. In this setting, our results suggest that the equilibrium output in the second stage of the game is socially excessive. Regulators can then induce firms to increase their equity shares, approaching them to the social optimum, by providing subsidies that lower their equity acquisition costs. In contrast, when (1) or (2) do not hold (i.e., large share of output is sold domestically and

[1] In the automobile industry, for example, Renault holds 44.3% equity shares in Nissan, while Nissan holds 15% in Renault; see Bárcena-Ruiz and Campo (2012) and www.nissan-global.com. Cross-ownership is also common in the financial sector, where Allianz AG owns 22.5% of Dresdner Bank, who owns 10% of Allianz AG; see La Porta et al. (1999). Other examples include only one firm holding equity shares on their rival's profits, such as Gillette, which owns 22.9% of the non-voting stock of Wilkison Sword, as reported in Gilo et al. (2006); and General Motors, which acquired 20% of Subaru's stock in 1999; see Ono et al. (2004).

[2] This result has been empirically confirmed in several industries where PCOs reduce output and increase prices, such as telecommunications, Parker and Roller (1997); energy sector in Northern Europe, Amundsen and Bergman (2002); and Italian banks, Trivieri (2007).

[3] The literature has also examined whether collusion becomes easier to sustain under PCOs. Specifically, Malueg (1992) considers a setting in which firms hold symmetric shares on each others profits, showing that collusive behavior becomes more difficult; whereas Gilo and Spiegel (2006) extend this model to a context of asymmetric equity shares, demonstrating that collusion can become easier to sustain under certain equity profiles.

its production generates little pollution), we demonstrate that the equilibrium output that emerges during the second stage becomes socially insufficient, like in a standard Cournot model without pollution. In this context, equity shares that firms choose in equilibrium are *excessive*; calling for a tax on share acquisition to increase firms' costs when purchasing equity. Our welfare analysis thus helps us examine a novel policy tool —taxes and subsidies on equity acquisition— which may be easier to implement, and more attractive for regulators, than trying to observe output or sales when monitoring technologies are imperfect and costly.

In the field of environmental economics, Ellis and Nouweland (2006) and Kanjilal and Munoz-Garcia (2017) also allow for endogenous equity shares, but in the context of common-pool resources (e.g., fishing grounds and forests) where fishing firms have been reported to hold equity shares on each other's profits in different regions.[4] These studies, however, consider a given market price (i.e., fishing vessels sell their appropriation in a perfectly competitive international market), and allow for every firm's exploitation of the resource to generate a cost externality on its rivals. In contrast, we examine endogenous equity shares under Cournot competition, where market price is not given but decreasing in aggregate output.

In summary, previous studies analyzing firms' choice of equity shares overlook welfare implications. In contrast, our study examines the welfare consequences of endogenous equity shares, both in markets where firms' production is sold entirely in the domestic market, in those were a proportion is sold overseas, in industries where production does not generate environmental externalities (pollution), in industries where it does; and in combinations of the above. Furthermore, our findings identify socially optimal taxes and subsidies that can be implemented to induce firms to hold welfare maximizing equity shares. Taxes on equity transactions are relatively small across countries, but they are not uncommon: 40 nations implement financial transaction taxes.[5] Our results in this paper suggest that such taxes can be used as a tool to induce a socially optimal output. While other policy tools, such as output subsidies and taxes, can also be implemented, directly taxing (or subsidizing) equity is less costly to monitor and implement. Moreover, traditional taxes on output could reduce firm profits in equilibrium. This occurs, for instance, when equilibrium production is socially excessive, and a per unit tax is implemented, reducing profits. However, the optimal policy tool we suggest in this case, an equity subsidy, increases the seller's profits while helping to implement the socially optimal outcome. Therefore, this policy tool should face less political resistance than output taxes.

For completeness, the appendix examines four extensions of our model where we allow for: (i) linear, rather than convex, cost of acquiring equity shares; (ii) convex production costs; (iii) firms jointly choosing their equity shares in each others' profits; and (iv) more than two firms. We show

[4] Examples include the Northeast Tilefish fishery, Kitts et al. (2007); the Alaskan Chignik Salmon fishery, Deacon et al. (2008); and the Pacific Whiting fishery, Sullivan (2001).

[5] For instance, the US Section 31 fee imposes $21.80 per million dollars for securities transactions; and the UK uses the Stamp Duty Reserve Tax at a rate of 0.5% on purchases of shares of companies headquartered in the UK, raising around $US4.4 billion per year. Similar equity taxes exist in Japan, Singapore, India, France, and Sweden. Worldwide, financial transaction taxes raise more than $US 38 billion. For a review of this taxation across different countries, see Anthony et al. (2012).

that our main results are qualitatively unaffected.

The following section describes the model. Section 3 identifies firms' equilibrium output (in the second stage), and equilibrium equity shares (in the first stage). Section 4 finds which are the socially optimal equity shares maximizing welfare, and compares them against equilibrium equity shares. Section 5 examines the equity taxes that induce firms to choose socially optimal equity shares. Finally, Section 6 discusses our results and offers policy implications.

2 Model

Consider a duopoly with two firms, 1 and 2, competing in quantities. They face an inverse demand function $p(q_i, q_j) = a - b(q_i + q_j)$, where $i = \{1, 2\}$, $j \neq i$, and $a, b > 0$. Firms have a common marginal cost c, where $a > c > 0$.[6] Every firm i's profit function is then

$$\pi_i \equiv [a - b(q_i + q_j)]\, q_i - cq_i.$$

We consider that firms i and j can hold shares in one another's profits. Shares held by firm i in firm j's profits are given by $\alpha_i \in \left[0, \frac{1}{2}\right]$. Similarly, $\alpha_j \in \left[0, \frac{1}{2}\right]$ is the share firm j holds in firm i's profits. Thus, firm i's objective function is given by:

$$V_i \equiv (1 - \alpha_j)\pi_i + \alpha_i \pi_j \tag{1}$$

When $\alpha_j = \alpha_i = 0$, firms do not share profits and the objective function in expression (1) collapses to π_i. In contrast, when $\alpha_i = \alpha_j = 1/2$, firm i's objective function coincides with that in a merger of symmetric firms $\frac{1}{2}(\pi_i + \pi_j)$. If $\alpha_j = 0$ but $\alpha_i > 0$, firm i's objective function reduces to $\pi_i + \alpha_i \pi_j$, indicating that firm i obtains a share α_i on firm j's profits, whereas firm j does not receive any profits from firm i. The opposite argument applies if $\alpha_i = 0$ but $\alpha_j > 0$.

In the first stage, firm i simultaneously and independently chooses its equity share on its rival's profit, α_i. In the second stage, firms observe the equity profile (α_i, α_j) chosen in the first period, and firm i responds selecting its output level q_i. We solve this game by backward induction.

3 Equilibrium analysis

3.1 Second stage - Optimal output

Lemma 1. *In the second period, firm i's the best response function is*

$$q_i(q_j) = \begin{cases} \frac{a-c}{2b} - \frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)} q_j & \textit{if } q_j \leq \frac{(a-c)(1-\alpha_j)}{b(1+\alpha_i-\alpha_j)} \\ 0 & \textit{otherwise.} \end{cases}.$$

[6] If firms exhibit convex production costs, Perry and Porter (1985) show that they may have stronger incentives to merge. In our setting, this could entail that firms have stronger incentives to acquire equity shares in each other's profits. For completeness, Appendix 2 examines how our results are affected by convex production costs.

Graphically, $q_i(q_j)$ originates at $\frac{a-c}{2b}$, which is constant in equity shares, and has a negative slope $\frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)}$, which is increasing in equity shares α_i and α_j. Therefore, $q_i(q_j)$ pivots inwards as either firm increases its equity share, i.e., as α_i and/or α_j increase, thus indicating that firms' output become more intense strategic substitutes. When firms hold no equity shares, $\alpha_i = \alpha_j = 0$, the slope of best response function $q_i(q_j)$ collapses to $\frac{1}{2}$, as in standard Cournot models. When only firm i holds equity shares on firm j, $\alpha_i > 0$ but $\alpha_j = 0$, the slope becomes $\frac{1+\alpha_i}{2}$, implying that the best response function is steeper than when no firm holds equity shares. Finally, when both firms hold equity shares, the slope becomes $\frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)}$, indicating that firm i's best response pivots inwards again.

Using $q_i(q_j)$ and $q_j(q_i)$ to simultaneously solve for q_i and q_j, we obtain the equilibrium output that firms choose in the second stage, as a function of the equity profile (α_i, α_j) selected in the first period, as follows.

Proposition 1. *In the second period, firm i selects an output level*

$$q_i^*(\alpha_i, \alpha_j) = \frac{(a-c)(1-\alpha_i)}{(3-\alpha_i-\alpha_j)b}$$

which is strictly positive for all parameter values. Output $q_i^*(\alpha_i, \alpha_j)$ is increasing in a and α_j, but decreasing in b and c and α_i, and satisfies $q_i^*(\alpha_i, \alpha_j) \geq q_j^*(\alpha_i, \alpha_j)$ if and only if $\alpha_i \leq \alpha_j$.

In addition, equilibrium output is decreasing in firm i's equity share on its rival's profit, α_i, since firm i internalizes a larger portion of the price reduction effect that its output produces on firm j's revenue. However, equilibrium output $q_i^*(\alpha_i, \alpha_j)$ is increasing in firm j's equity share on firm i's profits, α_j. Finally, firm i produces more units than its rival when it holds a smaller share of equity, $\alpha_i \leq \alpha_j$.

As special cases, note that if firms choose symmetric equity shares during the first stage of the game, i.e., $\alpha_i = \alpha_j = \alpha$, optimal output collapses to $q_i^*(\alpha) = \frac{(a-c)(1-\alpha)}{(3-2\alpha)b}$, which is decreasing in α. In this setting, if firms hold maximal equity in each other's profits, $\alpha = \frac{1}{2}$, firm i's equilibrium output becomes $q_i^*\left(\frac{1}{2}\right) = \frac{(a-c)}{4b}$, thus coinciding with its output under a merger. In addition, when firms hold no equity shares on each other's profits, $\alpha_i = \alpha_j = 0$, this output collapses to $\frac{a-c}{3b}$, as in standard duopoly models.

