Anticipatory Effects of Taxation in the Commons: When do taxes work, and when do they fail?[1]

Online Appendix

Appendix 1 - Cost externalities

In this appendix we use the second-period social welfare function to illustrate the effect of a marginal increase in q_i in terms of positive and negative externalities. First, we write the social welfare function in this period as follows

$$\begin{aligned}
SW_2(q_i, q_{-i}) &= CS_2(Q) + PS_2(q_i, q_{-i}) \\
&= \frac{b}{2}(Q)^2 + \sum_i^n \pi\left(q_i, q_{-i}\right) \\
&= \frac{b}{2}(q_i + q_{-i})^2 + \sum_i^n \left\{ \left[1 - b\left(q_i + q_{-i}\right)\right] q_i - \frac{q_i\left(q_i + \lambda q_{-i}\right)}{\theta(1+g) - X} \right\} \\
&= \underbrace{\frac{b}{2}(q_i + q_{-i})^2}_{\text{Consumer surplus}} + \underbrace{\left(\left[1 - b\left(q_i + q_{-i}\right)\right] q_i - \frac{q_i\left(q_i + \lambda q_{-i}\right)}{\theta(1+g) - X} \right)}_{\text{Firm } i\text{'s profits}} \\
&\quad \underbrace{+ \sum_{j \neq i} \left\{ \left[1 - b\left(q_j + q_i + q_{-j}\right)\right] q_j - \frac{q_j\left(q_j + \lambda\left(q_i + q_{-j}\right)\right)}{\theta(1+g) - X} \right\}}_{\text{Profits of firm } i\text{'s rivals}}.
\end{aligned}$$

Therefore, a marginal increase in firm i's second-period appropriation, q_i, produces: (1) an increase in consumer surplus (first term in the above expression), (2) a change in firm i's profits (second term), and (3) a decrease in the profits of firm i's rivals (third term). The first and second terms capture external effects of a marginal increase in q_i, the former being positive the latter being negative. Overall, in a symmetric outcome, $q_i = q_j = q$, the marginal external effects become

$$bNq - \left[b + \frac{\lambda}{\theta(1+g) - X}\right](N-1)q$$

Solving for λ, this expression is positive if and only if

$$\lambda < \overline{\lambda} \equiv \frac{b\left[\theta(1+g) - X\right]}{N-1}.$$

As described in Section 3.1, overall external effects are positive when the positive externality that a marginal increase in q_i generates on consumer surplus dominates the negative externality that q_i imposes on the profits of firm i's rivals (raising their production costs). In this context, optimal regulation leads to a subsidy per unit, $t^* > 0$. When $\lambda = \overline{\lambda}$, the positive

[1] Ana Espinola-Arredondo, Félix Muñoz-García, and Isaac Duah, School of Economic Sciences Washington State University Pullman, WA 99164.

consumer surplus externality coincides with the negative cost externality and the optimal policy is a zero fee, $t^* = 0$. Finally, when $\lambda > \overline{\lambda}$, the negative externality dominates and a positive tax $t^* > 0$ is socially optimal.
This result also implies that, starting from a setting where $\lambda > \overline{\lambda}$, a marginal increase in the extent of the cost externality, λ, increases taxes. However, if we start from a setting where $\lambda < \overline{\lambda}$, a marginal increase in λ produces a marginal decrease in the subsidy, until reaching $\lambda = \overline{\lambda}$ where the subsidy collapses to zero, and then becomes a subsidy for all $\lambda > \overline{\lambda}$.

Appendix 2 - Additional tables

The following tables report the PR for each (b, λ)-pair in Tables II, IIIa, IIIb, and IVa, respectively. For completeness, we provide first-period appropriation for each vector of parameter values, $x_i(t^*)$, as well as its corresponding profits, $\pi_i(t^*)$. We then report first-period appropriation under no regulation, $x_i(0)$, and its associated profits, $\pi_i(0)$. The table then lists the policy response $PR = x_i(t^*) - x_i(0)$. The sixth column of Table A1 includes the equilibrium tax t^* evaluated at each parameter values. The last column provides the value of cutoff $\overline{\lambda}$ evaluated in equilibrium. As predicted by Proposition 1, when λ satisfies $\lambda > \overline{\lambda}$, the emission fee is positive, while otherwise it is negative.