3.2 First stage - Optimal equity shares

In the first stage, we use the output profile that arises during the second stage, $q_i^*(\alpha_i, \alpha_j)$ and $q_j^*(\alpha_i, \alpha_j)$, to find firm i's equilibrium equity share α_i. For this, we first substitute these two terms into firm i's objective function, which yields[7]

$$\max_{0 \leq \alpha_i \leq \frac{1}{2}} V_i(q_i^*, q_j^*) + \alpha_j^{\beta} NW_i - C(\alpha_i) \tag{2}$$

[7] For completeness, Appendix 3 examines how our results are affected if firms jointly choose their equity shares (as in negotiations between both firms), as opposed to independently in the current setting.

where the first term

$$
\begin{aligned}
V_i(q_i^*, q_j^*) &= (1-\alpha_j)\pi_i(q_i^*, q_j^*) + \alpha_i \pi_j(q_j^*, q_i^*) \\
&= \frac{(a-c)^2(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^2 b}
\end{aligned}
$$

represents the equilibrium payoff that firm i obtains in the second period game, thus being evaluated at equilibrium output levels (q_i^*, q_j^*). This term coincides with that in Qin et al. (2017)[8] and collapses to $\frac{(a-c)^2}{9b}$ when firms hold no equity shares on each other's profits, $\alpha_i = \alpha_j = 0$.

The second term in the firm's program, $\alpha_j^\beta NW_i$, captures the revenue that firm i receives from firm j when the latter acquires α_j shares on firm i's net worth $NW_i \geq 0$. The third term $C(\alpha_i) = F + NW_j \alpha_i^\beta$, in contrast, represents the cost that firm i experiences from acquiring equity on firm j. In particular, $F \geq 0$ is a fixed cost of equity acquisition, being independent on the equity firm i acquires from its rival, α_i; while $NW_j \geq 0$ denotes firm j's net worth. When $\beta > 1$, cost $C(\alpha_i)$ is weakly increasing and convex in equity acquisition α_i, indicating that purchasing further equity on other firms becomes more costly as shares become more scarce. (For completeness, Appendix 1 studies the case in which cost $C(\alpha_i)$ is weakly concave in equity acquisition, showing that it leads to corner solutions where the firm purchases either no equity or maximal equity on its rival's profits.)

Differentiating with respect to α_i in problem (2), we obtain the following result.

Proposition 2. *In the first stage, firm i chooses the equity share α_i^* that solves*

$$
MB_i \equiv \frac{2(a-c)^2}{b} \frac{(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = \beta NW_j \alpha_i^{\beta-1} \equiv MC_i \tag{3}
$$

where the left term represents firm i's marginal benefit of increasing its equity share on j's profits, α_i, and the right side denotes its marginal cost from acquiring additional equity.

Intuitively, the firm weighs in the benefit of acquiring additional equity on its rival (in the form of profits and internalizing external effects of each other's production) against the cost of equity acquisition. Solving for equity share α_i^* in expression (3) produces an intractable root of firm i's best response function $\alpha_i(\alpha_j)$. We can, however, provide some comparative statics results as described in the next corollary.

Corollary 1. *Equilibrium equity solving expression (3) is increasing in firm i's profit margin, $a-c$, decreasing in the slope of the demand curve, b, and decreasing in firm j's net worth, NW_j, under all parameter conditions. In addition, marginal benefit MB_i is:*

[8] Expression (11) in Qin et al.'s (2017) paper, $\frac{\alpha_{ii}(a-c)^2}{\left(1+\sum_{j=1}^n \alpha_{jj}\right)^2}$ collapses to $\frac{\alpha_{ii}(a-c)^2}{\left(1+\alpha_{ii}+\alpha_{jj}\right)^2}$ in the case of $n=2$ firms. Furthermore, α_{ii} in Qin et al. (2017) can be interpreted as the share that firm i holds on its own profits. In the case of $n=2$ firms, share α_{ii} can then be rewritten as $(1-\alpha_j)$. Therefore, the term in their denominator, $(1+\alpha_{ii}+\alpha_{jj})^2$, can be expressed as $(1+(1-\alpha_j)+(1-\alpha_i))^2$ which simplifies to $(3-\alpha_i-\alpha_j)^2$. Finally, Qin et al. (2017) assume a demand function $p(Q) = a - Q$, where Q denotes aggregate output. As result, they consider that $b=1$, ultimately implying that our expression coincides with theirs.

a. *increasing and convex in firm i's equity share* α_i*;*

b. *increasing in firm j's equity share* α_j *if and only if* $\alpha_i > 2\alpha_j$*; and*

c. *satisfies* $MB_i > MC_i$ *for all* $NW_j \leq \overline{NW}_j \equiv \frac{1}{\beta(\beta-1)\alpha_i^{\beta-2}} \frac{6(1-\alpha_i)(a-c)^2}{b(3-\alpha_i-\alpha_j)^4}$.

The above corollary indicates that firm i acquires a larger equity when profit margins are higher, but less equity when market prices are relatively sensitive to output changes. Intuitively, when firm i has more resources to acquire equity, as captured by term $\frac{2(a-c)^2}{b}$ in expression (3), it holds a larger equity on its rival's profit. Furthermore, the corollary shows that firms' incentives to hold equity, as represented by marginal benefit MB_i, are increasing and convex in their own equity share on their rival's profit, α_i, and increasing in their rival's equity, α_j, only when firm i holds twice as much equity on its rival than j holds on firm i, $\alpha_i > 2\alpha_j$.

In point (c), the above corollary identifies that, when firm j is sufficiently small, $NW_j \leq \overline{NW}_j$ (or $NW_j = 0$ as a special case), firm i finds that the marginal benefit of equity lies above its marginal cost for all values of α_i, thus increasing its stock on firm j as much as possible, i.e., $\alpha_i^* = 1/2$ in a corner solution. This case can arise, for instance, in merged firms equally sharing profits. Intuitively, firm i can afford to purchase maximal equity on firm j.[9] The opposite corner solution, where the firm acquires no equity on its rival's profit, i.e., $\alpha_i^* = 0$, could only arise, however, if $NW_j \to \infty$, making the MC_i curve completely vertical; which cannot be sustained given that $\infty > NW_j \geq 0$ by definition.

Finally, when firm j's size is intermediate, $\overline{NW}_j < NW_j < \infty$, the firm holds an interior equity share $\alpha_i^* \in (0, 1/2)$ that solves the condition on Proposition 2. As described above, however, expression (3) produces an intractable root of firm i's best response function $\alpha_i(\alpha_j)$. We can nonetheless numerically approximate firm i's best response function $\alpha_i(\alpha_j)$; as depicted in Figure 1 below which assumes $a = b = 1$, $c = 0.3$, $\beta = 2$, and a symmetric setting where $NW_i = NW_j = NW = 0.1$.[10] The figure illustrates firm i's best response function, $\alpha_i(\alpha_j)$, firm j's, $\alpha_j(\alpha_i)$, and the point where they cross acquiring an equilibrium equity of $\alpha_i^* = \alpha_j^* = 0.23$ in this parametric

[9] This case embodies settings in which acquiring equity is costless, where firms would hold as much equity as possible (like in a merger where $\alpha_i = 1/2$) given that a larger equity produces an unambigous increase in profits during the subsequent stage when firms compete a la Cournot; as recurrently shown, for the case of a duopoly, in studies evaluating firms' incentives to merge such as Perry and Porter (1985) and Levin (1990).

[10] For each point in the figure, we first assumed a specific value for α_j in 0.001 increments; second, we evaluated firm i's profits in expression (2) for a specific equity share α_i, also in 0.001 increments; third, we identified the equity share α_i^* yielding the highest profit. These three steps provide us with one specific (α_i, α_j)-pair in firm i's best response function of figure 1. We then repeated the same three-step procedure, fixing one value for α_j at a time, in 0.001 increments, obtaining 1,000 different points.

example.

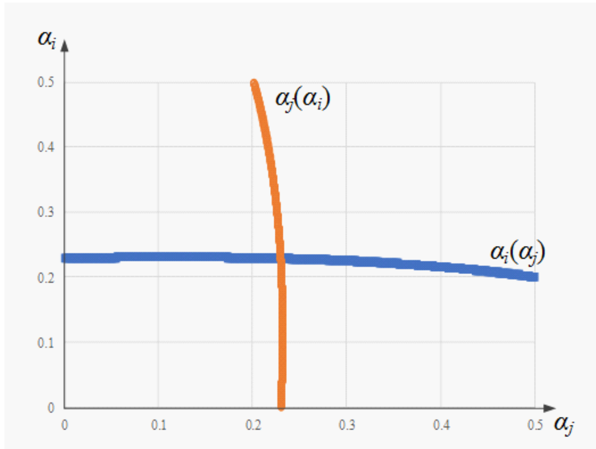

Figure 1. Numerical approximation of best response functions.

For completeness, Table I reports α_i^* for different net worth (in rows), and marginal cost of production (in columns).[11] Intuitively, the equilibrium equity share α_i^* decreases both in the net worth of firm i's rival, and in the production cost c.

Net worth NW / Marginal cost c	$c = 0$	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = 0.9$
$NW = 0.1$	0.5	0.5	0.23	0.10	0.03	0.004
$NW = 0.2$	0.24	0.18	0.10	0.05	0.02	0.002
$NW = 0.3$	0.14	0.11	0.06	0.03	0.01	0.001
$NW = 0.4$	0.10	0.08	0.05	0.02	0.01	0.001
$NW = 0.5$	0.08	0.06	0.03	0.02	0.01	0.001
$NW = 0.6$	0.07	0.05	0.03	0.02	0.006	0.001
$NW = 0.7$	0.06	0.04	0.02	0.01	0.005	0.001
$NW = 0.8$	0.05	0.04	0.02	0.01	0.004	0.0005
$NW = 0.9$	0.04	0.03	0.02	0.01	0.004	0.0004

Table I. Optimal equity share α_i^*.