		Regulation		No regulation		Policy response	Tax	Cutoff
b	λ	$x_i(t^*)$	$\pi_i(t^*)$	$x_i(0)$	$\pi_i(0)$	PR	t^*	$\overline{\lambda}$
0	0	0.375	0.359	0.375	0.359	0.000	0.000	-0.75
0	0.5	0.356	0.226	0.336	0.259	0.020	0.167	-0.712
0	1	0.313	0.156	0.296	0.194	0.016	0.250	-0.626
0	1.5	0.274	0.114	0.262	0.149	0.012	0.300	-0.548
0	2	0.243	0.087	0.234	0.118	0.009	0.333	-0.486
0.5	0	0.248	0.262	0.263	0.228	-0.015	-0.137	0.129
0.5	0.5	0.229	0.175	0.233	0.178	-0.004	0.027	0.167
0.5	1	0.209	0.128	0.210	0.143	0.000	0.121	0.207
0.5	1.5	0.192	0.099	0.191	0.17	0.001	0.182	0.241
0.5	2	0.176	0.079	0.174	0.098	0.002	0.225	0.273
1	0	0.186	0.199	0.194	0.157	-0.008	-0.234	0.878
1	0.5	0.171	0.143	0.176	0.131	-0.005	-0.085	0.908
1	1	0.159	0.110	0.162	0.111	-0.003	0.012	0.932
1	1.5	0.148	0.089	0.149	0.095	-0.002	0.079	0.954
1	2	0.138	0.079	0.139	0.082	-0.001	0.129	0.974
1.5	0	0.147	0.159	0.152	0.118	-0.004	-0.294	1.581
1.5	0.5	0.137	0.121	0.140	0.102	-0.004	-0.163	1.601
1.5	1	0.128	0.097	0.131	0.089	-0.003	-0.070	1.619
1.5	1.5	0.121	0.080	0.123	0.079	-0.002	-0.002	1.633
1.5	2	0.114	0.067	0.115	0.007	-0.001	0.052	1.647
2	0	0.122	0.131	0.124	0.094	-0.002	-0.334	2.256
2	0.5	0.114	0.104	0.116	0.084	-0.002	-0.218	2.272
2	1	0.108	0.086	0.110	0.075	-0.002	-0.131	2.284
2	1.5	0.102	0.073	0.104	0.068	-0.002	-0.064	2.296
2	2	0.097	0.062	0.099	0.061	-0.001	-0.011	2.306

Table A1. PR for $\theta = \delta = 1$, $g = 1/4$, and $N = 2$.

		Regulation		No regulation		
b	λ	$x_i(t^*)$	$\pi_i(t^*)$	$x_i(0)$	$\pi_i(0)$	PR
0	0	0.750	0.719	0.750	0.719	0
0	0.5	0.711	0.452	0.672	0.518	0.039
0	1	0.625	0.313	0.593	0.388	0.032
0	1.5	0.549	0.229	0.525	0.299	0.024
0	2	0.486	0.174	0.469	0.237	0.017
0.5	0	0.372	0.398	0.388	0.315	-0.016
0.5	0.5	0.342	0.287	0.352	0.261	-0.010
0.5	1	0.317	0.221	0.323	0.221	-0.006
0.5	1.5	0.296	0.177	0.299	0.190	-0.003
0.5	2	0.277	0.146	0.278	0.165	-0.001
1	0	0.243	0.262	0.248	0.188	-0.005
1	0.5	0.228	0.209	0.233	0.167	-0.005
1	1	0.215	0.172	0.220	0.150	-0.004
1	1.5	0.205	0.145	0.208	0.135	-0.003
1	2	0.195	0.125	0.197	0.122	-0.002
1.5	0	0.179	0.194	0.181	0.133	-0.002
1.5	0.5	0.171	0.163	0.173	0.122	-0.002
1.5	1	0.163	0.140	0.166	0.113	-0.002
1.5	1.5	0.157	0.122	0.153	0.104	-0.002
1.5	2	0.151	0.108	0.143	0.097	-0.002
2	0	0.142	0.153	0.138	0.103	-0.001
2	0.5	0.136	0.134	0.133	0.096	-0.001
2	1	0.131	0.118	0.090	0.090	-0.001
2	1.5	0.127	0.105	0.128	0.085	-0.001
2	2	0.123	0.095	0.124	0.080	-0.001

Table A2a. PR for $\theta = 2$, $\delta = 1$, $g = 1/4$, and $N = 2$.

		Regulation		No regulation		
b	λ	$x_i(t^*)$	$\pi_i(t^*)$	$x_i(0)$	$\pi_i(0)$	PR
0	0	1.125	1.078	1.125	1.078	0.000
0	0.5	1.067	0.677	1.008	0.777	0.059
0	1	0.938	0.469	0.889	0.581	0.049
0	1.5	0.823	0.343	0.787	0.448	0.036
0	2	0.729	0.261	0.703	0.355	0.026
0.5	0	0.442	0.476	0.455	0.354	-0.013
0.5	0.5	0.410	0.363	0.421	0.307	-0.011
0.5	1	0.385	0.290	0.393	0.268	-0.008
0.5	1.5	0.363	0.240	0.368	0.237	-0.005
0.5	2	0.343	0.202	0.346	0.211	-0.003
1	0	0.269	0.291	0.272	0.199	-0.003
1	0.5	0.256	0.245	0.260	0.183	-0.004
1	1	0.245	0.210	0.248	0.169	-0.003
1	1.5	0.235	0.184	0.238	0.156	-0.003
1	2	0.226	0.162	0.229	0.145	-0.003
1.5	0	0.192	0.208	0.193	0.138	-0.001
1.5	0.5	0.186	0.184	0.187	0.130	-0.001
1.5	1	0.180	0.164	0.181	0.123	-0.001
1.5	1.5	0.174	0.148	0.176	0.116	-0.002
1.5	2	0.169	0.134	0.171	0.110	-0.002
2	0	0.149	0.162	0.150	0.105	-0.001
2	0.5	0.145	0.147	0.146	0.101	-0.001
2	1	0.142	0.134	0.143	0.096	-0.001
2	1.5	0.138	0.123	0.139	0.092	-0.001
2	2	0.135	0.114	0.136	0.088	-0.001

Table A2b. PR for $\theta = 3$, $\delta = 1$, $g = 1/4$, and $N = 2$.

		Regulation		No regulation		
b	λ	$x_i(t^*)$	$\pi_i(t^*)$	$x_i(0)$	$\pi_i(0)$	PR
0	0	0.375	0.422	0.375	0.422	0.000
0	0.5	0.356	0.254	0.336	0.299	0.020
0	1	0.313	0.172	0.296	0.222	0.016
0	1.5	0.274	0.12	0.262	0.170	0.012
0	2	0.243	0.094	0.234	0.134	0.009
0.5	0	0.254	0.291	0.268	0.245	-0.015
0.5	0.5	0.231	0.194	0.237	0.192	-0.006
0.5	1	0.210	0.142	0.212	0.155	-0.002
0.5	1.5	0.192	0.110	0.192	0.128	0.000
0.5	2	0.176	0.087	0.175	0.108	0.001
1	0	0.189	0.214	0.196	0.163	-0.007
1	0.5	0.173	0.157	0.178	0.137	-0.005
1	1	0.159	0.121	0.163	0.117	-0.003
1	1.5	0.148	0.098	0.150	0.101	-0.002
1	2	0.139	0.081	0.140	0.088	-0.001
1.5	0	0.149	0.167	0.152	0.121	-0.003
1.5	0.5	0.138	0.130	0.141	0.106	-0.003
1.5	1	0.129	0.105	0.132	0.093	-0.003
1.5	1.5	0.121	0.088	0.123	0.083	-0.002
1.5	2	0.115	0.074	0.116	0.074	-0.001
2	0	0.122	0.137	0.124	0.096	-0.002
2	0.5	0.115	0.111	0.117	0.086	-0.002
2	1	0.108	0.093	0.110	0.077	-0.002
2	1.5	0.103	0.079	0.104	0.070	-0.002
2	2	0.098	0.068	0.099	0.064	-0.001

Table A3. PR for $\theta = \delta = 1$, $g = 1/2$, and $N = 2$.