When demand becomes stronger (higher a), the marginal benefit of acquiring equity increases, raising the equilibrium equity α_i^* that firms hold. When the demand function becomes steeper (higher parameter b), consumers are less price sensitive, which allows every firm to capture a larger profit margin for given equity shares. In this context, the marginal benefit from acquiring equity MB_i decreases, reducing firms' incentives to hold shares on each others' profits. In contrast,

[11] For convenience, Table I assumes that $a = b = 1$ and $\beta = 2$, but other parameter combinations yield similar results, and can be provided by the authors upon request.

when parameter b decreases (approaching zero), firms anticipate that their subsequent Cournot competition will be tougher, thus increasing their incentives to acquire equity shares during the first period to ameliorate posterior competition.

When firms exhibit asymmetric net worth, they acquire different equity shares in equilibrium. For instance, consider the same parameter values as in the second column of Table I (where $c = 0.3$), and assume a given $NW_i = 0.1$. When firm i's net worth is equal to j's, $NW_j = 0.1$, both firms acquire $\alpha^* = 0.23$. When this net worth increases to $NW_j = 0.7$, firm i acquires $\alpha_i^* = 0.02$ while firm j acquires $\alpha_j^* = 0.23$; as shown in Table I. Finally, when net worth further increases to $NW_j = 0.9$, firm i acquires $\alpha_i^* = 0.01$ while its rival has $\alpha_j^* = 0.23$.

Table Ia considers the same parameter values as Table I but assuming a stronger demand ($a = 2$ rather $a = 1$), illustrating that equilibrium equity shares weakly increase.

Net worth NW / Marginal cost c	$c = 0$	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = 0.9$
$NW = 0.1$	0.5	0.5	0.5	0.5	0.5	0.5
$NW = 0.2$	0.5	0.5	0.5	0.5	0.5	0.31
$NW = 0.3$	0.5	0.5	0.5	0.44	0.28	0.18
$NW = 0.4$	0.5	0.5	0.41	0.28	0.19	0.13
$NW = 0.5$	0.5	0.41	0.29	0.21	0.15	0.10
$NW = 0.6$	0.36	0.31	0.23	0.16	0.12	0.08
$NW = 0.7$	0.28	0.25	0.18	0.14	0.10	0.07
$NW = 0.8$	0.24	0.21	0.16	0.12	0.09	0.06
$NW = 0.9$	0.20	0.18	0.14	0.10	0.08	0.05

Table Ia. Optimal equity share α_i^* with stronger demand.

4 Welfare analysis

The social planner considers a welfare function

$$\max_{q_i, q_j} \; W = \gamma CS + PS - Env(q_i, q_j) \tag{4}$$

In the first term, $CS = \frac{b(q_i+q_j)^2}{2}$ denotes consumer surplus. For generality, parameter $\gamma \in [0, 1]$ represents the share of aggregate production $q_i + q_j$ sold domestically, which allows for $\gamma = 0$ and $\gamma = 1$ as special cases. The second term captures the producer surplus $PS = \left[V_i + \alpha_j^\beta NW_i - C(\alpha_i)\right] + \left[V_j + \alpha_i^\beta NW_j - C(\alpha_j)\right]$, which simplifies to $\pi_i + \pi_j - 2F$. Finally, $Env(q_i, q_j) = d(q_i + q_j)^2$ represents the environmental damage that firms' pollution generates, where $d \geq 0$. When $d = 0$, our setting collapses to the welfare function in standard duopoly models considering non-polluting industries. In this case, the regulator only deals with one output distortion (socially insufficient production in duopoly), whereas when $d > 0$ he also faces the output distortion stemming from pollution (which can lead to a socially excessive production).

Differentiating with respect to q_i we obtain that socially optimal output, as follows.

Proposition 3. *Socially optimal output is* $q_i^{SO} = \frac{a-c}{4(b+d)-2b\gamma}$*, which is positive for all admissible parameter values. In addition,* $q_i^* > q_i^{SO}$ *if and only if* $\alpha < \alpha^{SO}$*, where*

$$\alpha^{SO} \equiv 1 - \frac{b}{4d + 2b(1-\gamma)}.$$

Proposition 3 helps us compare equilibrium and socially optimal output, q_i^* and q_i^{SO}, by setting $q_i^* - q_i^{SO} = 0$, and solving for α. The proposition identifies that $q_i^* > q_i^{SO}$ if and only if $\alpha < \alpha^{SO}$, where cutoff α^{SO} is given by

$$\alpha^{SO} \equiv 1 - \frac{b}{4d + 2b(1-\gamma)}$$

Figure 2a depicts cutoff α^{SO} as a function of the proportion of output sold domestically, γ. When firms hold equity shares below cutoff α^{SO}, $\alpha < \alpha^{SO}$, they produce a socially excessive amount of output.[12] However, when firms hold equity shares above cutoff α^{SO} (which may include a total merger, where $\alpha = 1/2$, as a special case), equilibrium output becomes socially insufficient. In addition, cutoff α^{SO} decreases in γ, implying that the region of socially excessive production shrinks as firms sell a larger share of output domestically. As a special case, when $\gamma = 1$ and $d = 0$, social welfare only consider consumer and producer surplus, and output becomes unambiguously insufficient from a society viewpoint; as in standard Cournot models of non-polluting industries.[13]

[12] Like in Table I, we consider $a = b = 1$. We also assume now an environmental damage of $d = 0.2$. Other parameter values yield similar results and can be provided by the authors upon request.

[13] When $\gamma = 1$ and $d = 0$, cutoff α^{SO} collapses to $\alpha^{SO} = 1 - \frac{b}{0} = -\infty$. However, since equity shares are bounded between 0 and 1/2, cutoff $\alpha^{SO} = 0$ in this context. This case is depicted in Figure 2b, specifically by the curve evaluating cutoff α^{SO} at $d = 0$, where we can see that, at $\gamma = 1$ (on the right-hand side of the figure), cutoff α^{SO} becomes $\alpha^{SO} = 0$. Since all equity shares lie above cutoff α^{SO}, output is socially insufficient for every equity share $\alpha > \alpha^{SO}$.

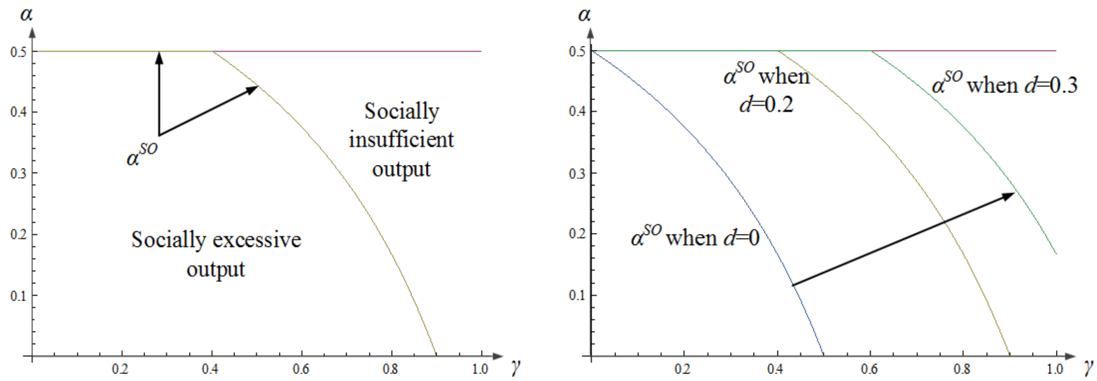

Fig. 2a. Cutoff α^{SO}.   Fig. 2b. Cutoff α^{SO} when d increases.

We next evaluate under which parameter conditions cutoff α^{SO} lies strictly inside its admissible range $[0, 1/2]$.

Corollary 2. *Cutoff α^{SO} satisfies $\alpha^{SO} > 0$ if and only if $\gamma < \gamma_1 \equiv \frac{1}{2} + \frac{2d}{b}$, and $\alpha^{SO} < 1/2$ if and only if $\gamma > \gamma_2 \equiv \frac{2d}{b}$, where $\gamma_1 > \gamma_2$. In addition, cutoff α^{SO} increases in d, but decreases in b and γ.*

Therefore, the proportion of output sold domestically must take intermediate values for cutoff α^{SO} to lie strictly inside its admissible range $[0, 1/2]$. However, when γ satisfies $\gamma > \gamma_1$, cutoff α^{SO} collapses to $\alpha^{SO} = 0$, meaning that the regulator seeks to discourage equity acquisition; whereas when $\gamma < \gamma_2$, cutoff α^{SO} becomes maximal at $\alpha^{SO} = 1/2$, entailing that the regulator seeks to promote maximal equity shares.[14]

Figure 2b illustrates that cutoff α^{SO} shifts upwards when the environmental damage from production, d, increases, thus expanding the region of socially excessive production.[15] For instance, when production does not generate pollution, $d = 0$, cutoff α^{SO} simplifies to $1 - \frac{1}{2(1-\gamma)}$; as depicted in Figure 2b. When firms sell no output domestically, $\gamma = 0$ (in the vertical intercept of cutoff

[14] Cutoff γ_1 is positive for all parameter values, and satisfies $\gamma_1 < 1$ for all $d < \frac{b}{4}$. Similarly, cutoff γ_2 is positive for all parameter values, and satisfies $\gamma_2 < 1$ for all $d < \frac{b}{2}$. Therefore, three cases can arise depending on the size of environmental damages. First, when environmental damages are low, $d < \frac{b}{4}$, cutoffs γ_1 and γ_2 satisfy $\gamma_1, \gamma_2 < 1$. In this case, the admissible range of $\gamma \in [0, 1]$ is divided into three regions: $\alpha^{SO} = 1/2$ for all $\gamma < \gamma_2$, $\alpha^{SO} \in (0, 1/2)$ for all $\gamma_2 \leq \gamma < \gamma_1$, and $\alpha^{SO} = 0$ otherwise. Second, when environmental damages are intermediate, $\frac{b}{4} \leq d < \frac{b}{2}$, cutoffs γ_1 and γ_2 satisfy $\gamma_1 > 1$ and $\gamma_2 < 1$. In this case, the admissible range of $\gamma \in [0, 1]$ is divided into two regions alone: $\alpha^{SO} = 1/2$ for all $\gamma < \gamma_2$, $\alpha^{SO} \in (0, 1/2)$ otherwise. Finally, when environmental damages are large, $d \geq \frac{b}{2}$, cutoffs $\gamma_1, \gamma_2 > 1$, implying that $\alpha^{SO} = 1/2$ for all values of γ.

[15] Furthermore, cutoff α^{SO} shifts downward as b increases, which captures the magnitude of the slope of the demand curve; producing a shrink in the region where socially excessive production can be sustained. Differentiating cutoff α^{SO} with respect to the parameter values, we obtain that $\frac{\partial \alpha^{SO}}{\partial d} = \frac{b}{(b+2d-b\gamma)^2} > 0$, $\frac{\partial \alpha^{SO}}{\partial b} = -\frac{d}{(b+2d-b\gamma)^2} < 0$, and $\frac{\partial \alpha^{SO}}{\partial \gamma} = -\frac{b^2}{(b+2d-b\gamma)^2} < 0$.

α^{SO} in Figure 2b), $\alpha^{SO} = 1/2$, which implies that output is socially excessive for all equity shares firms choose; unless firms merge which implies $\alpha = 1/2$. However, when firms sell more than 50% of their output domestically (i.e., if $\gamma > 1/2$), equilibrium output becomes socially insufficient for all parameter values. As a reference, this is illustrated by the horizontal intercept of cutoff α^{SO} when $d = 0$, which occurs at $\gamma = 1/2$. For all $\gamma \geq 1/2$, cutoff α^{SO} collapses to $\alpha^{SO} = 0$, entailing a socially insufficient output.[16]

Comparing equilibrium and socially optimal equity shares, we obtain that they do not necessarily coincide, thus yielding a socially excessive equity acquisition if $\alpha_i^* > \alpha^{SO}$, or socially insufficient equity holdings if $\alpha_i^* < \alpha^{SO}$. Figure 3 superimposes equilibrium equity $\alpha_i^* = 0.23$ on Figure 2a, which occurs when $c = 0.3$ and $NW = 0.1$; as reported in the top row of Table I.

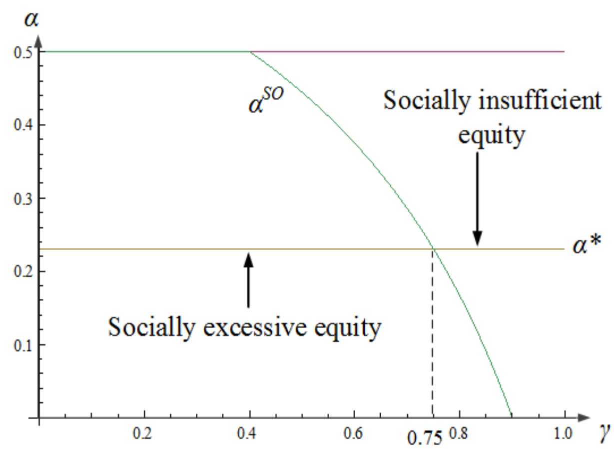

Fig. 3. Comparing α_i^* and α^{SO}.

5 Equity share taxes

In this section, we examine how government agencies can design taxes (or subsidies) inducing firm i to hold an equilibrium equity share α_i^* that coincides with the socially optimal equity share α^{SO} found above. When considering the optimal equity tax, the regulator anticipates the marginal cost that the firm faces, $NW_j \beta \alpha_i^{\beta-1}$. The social planner then increases the firm's marginal cost of acquiring equity by setting a tax T that solves

$$MB_i\left(\alpha_i^{SO}\right) = NW_j(1+t)\beta\left(\alpha_i^{SO}\right)^{\beta-1} \tag{5}$$

[16] Our results then connect with Levin (1990), who considers an industry with N firms, and examines their incentives to merge as well as the resulting social welfare; where his welfare function only considers consumer and producer surplus (i.e., he assumes $\gamma = 1$ and $d = 0$). If we evaluate Levin's (1990) results in the context of two firms, he shows that mergers are either profit enhancing or welfare increasing, but not both. As discussed above, in the case that $\gamma = 1$ and $d = 0$ our results show that mergers are profit enhancing but welfare reducing, in line with Levin (1990). However, we also demonstrate that equity share acquisition (and mergers) can lead to an increase in both profits and social welfare when assumptions $\gamma = 1$ and $d = 0$ are relaxed.

where both marginal benefit and cost are evaluated at the socially optimal equity α_i^{SO}, and the price of equity increased from NW_j to $NW_j(1+t)$. Monitoring equity transactions is often done for legal and accounting reasons, thus being a policy easier to implement than directly monitoring output or sales. Table II reports, for different values of γ (in rows), the socially optimal equity, α^{SO}, equity shares without taxes, α_i^*, and the tax t that induces firms to choose α^{SO} in equilibrium.[17]

In this setting, the unregulated equilibrium yields $\alpha^* = 0.44$ while $\alpha^{SO} = 0.34$. Therefore, the regulator can set a tax $T = 0.17$ that decreases equilibrium equity shares towards its socially optimal level. A similar approach applies to higher environmental damages, as reported in the left panel of Table II. Recall that the equilibrium equity share α^* unaffected by changes in d, whereas α^{SO} increases in d. When $d < 0.36$, equilibrium equities are socially excessive, $\alpha^* > \alpha^{SO}$, and the regulator sets a tax $T > 0$ on equity acquisition, which induces firms to choose α^{SO}. When $d = 0.36$, equilibrium equities are socially optimal, $\alpha^* = \alpha^{SO}$, entailing a zero tax $T = 0$. Last, when $d > 0.36$, equilibrium equities are socially insufficient, $\alpha^* < \alpha^{SO}$, and the regulator provides a subsidy $T < 0$ to increase equity holdings towards α^{SO}.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Tax t
$\gamma = 0$	0.5	0.23	-0.38
$\gamma = 0.1$	0.5	0.23	-0.38
$\gamma = 0.2$	0.5	0.23	-0.38
$\gamma = 0.3$	0.5	0.23	-0.38
$\gamma = 0.4$	0.5	0.23	-0.38
$\gamma = 0.5$	0.44	0.23	-0.38
$\gamma = 0.6$	0.38	0.23	-0.29
$\gamma = 0.7$	0.29	0.23	-0.15
$\gamma = 0.8$	0.17	0.23	0.27
$\gamma = 0.9$	0	0.23	180.6
$\gamma = 1$	0	0.23	180.6

Table II. Optimal equity subsidies.

When γ is low (i.e., most output is sold overseas), we have that $\alpha^{SO} > \alpha_i^*$ (see first four rows in Table II). In this context, the social planner can subsidize firms so they acquire more equity (negative tax, $t < 0$). For instance, when $\gamma = 0.7$, socially optimal equity is $\alpha^{SO} = 0.29$, whereas equilibrium equity is only $\alpha_i^* = 0.23$. A subsidy $t < 0$ solving expression (5) induces firms to choose α^{SO}, which yields $t = -0.15$. In contrast, when γ is relatively high, we obtain that $\alpha^{SO} < \alpha_i^*$, and the social planner can tax equity acquisition (see bottom rows in Table II, where $t > 0$).

If the net worth NW, increases, equilibrium equity α_i^* decreases (as shown in Table I) while optimal equity α^{SO} is unaffected. In this setting, the tax t that the planner offers to induce

[17] Like in Table I, we consider $a = b = 1$. We also assume now an environmental damage $d = 0.2$, production cost $c = 0.3$ and a symmetric net worth of $NW = 0.1$. Other parameter values yield similar results and can be provided by the authors upon request.

optimal equity acquisition must be less severe (or subsidies become more generous). A similar argument applies when production cost c increases since α_i^* decreases whereas optimal equity α^{SO} is unchanged.

In contrast, when demand becomes stronger (higher a), firms increase their equilibrium equity α_i^* since sharing profits becomes more attractive, but optimal equity α^{SO} remains unaffected relative to Table I since α^{SO} is not a function of parameter a. Table III considers that a increases from $a = 1$ in Tables I-II to $a = 2$. In this setting, the regulator finds that equilibrium equity shares, α_i^*, are socially excessive under larger conditions, $\alpha_i^* \geq \alpha^{SO}$, yielding a tax to curb equity acquisition in more cases.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Tax t
$\gamma = 0$	0.5	0.5	0
$\gamma = 0.1$	0.5	0.5	0
$\gamma = 0.2$	0.5	0.5	0
$\gamma = 0.3$	0.5	0.5	0
$\gamma = 0.4$	0.5	0.5	0
$\gamma = 0.5$	0.44	0.5	2.86
$\gamma = 0.6$	0.38	0.5	3.20
$\gamma = 0.7$	0.29	0.5	3.99
$\gamma = 0.8$	0.17	0.5	6.49
$\gamma = 0.9$	0	0.5	1070.44
$\gamma = 1$	0	0.5	1070.44

Table III. Optimal equity subsidies with stronger demand.

When demand becomes steeper (higher b), firms reduce their equilibrium equity α_i^* (from $\alpha_i^* = 0.23$ in Table II to $\alpha_i^* = 0.10$ in Table IV), and so does optimal equity α^{SO} (compare the second column in Tables II and IV). As a consequence, $\alpha^{SO} > \alpha_i^*$ when most output is sold overseas, leading the regulator to offer a subsidy on equity acquisition; whereas $\alpha^{SO} < \alpha_i^*$ when most output is sold domestically, which yields a tax on equity acquisition.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Tax t
$\gamma = 0$	0.5	0.10	−0.69
$\gamma = 0.1$	0.5	0.10	−0.69
$\gamma = 0.2$	0.5	0.10	−0.69
$\gamma = 0.3$	0.44	0.10	−0.67
$\gamma = 0.4$	0.38	0.10	−0.64
$\gamma = 0.5$	0.29	0.10	−0.58
$\gamma = 0.6$	0.17	0.10	−0.36
$\gamma = 0.7$	0	0.10	89.83
$\gamma = 0.8$	0	0.10	89.83
$\gamma = 0.9$	0	0.10	89.83
$\gamma = 1$	0	0.10	89.83

Table IV. Optimal equity subsidies with steeper demand.