Appendix 3 - Cartel appropriation

In this appendix, we explore how our results are affected when firms coordinate their appropriation decisions in a cartel seeking to maximize their joint profits during the first and second periods. In the second period, the cartel solves

$$\pi^C(t) \equiv \max_{q_1,\dots,q_N \geq 0} \sum_{i=1}^{N} \left[\left[1 - b\left(q_i + Q_{-i}\right)\right] q_i - \frac{q_i(q_i + \lambda Q_{-i})}{\theta(1+g) - X} - tq_i \right] \tag{A1}$$

where superscript C denotes cartel. Differentiating with respect to appropriation q_i and solving, we obtain profit-maximizing appropriation for every firm i of

$$q_i^C(t) = \frac{(2-t)\left[\theta(1+g) - X\right]}{2(N-1)\left[b(\theta(1+g) - X) + \lambda\right]},$$

which yields second-period profits of $\pi_i(t) = \frac{(1-t)^2[\theta(1+g)-X]}{4[1+bN[\theta(1+g)-X]+(N-1)\lambda]}$.
Socially optimal appropriation in this setting coincides with that found in problem (4) in the main body of the paper, Q^{SO}. The optimal fee under cartel t^C, however, now solves $Q^{SO} = Q^C(t)$, where $Q^C(t) \equiv \sum_{i=1}^N q_i^C(t)$ denotes aggregate second-period cartel appropriation, which yields

$$t^C = \frac{N-1}{N} - \frac{b\left[\theta(1+g)-X\right]}{2+bN\left[\theta(1+g)-X\right]+2\lambda(N-1)}$$

entailing second-period profits for the entire cartel (evaluated at fee t^C) of

$$\pi^C(t^C) = N\frac{\left[\theta(1+g)-(x_i+X_{-i})\right]\left[1+bN(\theta(1+g)-(x_i+X_{-i})+\lambda(N-1))\right]}{N^2\left[2+bN\left[\theta(1+g)-(x_i+X_{-i})\right]+2\lambda(N-1)\right]^2},$$

so every firm i earns second-period profit $\frac{\pi^C(t^C)}{N}$.
In the first period, the cartel solves

$$\max_{x_1,\ldots,x_N\geq 0} \sum_{i=1}^{N}\left[\left[1-b(x_i+X_{-i})\right]x_i - \frac{x_i(x_i+\lambda X_{-i})}{\theta} + \delta\frac{\pi^C(t^C)}{N}\right] \tag{A2}$$

where profit $\pi^C(t^C)$ was defined above. As in the main body of the paper, differentiating for every firm's first-period appropriation x_i produces a highly non-linear equation, which does not allow for analytical solutions of the optimal x_i^C. For comparison purposes, Table VII in the main body of the paper evaluates $x_i^C(t^C)$, $x_i^C(0)$, and their difference (PR) at the same parameter values as Table I.

Appendix 4 - Allowing for environmental damages

In this appendix, we solve the sequential-move game again, but allowing for environmental damages, as presented in Section 4.2.

Second period. In the second period, every firm solves problem (3), thus yielding the same profit-maximizing appropriation $q_i(t)$ and second-period profits $\pi_i(t)$ as in Section 3.1. The regulator then solves a problem analogous to (4), but including the environmental damage from appropriation, as follows

$$\max_{Q} SW_2(q_i,q_{-i}) = CS_2(Q) + PS_2(q_i,q_{-i}) - d(Q+\gamma X)^2 \tag{A3}$$

Solving for socially optimal appropriation Q^{SO}, yields

$$Q^{SO} = \frac{N(1-2dX\gamma)\left[\theta-(1-\beta)X\right]}{2+(b+2d)N\left[\theta-(1-\beta)X\right]+2N\lambda(N-1)}.$$

When environmental damages are absent, $d=0$, this expression simplifies to the socially optimal appropriation we found in Section 3.1
Therefore, the optimal emission fee t^* solves $Q^{SO} = Q(t)$, where $Q(t) \equiv \sum_{i=1}^N q_i(t)$ denotes aggregate second-period appropriation. Solving for tax t, yields

$$t^* = \frac{4dX\gamma + 2dN\left[\theta(1+g)-X\right] + b(2d(N+1)X\gamma)(\theta(1+g)-X-1) + (N-1)(2N-1+2dX\gamma)\lambda}{2+(b+2d)N\left[\theta(1+g)-X\right]+(N-1)2N\lambda}$$

Differentiating the optimal emission fee t^* with respect to γ, we find

$$\frac{dt^*}{d\gamma} = \frac{2dX(2 + b(N+1)\left[\theta(1+g) - X\right] + (N-1)\lambda)}{2 + (b+2d)N\left[\theta(1+g) - X\right] + 2N\lambda(N-1)} > 0$$

which is positive since, in the numerator, term $[\theta(1+g) - X] + (N-1)\lambda$ is positive because $\theta > X$ and $(N-1)\lambda \geq 0$. Therefore, a larger damage persistence of first-period appropriation into the second period induces the regulator to set a more stringent emission fee t^*.

Differentiating the optimal emission fee t^* with respect to d, we obtain

$$\frac{\delta t^*}{\delta d} = \frac{2(2 + b(N+1)\left[\theta(1+g) - X\right] + (N-1)\lambda)(2X\gamma + N(1 + bX\gamma)\left[\theta(1+g) - X\right] + 2NX\gamma\lambda(N-1)}{\left[2 + (b+2d)N\left[\theta(1+g) - X\right] + 2N\lambda(N-1)\right]^2} >$$

which is positive since, in the numerator, $\theta > X$, and $(N-1)\lambda \geq 0$ and $2NX\gamma\lambda(N-1) \geq 0$, implying that the numerator is positive. The denominator is squared so it is also positive. As a consequence, a larger environmental damage from first- or second-period appropriation, d, leads the regulator to set a more stringent emission fee.

First period. In the first period, every firm i solves

$$\max_{x_i \geq 0} \; \left[1 - b(x_i + X_{-i})\right] x_i - \frac{x_i(x_i + \lambda X_{-i})}{\theta} + \delta\pi_i(t^*) \tag{A4}$$

which is analogous to problem (5) in Section 3.2, except for the fact that the firm's second-period equilibrium profit under regulation is now evaluated at a different emission fee t^* , that is,

$$\pi_i(t^*) = \frac{(1 - 2dX\gamma)^2\left[\theta(1+g) - X\right](1 + b\left[\theta(1+g) - X\right])}{\left[2 + (b+2d)N\left[\theta(1+g) - X\right] + 2N\lambda(N-1)\right]^2}.$$

Differentiating problem (A4) with respect to x_i, we obtain a rather intractable first-order condition which does not allow us to explicitly solve for x_i to obtain the equilibrium first-period appropriation. For comparison purposes, we evaluate the resulting first-order conditions at the same parameter values as Table I, yielding tables VIII and IX, as reported in Section 4.2.

Appendix 5 - Allowing for asymmetric firms

In this appendix, we examine how our results in Sections 3.1 and 3.2 are affected when we assume that firms are asymmetric in their production costs. In particular, we consider that firm i's costs are still given by $\frac{x_i(x_i+\lambda x_j)}{\theta}$ while firm j's are $\alpha\frac{x_j(x_j+\lambda x_i)}{\theta}$, where parameter $\alpha \in [0,1]$ represents firm j's cost advantage relative to firm i. When $\alpha = 1$, firm i and j exhibit the same cost function, as in the main body of the paper, while when $\alpha < 1$ firm j benefits from a cost advantage. A similar argument applies to second-period costs. For simplicity, we consider only two firms.