6 Extensions

Appendices 2-4 consider different extensions of our main model, identifying equilibrium results in each case. For compactness, we next verbally summarize how our results are affected.

Convex production costs. Appendix 2 extends our previous analysis to a setting where firms face convex production cost $C(q_i) = c\,(q_i)^2$, where $c > 0$. Overall, our results are qualitatively unaffected, but we find that equilibrium equity share α_i^* is larger when firms face convex than linear production costs. This occurs because firms have higher marginal costs, and thus face more incentives to acquire equity on their rival's profits. We also demonstrate that the socially optimal output q^{SO} is lower when firms have convex production costs, relative to linear costs. However, the socially optimal equity share α^{SO} is unaffected by the firm's cost structure, since production costs symmetrically affect the firm, and thus q_i^*, and the planner's social welfare function, yielding q^{SO}. Therefore, when setting $q_i^* = q^{SO}$, we obtain the same equity share α^{SO} as under linear production costs.

Joint equity acquisition. Appendix 3 consider a setting where firms coordinate their equity share in each other's profits, α_i and α_j, to maximize their joint profits. We show that, in that context, a corner solution emerges where firms choose maximal equity in its rival's profit, $\alpha_i = \alpha_j = 1/2$ (a full merger), which weakly exceeds the equilibrium equity each firm acquires when they independently solve the problem in section 3.2. Intuitively, when firms maximize their joint profits, they face a zero cost of acquiring equity, since the cost each of them incurs coincides with the revenue that its rival receives from selling equity. Intuitively, firms benefit from acquiring equity but now suffer no costs from doing so, leading them to acquire maximal equity. As a consequence, regulators set equity share taxes under larger parameter combinations than when firms independently choose their equity holdings.

Sequential equity acquisition. Finally, Appendix 4 considers that, in the first stage, firms sequentially acquire equity, with firm i choosing α_i followed by firm j responding with α_j. In the second stage, for a profile of equity shares (α_i, α_j), firms compete a la Cournot. The second-stage game is, then, unaffected, while the first-stage game now gives rise to a new strategic effect. Specifically, the leader acquires less equity than in the simultaneous-move game we consider in previous sections, inducing the follower to acquire more equity. Intuitively, when the follower increases its equity holding in the leader, it internalizes output decisions more intensively in the subsequent stage, benefiting both firms. Therefore, the leader free-rides off the follower by inducing the latter to increase its equity holdings.

7 Discussion

An alternative policy. Governments commonly use price-based policy tools, such as per-unit subsidies and taxes to alter firms' production decisions. Output subsidies, for instance, induce firms to increase their production, which is a common policy in industries where few companies compete, and thus unregulated output is naturally low. In contrast, per-unit taxes seek to curb excessive production, and thus are often introduced in polluting industries (e.g., emission fees). Alternatively, regulatory agencies can set quantity-based systems, such as production quotas, that firms must comply with. While these policy tools are usually effective, they entail monitoring and supervision costs, which are often substantial.[18] Our paper suggests an alternative policy tool, subsidies or taxes on equity share acquisition. Since firms must regularly inform about their equity holdings on other firms, this policy can be easier to monitor than quantity-based policies. Importantly, equity share policies operate before firms choose their equity and, as a consequence, prior to their competition with other firms in subsequent periods. In addition, this policy does not require ex-post monitoring of firms' output or prices. Intuitively, the policy provides every firm with the incentives to acquire the socially optimal level of equity shares, α^{SO}, which implies that firms' production decisions in subsequent stages are socially optimal as well.

High or low equity taxes? Our results also help us understand the size of equity share taxes in different settings. First, when a large proportion of the good is sold domestically, the social planner seeks a higher output level which, in turn, entails a lower level of socially optimal equity. In this case, equity taxes are likely to be positive, especially if the cost of acquiring equity is low and/or their production does not generates large negative externalities (e.g., pollution). Conversely, when a small proportion of the good is sold domestically, taxes are likely to be low if pollution is substantial and/or if demand is relatively weak.

Pollution effects. More damaging pollution produces the opposite effect than a larger proportion of goods being sold domestically. Intuitively, since pollution reduces social welfare, the social planner seeks to induce lower production levels, which in our findings can be done via subsidies on

[18] For empirical estimations of the monitoring costs of production quotas in Canada, see Grafton (1996). Hatcher (1998) provides a similar estimation of quotas for the UK, and Milazzo (1998) for the U.S.

equity shares. Conversely, if the production process does not generate much pollution (relative to its consumer and producer surplus), the regulator seeks to induce a larger production by taxing equity share acquisition.

8 Appendix

8.1 Appendix 1 - Weakly concave cost of equity

In this appendix we show that when $\beta \leq 1$, firm i acquires no equity on firm j, $\alpha_i^* = 0$, if $\frac{(a-c)^2(1-\alpha_j)}{(3-\alpha_j)^2 b} \geq \left(\frac{1}{2}\right)^\beta NW_j - F$ or maximal equity, $\alpha_i^* = \frac{1}{2}$, otherwise. Therefore, the firm's equilibrium equity acquisition is at a corner solution ($\alpha_i^* = 0$ or $\alpha_i^* = \frac{1}{2}$) if the cost of acquiring equity is weakly concave, that is, acquiring further shares becomes relatively cheaper as the firm owns a larger equity on its rival. Otherwise, the firm acquires a positive amount of equity as long as its cost is not extremely low or high. We prove this result below.

Proof. When $\beta \leq 1$, the marginal cost of acquiring equity satisfies

$$\frac{\partial^2 MC}{\partial \alpha_i^2} = (\beta - 1)\beta\alpha^{\beta-2} NW_j$$

Since $\beta \leq 1$, the above expression satisfies $\frac{\partial^2 MC}{\partial \alpha_i^2} \leq 0$. The second-order condition yields $\frac{\partial^2 MB}{\partial \alpha_i^2} - \frac{\partial^2 MC}{\partial \alpha_i^2}$. In addition, the marginal benefit from acquiring equity is increasing since $\frac{\partial^2 MB}{\partial \alpha_i^2} = \frac{24(1-\alpha_i)(a-c)^2}{b(3-\alpha_i-\alpha_j)^5} > 0$, implying that

$$\frac{\partial^2 MB}{\partial \alpha_i^2} - \frac{\partial^2 MC}{\partial \alpha_i^2} > 0.$$

Therefore, the value of α_i that solves $\frac{\partial MB}{\partial \alpha_i} = \frac{\partial MC}{\partial \alpha_i}$ is a minimum and not a maximum. In such a case, the optimal α^* is given by a corner solution, i.e., $\alpha^* = 0$ or $\alpha^* = \frac{1}{2}$. The profits for firm i when acquiring no equity, $\alpha_i = 0$, are given by,

$$\frac{(a-c)^2(1-\alpha_j)}{(3-\alpha_j)^2 b} + \alpha_j^\beta NW_i$$

while the profits for firm i when acquiring maximal equity, $\alpha_i = \frac{1}{2}$, are

$$\frac{(a-c)^2(1-\alpha_j)}{(2.5-\alpha_j)^2 b} - \left(\frac{1}{2}\right)^\beta NW_j - F + \alpha_j^\beta NW_i.$$

Summarizing, when $\beta \leq 1$, the firm chooses no equity, $\alpha_i^* = 0$, if

$$\frac{(a-c)^2(1-\alpha_j)}{(3-\alpha_j)^2 b} \geq \frac{(a-c)^2(1-\alpha_j)}{(2.5-\alpha_j)^2 b} - \left(\frac{1}{2}\right)^\beta NW_j - F$$

holds, while otherwise it chooses maximal equity, $\alpha_i^* = \frac{1}{2}$.

8.2 Appendix 2 - Extension to convex production costs

In this appendix, we explore how our results change when firms face a convex production cost $C(q_i) = c\,(q_i)^2$, where $c > 0$. For simplicity, we assume $a = b = 1$. In this context, firm i's profit π_i is

$$\pi_i \equiv [1 - (q_i + q_j)]\, q_i - c\,(q_i)^2\,.$$

Equilibrium output - Second stage. Using this definition of π_i in problem (2), and differentiating with respect to q_i, we obtain best response function

$$q_i(q_j) = \begin{cases} \frac{1}{2(1+c)} - \frac{(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)(1+c)} q_j & \text{if } q_j \leq \frac{(1-\alpha_j)}{(1+\alpha_i-\alpha_j)} \\ 0 \quad \text{otherwise.} \end{cases}.$$

where the vertical axis is $\frac{1}{2(1+c)}$ thus being unaffected by firms' equity shares. Like in the main body of the paper, when firms hold no equity shares, $\alpha_i = \alpha_j = 0$, the best response function collapses to $\frac{1}{2(1+c)} - \frac{1}{2(1+c)} q_j$. When only firm i holds equity shares on firm j's profits, $\alpha_i > 0$ but $\alpha_j = 0$, the best response function pivots inwards becoming $\frac{1}{2(1+c)} - \frac{(1+\alpha_i)}{2(1+c)} q_j$. Finally, when both firms sustain positive equity shares, $\alpha_i, \alpha_j > 0$, the best response function pivots inwards even further.

Simultaneously solving for output levels q_i and q_j, we find

$$q_i^* = \frac{(1-\alpha_i)\,[(1-\alpha_i-\alpha_j) + 2(1-\alpha_j)c]}{(3-\alpha_i-\alpha_j)(1-\alpha_i-\alpha_j) + 8(1-\alpha_i)(1-\alpha_j)c + 4(1-\alpha_i)(1-\alpha_j)c^2}$$

which is positive under all parameter values.

Equilibrium equity - First stage. We now substitute the solution for q_i^* and q_j^* into the profit function of firm i, π_i, and obtain $\pi_i(\alpha_i, \alpha_j)$, which represents the profit that firm i earns during the second stage as a function of equity shares α_i and α_j. We can now insert profit $\pi_i(\alpha_i, \alpha_j)$ into firm i's equity choice in the first-period game, as follows.

$$\max_{0 \leq \alpha_i \leq \frac{1}{2}} \ (1-\alpha_j)\pi_i + \alpha_i \pi_j + \alpha_j^{\beta} N W_i - C\,(\alpha_i)$$

where, similarly as in expression (3), $C\,(\alpha_i) = \delta \alpha_i^2$ captures the cost of acquiring equity. Differentiating the above expression with respect to equity share α_i to obtain firm i's marginal benefit of acquiring equity, MB_i. This marginal benefit is, however, very large in this setting of convex production costs. For tractability, we do not provide the expression of MB_i here. However, we set $MB_i = MC_i$, and Table A-III below provides an analogous simulation of optimal equity share α_i^* as in Table I of the paper using the same parameter values.

Net Worth NW / Marginal cost c	$c = 0$	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = 0.9$
$NW = 0.1$	0.5	0.5	0.5	0.5	0.47	0.45
$NW = 0.2$	0.23	0.25	0.25	0.24	0.22	0.21
$NW = 0.3$	0.14	0.15	0.16	0.15	0.14	0.14
$NW = 0.4$	0.10	0.11	0.12	0.11	0.11	0.10
$NW = 0.5$	0.08	0.09	0.09	0.09	0.09	0.08
$NW = 0.6$	0.07	0.07	0.08	0.07	0.07	0.07
$NW = 0.7$	0.06	0.06	0.06	0.06	0.06	0.06
$NW = 0.8$	0.05	0.05	0.06	0.05	0.07	0.07
$NW = 0.9$	0.04	0.05	0.05	0.05	0.05	0.04

Table A-III. Optimal equity share α_i^* with convex production costs.

Overall, equilibrium equity share α_i^* is larger when firms face convex than linear production costs. Intuitively, firms have stronger incentives to acquire equity on their rivals' profits when facing convex production costs, since this allows the firm to reduce its total costs. In addition, equilibrium equity α_i^* decreases as the net worth of firm i's rival, NW, increases.

Welfare analysis. Like in the paper, we next identify the socially optimal output in this context with convex production costs, q^{SO}, and find the corresponding cutoff α^{SO}. The welfare function is given by expression (4), but where firm i's profit is now $\pi_i \equiv [1 - (q_i + q_j)]\, q_i - c\,(q_i)^2$. This yields a socially optimal output of

$$q^{SO} = \frac{1}{2\,(2 + c + 2d - \gamma)}$$

which is positive under all parameter values. As a next step, we evaluate equilibrium output q_i^* in the case of a symmetric equilibrium in the first stage, $\alpha_i = \alpha_j = \alpha$, as described above. This yields an equilibrium output

$$q_i^* = \frac{(1 - \alpha_i)}{3 + 2c - 2\alpha_i(1 + c)}$$

Thus, the condition for which equilibrium output is socially excessive, $q_i^* > q^{SO}$, is

$$\alpha < \alpha^{SO} \equiv 1 - \frac{1}{2(1 + 2d - \gamma)}$$

Cutoff α^{SO} is identical to the cutoff we found with linear production costs and $b = 1$. Intuitively, the welfare function in expression (4) considers consumer surplus, producer surplus, and environmental damage from production. However, producer surplus collapses to $\pi_i + \pi_j - 2F$, which coincides with $\left[V_i + \alpha_j^\beta NW_i - C\,(\alpha_i)\right] + \left[V_j + \alpha_i^\beta NW_j - C\,(\alpha_j)\right]$, where $V_i = (1 - \alpha_j)\pi_i + \alpha_i \pi_j$ for firm i. Therefore, convex costs symmetrically affect the firm's and the social planner's problem. In contrast, consumer surplus and environmental damage are not affected by the convexity of production costs, ultimately implying that cutoff α^{SO} is unaffected by the type of production costs (linear or convex) that the firm faces. Therefore, the comparative statics of this cutoff remain the same.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Tax t
$\gamma = 0$	0.5	0.5	0
$\gamma = 0.1$	0.5	0.5	0
$\gamma = 0.2$	0.5	0.5	0
$\gamma = 0.3$	0.5	0.5	0
$\gamma = 0.4$	0.5	0.5	0
$\gamma = 0.5$	0.44	0.5	0.25
$\gamma = 0.6$	0.38	0.5	0.42
$\gamma = 0.7$	0.29	0.5	0.77
$\gamma = 0.8$	0.17	0.5	1.81
$\gamma = 0.9$	0	0.5	433.79
$\gamma = 1$	0	0.5	433.79

Table A-IV. Optimal equity subsidies with convex production costs.

Relative to the setting with linear production costs (Table II in the main body of the paper), socially optimal equity α^{SO} does not change, while equilibrium equity α_i^* is higher, ultimately yielding lower equity taxes when γ is relatively low, or more stringent taxes when γ is high.

8.3 Appendix 3 - Extension to joint equity share acquisition

Previous sections considered that every firm independently chooses its equity shares. In some settings, however, firms may negotiate with each other their equity holding. In this appendix, we explore how our findings are affected when firms jointly choose their equity shares in each other's profits, α_i and α_j, solving the following joint-maximization problem

$$\max_{\alpha_i,\alpha_j \geq 0} \; (1-\alpha_j)\pi_i\left(q_i^*, q_j^*\right) + \alpha_i \pi_j\left(q_i^*, q_j^*\right) + \alpha_j^{\beta} NW_i - (F + NW_j \alpha_i^{\beta})$$
$$+(1-\alpha_i)\pi_j\left(q_i^*, q_j^*\right) + \alpha_j \pi_i\left(q_i^*, q_j^*\right) + \alpha_i^{\beta} NW_j - (F + NW_i \alpha_j^{\beta})$$

Differentiating with respect to α_i yields

$$\frac{(a-c)^2(1-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3 b} > 0$$

and similarly after differentiating with respect to α_j. As a result, firm i increases its equity α_i as much as possible, $\alpha_i = 0.5$, implying that firms acquire weakly more equity when they jointly choose their equity holdings than when they independently do (relative to Table I in Section 3.2). Intuitively, under joint equity decisions, every firm's cost of equity is exactly offset by the equity revenue that its rival receives, ultimately implying that the cost of acquiring equity is nil. This result leads firms to acquire maximal equity on each other, that is, a full merger when firms jointly maximize profits.

Equity taxes. Suppose that the regulator sets a tax $t > 0$ increasing firms' cost of equity acquisition. Then, the above problem becomes

$$\max_{\alpha_i,\alpha_j \geq 0} (1-\alpha_j)\pi_i\left(q_i^*, q_j^*\right) + \alpha_i \pi_j\left(q_i^*, q_j^*\right) + \alpha_j^\beta NW_i - (F + (1+t)NW_j\alpha_i^\beta)$$
$$+(1-\alpha_i)\pi_j\left(q_i^*, q_j^*\right) + \alpha_j \pi_i\left(q_i^*, q_j^*\right) + \alpha_i^\beta NW_j - (F + (1+t)NW_i\alpha_j^\beta)$$

Differentiating with respect to α_i yields

$$\frac{(a-c)^2(1-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3 b} = t\beta\alpha_j^{\beta-1}NW_i$$

Every firm i now faces a positive marginal cost from acquiring equity, thus decreasing its equilibrium equity holdings. Since firms acquire maximal equity in equilibrium, our results imply that regulators would set weakly positive taxes under all parameter conditions; as illustrated in Table A-V. This table considers the same parameter values as Table II showing that, relative to the setting where firms independently acquire equity (Table II), regulators need to set more severe taxes to induce firms to choose α^{SO}.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Tax t
$\gamma = 0$	0.5	0.5	0
$\gamma = 0.1$	0.5	0.5	0
$\gamma = 0.2$	0.5	0.5	0
$\gamma = 0.3$	0.5	0.5	0
$\gamma = 0.4$	0.5	0.5	0
$\gamma = 0.5$	0.44	0.5	0.24
$\gamma = 0.6$	0.38	0.5	0.14
$\gamma = 0.7$	0.29	0.5	0.25
$\gamma = 0.8$	0.17	0.5	0.51
$\gamma = 0.9$	0	0.5	90.74
$\gamma = 1$	0	0.5	90.74

Table A-V. Optimal equity taxes under joint profit maximization.

8.4 Appendix 4 - Extension to sequential equity acquisition

In this appendix, we consider an alternative equity acquisition game. Here, firms sequentially purchase equity on each others profits. In the first stage, firm i acquires α_i on firm j; in the second stage, firm j purchases α_j shares on firm i; and in the third stage, based on the profile of equity shares (α_i, α_j) arising in previous stages, firms compete in Cournot.

We solve this game by backward induction. First, we insert the equilibrium output levels from the last stage to find equilibrium profits, as a function of equity shares α_i and α_j. Second, we use our condition $MB_j = MC_j$ as defined in equation (3) to numerically approximate the follower's best response function, $\alpha_j(\alpha_i)$. Specifically, we consider values of α_i in 0.01 increments, within the admissible range of $\alpha_i \in [0, 0.5]$, finding the value of α_j that maximizes the follower's profits for every given α_i. We then calculate the leader's profit for each equity level α_i, evaluated at the corresponding optimal equity of the follower, $\alpha_j(\alpha_i)$, identifying which value of α_i maximizes the leader's profits. For comparison purposes, Table IV evaluates our results using the same parameter values as in the first column of Table A-VI. In column one in Table A-VI, we present the leader's optimal equity share when firms choose their equity sequentially. In column two, we summarize the follower's optimal equity in this context. Finally, in column three, we show equilibrium equity shares in the simultaneous-move version of the game (as shown in Table I).

Net worth NW	Leader, α_i^*	Follower, α_j^*	Simultaneous game, α^*
$NW = 0.1$	0.11	0.20	0.23
$NW = 0.2$	0.04	0.09	0.10
$NW = 0.3$	0.03	0.06	0.06
$NW = 0.4$	0.02	0.05	0.05
$NW = 0.5$	0.02	0.03	0.03
$NW = 0.6$	0.02	0.03	0.03
$NW = 0.7$	0.01	0.05	0.02
$NW = 0.8$	0.01	0.05	0.02
$NW = 0.9$	0.01	0.05	0.02

Table A-VI. Equilibrium equity share α^* when $c = 0.3$.