Second stage

Second-period appropriation. In the second period, every firm i solves

$$\pi_i(t) \equiv \max_{q_i \geq 0} \left[1 - b\left(q_i + q_j\right)\right] q_i - \frac{q_i(q_i + \lambda q_j)}{\theta(1+g) - X} - tq_i \tag{A5}$$

and similarly for firm j, which solves

$$\pi_j(t) \equiv \max_{q_j \geq 0} \left[1 - b\left(q_i + q_{-i}\right)\right] q_j - \alpha \frac{q_j(q_j + \lambda q_i)}{\theta(1+g) - X} - tq_j. \tag{A6}$$

Differentiating with respect to appropriation q_i in A5, with respect to appropriation q_j in A6, and simultaneously solving for q_i and q_j, we obtain

$$\begin{aligned} q_i(t) &= \frac{(1-t)\left[\theta(1+g) - X\right]\left[2\alpha - \lambda + b\left[\theta(1+g) - X\right]\right]}{3b^2\left[\theta(1+g) - X\right]^2 + b(4-\lambda)(1+\alpha)\left[\theta(1+g) - X\right] + \alpha(4-\lambda^2)}, \text{ and} \\ q_j(t) &= \frac{(1-t)\left[\theta(1+g) - X\right]\left[2 - \alpha\lambda + b\left[\theta(1+g) - X\right]\right]}{3b^2\left[\theta(1+g) - X\right]^2 + b(4-\lambda)(1+\alpha)\left[\theta(1+g) - X\right] + \alpha(4-\lambda^2)}. \end{aligned}$$

In the case that firms face symmetric costs, $\alpha = 1$, the above equilibrium appropriations collapse to

$$q_i(t) = q_j(t) = \frac{(1-t)\left[\theta(1+g) - X\right]}{2 + 3b\left[\theta(1+g) - X\right] + \lambda},$$

which coincides with our equilibrium appropriation in Section 3.1, evaluated at $N = 2$ firms. Second-period profits for firm i are,

$$\pi_i(t) = \frac{(1-t)^2\left[\theta(1+g) - X\right]\left[1 + b(\theta(1+g) - X)\right]\left[2\alpha - \lambda + b\left[\theta(1+g) - X\right]\right]^2}{\left[3b^2\left[\theta(1+g) - X\right]^2 + b(4-\lambda)(1+\alpha)\left[\theta(1+g) - X\right] + \alpha(4-\lambda^2)\right]^2}$$

while those of firm j are.

$$\pi_j(t) = \frac{(1-t)^2\left[\theta(1+g) - X\right]\left[\alpha + b(\theta(1+g) - X)\right]\left[2 - \alpha\lambda + b\left[\theta(1+g) - X\right]\right]^2}{\left[3b^2\left[\theta(1+g) - X\right]^2 + b(4-\lambda)(1+\alpha)\left[\theta(1+g) - X\right] + \alpha(4-\lambda^2)\right]^2}.$$

Optimal fee. The socially optimal appropriation solves

$$\max_{q_i, q_j} SW_2(q_i, q_j) = CS_2(q_i, q_j) + PS_2(q_i, q_j) \tag{A7}$$

where $CS_2(q_i, q_j) = \frac{1}{2}b(q_i + q_j)^2$ denotes second-period consumer surplus, and producer surplus is

$$PS_2(q_i, q_j) = \left(\left[1 - b\left(q_i + q_j\right)\right] q_i - \frac{q_i(q_i + \lambda q_j)}{\theta(1+g) - X}\right) + \left(\left[1 - b\left(q_i + q_{-i}\right)\right] q_j - \alpha \frac{q_j(q_j + \lambda q_i)}{\theta(1+g) - X}\right).$$

Differentiating problem (A7) with respect to q_i and q_j, and then simultaneously solving for q_i and q_j, we find socially optimal appropriation levels for each firm

$$\begin{aligned} q_i^{SO} &= \frac{\left[\theta(1+g) - X\right]\left[\alpha(2-\lambda) - \lambda\right]}{4\alpha + 2b(1+\alpha)(1-\lambda)\left[\theta(1+g) - X\right] - (1+\alpha)^2\lambda^2}, \text{ and} \\ q_j^{SO} &= \frac{\left[\theta(1+g) - X\right]\left[2 - \lambda(1+\alpha)\right]}{4\alpha + 2b(1+\alpha)(1-\lambda)\left[\theta(1+g) - X\right] - (1+\alpha)^2\lambda^2}. \end{aligned}$$