Table A-VI suggests that the leader acquires less equity than in the simultaneous-move game considered in previous sections of the paper, which, in turn, induces the follower to acquire weakly more equity than in the simultaneous game. Intuitively, the leader free rides off the follower's equity acquisition, thus reducing its equity acquisition. This is because the leader anticipates that the follower will respond by increasing its equity purchases relative to the simultaneous setting of subsection 3.2. Additionally, both firms decrease their equity acquisition as the net worth of the company increases, i.e., as we move to lower rows in Table A-VI.[19]

8.5 Appendix 5 - Allowing for more firms

In this appendix, we extend our model to a setting with three firms.

[19] Larger values of parameter c produce lesser equity acquisition by both the leader and the follower; an analogous result to that in the simultaneous-move version of the game in Table I.

Second stage. In this case, firm i solves

$$\max_{q_i \geq 0} \quad V_i = (1 - \alpha_{ji} - \alpha_{ki})\pi_i + \alpha_{ij}\pi_j + \alpha_{ik}\pi_k$$

where $\pi_i = (a - bQ)q_i - cq_i$ denotes firm i's profit and $Q \equiv q_i + q_j + q_k$ represents aggregate output and $i \neq j \neq k$. Intuitively, the first term represent firm i's share in its own profits, while the second (third) term captures firm i's share in firm j (firm k, respectively) profits. Differentiating with respect to q_i, we obtain best response function

$$q_i(q_j, q_k) = \frac{a-c}{2b} - \frac{1 + \alpha_{ij} - \alpha_{ji} - \alpha_{ki}}{2(1 - \alpha_{ji} - \alpha_{ki})} q_j - \frac{1 + \alpha_{ik} - \alpha_{ji} - \alpha_{ki}}{2(1 - \alpha_{ji} - \alpha_{ki})} q_k$$

or, more compactly,

$$q_i(q_j, q_k) = \frac{a-c}{2b} - \frac{(1 + \alpha_{ij} - r_i)\, q_j + (1 + \alpha_{ik} - r_i)\, q_k}{2(1 - r_i)}$$

where $s_i \equiv \alpha_{ij} + \alpha_{ik}$ denotes firm i's equity share in its rivals (firm j and k), and $r_i \equiv \alpha_{ji} + \alpha_{ki}$ represents the share that all firm i's rivals hold on this firm's profits. We find symmetric best response functions for firm j, $q_j(q_i, q_k)$, and for firm k, $q_k(q_i, q_j)$.

Simultaneously solving for q_i, q_j and q_k, yields output

$$q_i^* = \frac{(a-c)\left[(r_k - 1)\left(1 - \alpha_{ik} - r_i + \alpha_{ij}\left(s_i + s_j + \alpha_{ki} - 2\right)\right)\right]}{(s_i s_j + s_k - 4)\left[\alpha_{ij}(r_k + \alpha_{ki} - 1) + \alpha_{ik}(\alpha_{ji} + \alpha_{kj} - 1) + (r_1 - 1)(\alpha_{jk} + \alpha_{kj})\right] b}$$

where $A \equiv \alpha_{kj}\left[1 - \alpha_{ji} + \alpha_{ij}(r_k + \alpha_{ik} - 1) - \alpha_{ki} + \alpha_{ik}(r_i + r_k - 3)\right]$, and similar expressions apply for output levels q_j^* and q_k^*. Substituting q_i^*, q_j^* and q_k^* into firm i's objective function, V_i, gives us

$$V_i\left(q_i^*, q_j^*, q_k^*\right) = \frac{(a-c)^2(1 - \alpha_{ji} - \alpha_{ki})}{b(4 - \alpha_{ij} - \alpha_{ik} - \alpha_{ji} - \alpha_{ki} - \alpha_{jk} - \alpha_{kj})^2}.$$

First stage. Anticipating the above profits in the second stage, firm i solves

$$\max_{\alpha_{ij}, \alpha_{ik}} \quad V_i\left(q_i^*, q_j^*, q_k^*\right) - (F + \alpha_{ij}^{\beta} NW_j) - (F + \alpha_{ik}^{\beta} NW_k) + (\alpha_{ji} + \alpha_{ki}) NW_i$$

Differentiating with respect to α_{ij} yields

$$\frac{2(a-c)^2}{b} \frac{1 - r_i}{(4 - r_i - r_j - r_k)^3} = \beta NW_j \alpha_{ij}^{\beta - 1} \tag{A1}$$

and differentiating with respect to α_{ik} we find

$$\frac{2(a-c)^2}{b} \frac{1 - r_i}{(4 - r_i - r_j - r_k)^3} = \beta NW_k \alpha_{ik}^{\beta - 1}. \tag{A2}$$

Therefore, firm i faces two first-order conditions, A1 and A2, and a similar pair of expressions applies to firms j and k, producing a total of six first-order conditions, i.e., generally $N(N-1)$ expressions where N denotes the number of firms. When firms are symmetric, $NW_j = NW_k = NW$ for every two firms j and k, expression A1 and A2 coincide and, in addition, all three firms faces the exact same first-order condition, i.e., A1 evaluated at $NW_j = NW$. As a result, all three firms hold the same equity share in equilibrium on their rivals' profits, that is, $\alpha_{ij}^* = \alpha_{ik}^* = \alpha^*$ for every firm i and $j \neq k \neq i$.

We next extend our model to a setting with N firms. For simplicity, we consider symmetric equity shares $\alpha_i = \alpha_j = \alpha$ for every two firms i and j, as in the symmetric result with three firms presented above. Following the same approach as in the main text, we first identify equilibrium output in this context, then the socially optimal output, and finally compare these two findings.

Equilibrium output. Every firm i solves

$$\max_{q_i \geq 0} V_i = [1-(N-1)\,\alpha]\,\pi_i + \sum_{j \neq i} \alpha \pi_j$$

where $\pi_i \equiv [a - b(Q_{-i}+q_i)]q_i - cq_i$ denotes firm i's profit, $Q_{-i} \equiv \sum_{j\neq i} q_j$ represents the aggregate output from all firms other than i, and $\pi_j \equiv [a - b(Q_{-j}+q_j)]q_j - cq_j$ denotes firm j's profit. Differentiating with respect to q_i yields best response function

$$q_i(Q_{-i}) = \frac{a-c}{2b} - \left(\frac{\alpha(N-2)-1}{2[\alpha(N-1)-1]}\right) Q_{-i}$$

Relative to the best response function identified in Lemma 1 for two firms, $q_i(Q_{-i})$ has the same vertical intercept, $\frac{a-c}{2b}$, and decreases in its rivals' appropriation, Q_{-i}; but has a different slope, $\frac{\alpha(N-2)-1}{2[\alpha(N-1)-1]}$. Since $\frac{\partial \frac{\alpha(N-2)-1}{2[\alpha(N-1)-1]}}{\partial N} = \frac{\alpha^2}{2(1-\alpha(N-1)^2} > 0$, the slope is increasing in magnitude in N. Intuitively, competition becomes tougher, and every individual firm reduces its own appropriation more significantly, i.e., firms' decisions are strategic substitutes to a greater extent. Graphically, the best response function rotates inwards, becoming steeper.

Invoking symmetry in equilibrium, $q_i^* = q_j^* = q^*$, we obtain that $Q_{-i}^* = (N-1)q^*$. Solving for q^*, yields the equilibrium appropriation in this N-firm setting

$$q_i^*(N) = \frac{\theta[\alpha(N-1)-1]}{[N[\alpha(N-1)-1]-1]\,b}$$

When only two firms compete, $N=2$, equilibrium output simplifies to $q_i^*(2) = \frac{(a-c)(1-\alpha)}{(3-2\alpha)b}$, which coincides with that in Proposition 1 when equity shares are equal across firms.

We next examine special cases, showing that equilibrium output q_i^* becomes:

1. *Cournot model without equity shares*: $q_i^* = \frac{a-c}{(N+1)b}$, i.e., $\alpha_i = 0$ for every firm i.

2. *Cournot model with equally shared equity*: $q_i^* = \frac{a-c}{2Nb}$, i.e., $\alpha_i = \alpha_j = \frac{1}{N}$.

Social optimum. The social planner solves

$$\begin{aligned}\max_{q_1,,,,q_N} W &= \gamma CS(Q) + PS(Q) - dQ^2 \\ &= \gamma \frac{bQ^2}{2} + \sum_{i=1}^{N} \pi_i - dQ^2\end{aligned}$$

where $Q \equiv \sum_{i=1}^{N} q_i$ denotes aggregate output. Note that, since equity shares are symmetric in this setting, they cancel out from the producer surplus, i.e., $PS(Q) = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \pi_i$. Differentiating with respect to q_i, we obtain

$$q_i(Q_{-i}, Q) = \frac{[(a-c) - 2dQ]}{b(2-\gamma)} - Q_{-i}$$

Invoking symmetry, we find that the socially optimal output level becomes

$$q_i^{SO}(N) = \frac{(a-c)}{N[b(2-\gamma) + 2d]}$$

which collapses to $q_i^{SO}(2) = \frac{(a-c)}{4(b+d)-2b\gamma}$ when only two firms operate, $N = 2$, thus coinciding with our result in Proposition 3. Socially optimal output $q_i^{SO}(N)$ decreases in the number of firms exploiting the resource, N since $\frac{\partial q_i^{SO}(N)}{\partial N} = -\frac{a-c}{N^2[b(2-\gamma)+2d]} < 0$. Finally, equilibrium appropriation is socially excessive, $q_i^{SO}(N) \leq q_i^*(N)$, if and only if

$$\alpha < \alpha^{SO}(N) \equiv \frac{N[2d - b(1-\gamma)] - b}{N(N-1)[2d + b(1-\gamma)]}$$

In the case that only two firms compete in the industry, $N = 2$, cutoff $\alpha^{SO}(2)$ collapses to $\alpha^{SO}(2) \equiv 1 - \frac{b}{4d+2b(1-\gamma)}$, thus coinciding with cutoff α^{SO} in Proposition 3.

8.6 *Proof of Lemma 1*

Firm i solves problem (1), which we can more explicitly write as follows

$$\max_{q_i \geq 0} \; (1-\alpha_j)\pi_i + \alpha_i \pi_j$$

where $\pi_i = [a - b(q_i + q_j)]\, q_i - cq_i$ and, similarly, $\pi_j = [a - b(q_i + q_j)]\, q_j - cq_j$. Differentiating with respect to q_i yields,

$$(1-\alpha_j)\,(a - c - 2bq_i - bq_j) + \alpha_i(-bq_j) = 0$$

Solving for q_i, we obtain

$$q_i = \frac{a-c}{2b} - \frac{1+\alpha_i-\alpha_j}{2\,(1-\alpha_j)} q_j$$

Since firms produce weakly positive amounts, we can set the above expression greater or equal to zero, and solve for q_j, finding $q_j \leq \frac{(a-c)(1-\alpha_j)}{b(1+\alpha_i-\alpha_j)}$. Therefore, firm i's best response function is

$$q_i(q_j) = \begin{cases} \frac{a-c}{2b} - \frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)} q_j & \text{if } q_j \leq \frac{(a-c)(1-\alpha_j)}{b(1+\alpha_i-\alpha_j)} \\ 0 & \text{otherwise.} \end{cases}$$

8.7 *Proof of Proposition 1*

From Lemma 1, we found the best response function for firms i and j. Simultaneously solving for q_i and q_j in $q_i(q_j)$ and $q_j(q_i)$, we obtain that the optimal appropriation for every firm i is

$$q_i^* = \frac{(a-c)(1-\alpha_i)}{b\,(3-\alpha_i-\alpha_j)}$$

This output level is strictly positive since $a > c$ and $\alpha_i, \alpha_j \in \left[0, \frac{1}{2}\right]$ by definition. We can now differentiate q_i^* with respect to parameters. We can now differentiate q_i^* with respect to parameters. First,

$$\frac{\partial q_i^*}{\partial a} = \frac{1-\alpha_i}{b\,(3-\alpha_i-\alpha_j)} > 0$$

thus indicating that q_i^* is increasing in a. Second,

$$\frac{\partial q_i^*}{\partial \alpha_j} = -\frac{(a-c)(1-\alpha_i)}{b\,(3-\alpha_i-\alpha_j)^2} < 0$$

meaning that q_i^* is increasing in firm j's equity holding on firm i. Third,

$$\frac{\partial q_i^*}{\partial c} = -\frac{1-\alpha_i}{b\,(3-\alpha_i-\alpha_j)} < 0$$

which says that q_i^* is decreasing in its production cost c. Fourth,

$$\frac{\partial q_i^*}{\partial \alpha_i} = -\frac{(a-c)(2-\alpha_j)}{b\,(3-\alpha_i-\alpha_j)^2} < 0$$

which reflects that q_i^* is decreasing in firm i's equity share on firm j's profit, α_i. Fifth,

$$\frac{\partial q_i^*}{\partial b} = -\frac{(a-c)(1-\alpha_i)}{b^2\,(3-\alpha_i-\alpha_j)} < 0$$

which implies that q_i^* is decreasing in the slope of the demand curve, b. Finally, the output difference $q_i^* - q_j^*$ is

$$\frac{(a-c)(1-\alpha_i)}{b\,(3-\alpha_i-\alpha_j)} - \frac{(a-c)(1-\alpha_j)}{b\,(3-\alpha_j-\alpha_i)} = \frac{(a-c)(\alpha_j-\alpha_i)}{b\,(3-\alpha_j-\alpha_i)}$$

is weakly positive if and only if $\alpha_i \leq \alpha_j$.

8.8 *Proof of Proposition 2*

We first evaluate equilibrium profits in the second stage of the game, $\pi_i\left(q_i^*, q_j^*\right)$, by inserting equilibrium appropriation levels found in Proposition 1, $q_i^* = \frac{(a-c)(1-\alpha_i)}{b(3-\alpha_i-\alpha_j)}$ and $q_j^* = \frac{(a-c)(1-\alpha_j)}{b(3-\alpha_i-\alpha_j)}$, which yields $\pi_i\left(q_i^*, q_j^*\right) = \frac{(a-c)^2(1-\alpha_i)}{b(3-\alpha_i-\alpha_j)^2}$. Operating similarly for the equilibrium profit of firm j, we obtain $\pi_j\left(q_i^*, q_j^*\right) = \frac{(a-c)^2(1-\alpha_j)}{b(3-\alpha_i-\alpha_j)^2}$. Therefore, every firm i solves

$$\begin{aligned} & \max_{\alpha_i \geq 0} \ (1-\alpha_j)\pi_i\left(q_i^*, q_j^*\right) + \alpha_i \pi_j\left(q_i^*, q_j^*\right) - (F + NW\alpha_i^\beta) \\ = \ & \frac{(a-c)^2(1-\alpha_j)}{b\left(3-\alpha_i-\alpha_j\right)^2} - (F + NW\alpha_i^\beta) \end{aligned}$$

Differentiating with respect to α_i yields

$$\frac{2(a-c)^2(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3 b} - NW\beta\alpha_i^{\beta-1} \leq 0$$

with equality if $\alpha_i^* > 0$.

8.9 *Proof of Corollary 1*

The first term in expression (3) can be interpreted as the marginal benefit that firm i obtains from marginally increasing its equity share in firm j's profits, α_i, that is, $MB_i \equiv \frac{2(a-c)^2(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3 b}$. This term is increasing in a, decreasing in b and c, and increasing in α_j when

$$\frac{\partial MB_i}{\partial \alpha_j} = \frac{2(\alpha_i - 2\alpha_j)(a-c)^2}{b(3-\alpha_i-\alpha_j)^4} > 0$$

which holds as long as $\alpha_i > 2\alpha_j$. Otherwise, MB_i decreases in α_j.

We next show that MB_i is increasing and convex in α_i. Differentiating MB_i with respect to α_i, we find

$$\frac{\partial MB_i}{\partial \alpha_i} = \frac{6(1-\alpha_i)(a-c)^2}{b(3-\alpha_i-\alpha_j)^4} > 0.$$

Moreover, differentiating MB_i with respect to α_i again yields

$$\frac{\partial MB_i^2}{\partial \alpha_i^2} = \frac{24(1-\alpha_i)(a-c)^2}{b(3-\alpha_i-\alpha_j)^5} > 0.$$

In addition, evaluating MB_i at $\alpha_i = 0$, we obtain $MB_i = \frac{2(a-c)^2(1-\alpha_j)}{b(3-\alpha_j)^3}$, while evaluating MB_i at its upper bound, $\alpha_i = \frac{1}{2}$, yields $MB_i = \frac{2(a-c)^2(1-\alpha_j)}{b\left(\frac{5}{2}-\alpha_j\right)^3}$.

Finally, we investigate under which conditions $MB_i \geq MC_i$ holds for all values of α_i. Since MB_i is increasing and convex in α_i, this is equivalent to the slope of MC_i being lower than that of MB_i. In particular, this entails $\frac{\partial MC_i}{\partial \alpha_i} \leq \frac{\partial MB_i}{\partial \alpha_i}$, or $NW_j\beta(\beta-1)\alpha_i^{\beta-2} \leq \frac{6(1-\alpha_i)(a-c)^2}{b(3-\alpha_i-\alpha_j)^4}$. After solving for NW_j, yields $NW_j \leq \frac{1}{\beta(\beta-1)\alpha_i^{\beta-2}} \frac{6(1-\alpha_i)(a-c)^2}{b(3-\alpha_i-\alpha_j)^4}$. Note that when firm i holds no equity on firm j, $\alpha_i = 0$, the above inequality collapses to $NW_j \leq +\infty$, thus holding for all parameter values.

8.10 *Proof of Proposition 3*

Differentiating with respect to q_i in problem (4), we find that $q_i(q_j) = \frac{a-c}{2(b+d)-b\gamma} - q_j$. A symmetric expression applies when differentiating with respect to q_j. In a symmetric output profile, we obtain socially optimal output

$$q_i^{SO} = \frac{a-c}{4(b+d)-2b\gamma}.$$

The numerator of q_i^{SO} is positive since $a > c$ by definition. The denominator is positive for all $\gamma < 2 + \frac{2d}{b}$, which holds for all $d, b \geq 0$ since $\gamma \in [0,1]$ by definition. Therefore, q_i^{SO} is positive for all admissible parameter values.

We can now compare equilibrium and socially optimal output, q_i^* and q_i^{SO}, by setting $q_i^* - q_i^{SO} = 0$, and solving for α. We find that $q_i^* > q_i^{SO}$ if and only if $\alpha < \alpha^{SO}$, where cutoff α^{SO} is given by

$$\alpha^{SO} \equiv 1 - \frac{b}{4d + 2b(1-\gamma)}.$$

8.11 *Proof of Corollary 2*

Setting cutoff $\alpha^{SO} > 0$ and solving for γ, we find $\gamma < \gamma_1 \equiv \frac{1}{2} + \frac{2d}{b}$. Setting $\alpha^{SO} < 1/2$ and solving for γ, we obtain that $\gamma > \gamma_2 \equiv \frac{2d}{b}$. In addition, the difference between cutoff γ_1 and γ_2 is $\gamma_1 - \gamma_2 = \left(\frac{1}{2} + \frac{2d}{b}\right) - \frac{2d}{b} = \frac{1}{2}$, implying that $\gamma_1 > \gamma_2$ under all parameter values.

Differentiating cutoff α^{SO} with respect to its parameter values, we obtain

$$\begin{aligned}
\frac{\partial \alpha^{SO}}{\partial d} &= \frac{b}{(b+2d-b\gamma)^2} > 0, \\
\frac{\partial \alpha^{SO}}{\partial b} &= -\frac{d}{(b+2d-b\gamma)^2} < 0, \text{ and} \\
\frac{\partial \alpha^{SO}}{\partial \gamma} &= -\frac{b^2}{(b+2d-b\gamma)^2} < 0.
\end{aligned}$$

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