In the case that firms face symmetric costs, $\alpha = 1$, the above socially optimal appropriation collapse to

$$q_i^{SO} = q_j^{SO} = \frac{\theta(1+g) - X}{2\left[1 + \lambda + b\left[\theta(1+g) - X\right] + \lambda\right]},$$

which coincides with our results in Section 3.1, i.e., $q_i^{SO} = \frac{Q^{SO}}{N}$, when evaluated at $N = 2$ firms.

Therefore, the optimal fee t_i^* for firm i solves $q_i^{SO} + q_j^{SO} = q_i(t) + q_j(t)$, as found above. Intuitively, fee t induces firms to produce an aggregate appropriation that coincides with the aggregate socially optimal appropriation. For compactness, we do not include here the expression of the optimal fee, but it can be provided by the authors upon request.

First period

In the first period, every firm i solves

$$\max_{x_i \geq 0} \quad \left[1 - b(x_i + x_j)\right] x_i - \frac{x_i(x_i + \lambda x_j)}{\theta} + \delta \pi_i(t^*) \tag{A8}$$

where $\delta \in [0,1]$ denotes the discount factor. The profit function in problem (A8), $\pi_i(t^*)$ —the value function of firm i's second-period problem— is evaluated at the optimal fee t^* found above, since the firm can anticipate the fee effective in the subsequent stage of the game, that is,

$$\pi_i(t^*) = \frac{\left[\theta(1+g) - (x_i + x_j)\right]\left[1 + b(\theta(1+g) - (x_i + x_j))\right]\left[\alpha(2-\lambda) - \lambda\right]^2}{\left[4\alpha + 2b(1+\alpha)(1-\lambda)\left[\theta(1+g) - (x_i + x_j)\right] - (1+\alpha)^2\lambda^2\right]^2}.$$

A similar argument applies for firm j, which solves

$$\max_{x_j \geq 0} \quad \left[1 - b(x_i + x_j)\right] x_j - \alpha \frac{x_j(x_j + \lambda x_i)}{\theta} + \delta \pi_j(t_j^*) \tag{A9}$$

where $\pi_j(t^*)$ denotes firm j's value function in the second-period problem, which simplifies to

$$\pi_j(t^*) = \frac{\left[\theta(1+g) - (x_i + x_j)\right]\left[\alpha + b(\theta(1+g) - (x_i + x_j))\right]\left[2 - \lambda(1+\alpha)\right]^2}{\left[4\alpha + 2b(1+\alpha)(1-\lambda)\left[\theta(1+g) - (x_i + x_j)\right] - (1+\alpha)^2\lambda^2\right]^2}.$$

Appendix 6 - Proofs

Proof of Lemma 1

Differentiating problem (3) with respect to appropriation q_i and solving, we obtain the best response function

$$q_i(q_{-i}) = \frac{(1-t)\left[\theta(1+g) - X\right]}{2\left[1 + b\left[\theta(1+g) - X\right]\right]} - \frac{b\left[\theta(1+g) - X\right] + \lambda}{2\left[1 + b\left[\theta(1+g) - X\right]\right]} q_{-i}$$

which is decreasing in the aggregate appropriation by firm i's rivals, q_{-i}. In a symmetric equilibrium, $q_i = q_j$ for all $j \neq i$, we obtain a profit-maximizing appropriation

$$q_i(t) = q_j(t) = \frac{(1-t)\left[\theta(1+g) - X\right]}{2 + b(N+1)\left[\theta(1+g) - X\right] - (N-1)\lambda}.$$

The above equilibrium appropriation yields second-period equilibrium profits of

$$\begin{aligned}\pi_i(t) &= \pi_j(t) = \left[1 - b\left(q_i(t) + (N-1)q_j(t)\right)\right] q_i(t) - \frac{q_i(t)(q_i(t) + \lambda(N-1)q_j(t))}{\theta(1+g) - X} - tq_i(t) \\ &= \frac{(1-t)^2 \left[\theta(1+g) - X\right]\left[1 + b\left[\theta(1+g) - X\right]\right]}{\left[2 + b(N+1)\left[\theta(1+g) - X\right] - (N-1)\lambda\right]^2}.\end{aligned}$$

Proof of Lemma 2

Differentiating with respect to aggregate appropriation Q in problem (4), and solving for Q, we obtain a socially optimal appropriation

$$Q^{SO} = \frac{N\left[\theta(1+g) - X\right]}{2 + bN\left[\theta(1+g) - X\right] + 2(N-1)\lambda}$$

where comparative statics satisfy

$$\frac{\partial Q^{SO}}{\partial X} = -\frac{2N\left[(N-1)\lambda + 1\right]}{\left[2 + bN\left[\theta(1+g) - X\right] + 2(N-1)\lambda\right]^2}$$

which is negative since $N > 1$ by assumption. The derivative with respect to the number of firms competing for the resource, N, is

$$\frac{\partial Q^{SO}}{\partial N} = \frac{2\left[\theta(1+g) - X\right](1-\lambda)}{\left[2 + bN\left[\theta(1+g) - X\right] + 2(N-1)\lambda\right]^2}$$

which is negative if $\lambda > 1$. The derivative with respect to the growth rate, g, is unambiguously positive since

$$\frac{\partial Q^{SO}}{\partial g} = \frac{2N\theta\left[(N-1)\lambda + 1\right]}{\left[2 + bN\left[\theta(1+g) - X\right] + 2(N-1)\lambda\right]^2}$$

and the derivative with respect to the initial stock, θ, is also positive since

$$\frac{\partial Q^{SO}}{\partial \theta} = \frac{2N(1+g)\left[(N-1)\lambda + 1\right]}{\left[2 + bN\left[\theta(1+g) - X\right] + 2(N-1)\lambda\right]^2}$$

given that $N > 1$, whereas the derivative with respect to the cost externality, λ, is negative since

$$\frac{\partial Q^{SO}}{\partial \lambda} = -\frac{2N(N-1)\left[\theta(1+g) - X\right]}{\left[2 + bN\left[\theta(1+g) - X\right] + 2(N-1)\lambda\right]^2}.$$

Proof of Proposition 1

The derivative of fee t^* with respect to X is

$$\frac{\partial t^*}{\partial X} = \frac{b\left[2 + \lambda(N^2 + N - 2)\right]}{\left[2\lambda(N-1) + 2 + bN\left[\theta(1+g) - X\right]\right]^2},$$

which is positive since $N > 1$ by definition. Similarly,

$$\frac{\partial t^*}{\partial N} = \frac{2\lambda + b\left[\theta(1+g) - X\right]\left[b\left[\theta(1+g) - X\right] + 3\lambda\right]}{\left[2\lambda(N-1) + 2 + bN\left[\theta(1+g) - X\right]\right]^2}$$

is also positive since $\theta > X$ (i.e., exploitation does not exceed the available stock). In contrast,

$$\frac{\partial t^*}{\partial g} = -\frac{b\theta\left[2 + \lambda(N^2 + N - 2)\right]}{\left[2\lambda(N-1) + 2 + bN\left[\theta(1+g) - X\right]\right]^2},$$

$$\frac{\partial t^*}{\partial \theta} = -\frac{b(1+g)\left[2 + \lambda(N^2 + N - 2)\right]}{\left[2\lambda(N-1) + 2 + bN\left[\theta(1+g) - X\right]\right]^2},$$

and

$$\frac{\partial t^*}{\partial \lambda} = -\frac{(N-1)\left[2 + b(N+2)\left[\theta(1+g) - X\right]\right]}{\left[2\lambda(N-1) + 2 + bN\left[\theta(1+g) - X\right]\right]^2}$$

are all negative by definition since $N > 1$.