

Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 14: Imperfect Competition

Outline

- Summary of Market Structures
- Measuring Market Power
- Models of Imperfect Competition
 - Cournot Model – Simultaneous Quantity Competition
 - Bertrand Model – Simultaneous Price Competition
 - Cartels and Collusion
 - Stackelberg Model – Sequential Quantity Competition
- Product Differentiation
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Summary of Market Structures

Table 14.1

Industry	N of firms	Type of Good	Price-takers?	Entry Barriers?
Perfect competition	Many	Homogeneous	Yes	No
Monopoly	One	No close substitutes	No	Yes
Oligopoly	Some	Homogeneous or heterogeneous	No	Yes

Measuring Market Power

Measuring Market Power

- A common measure of market power is the number of firms in an industry, $N \geq 1$.
 - It does not inform about market shares.
 - *Example:*
 - Consider two industries, A and B , with $N = 3$ firms each.
 - In industry A , one of the firms enjoys a 98% market share.
 - In industry B , market share is evenly distributed, each firm holds 33.33%.
- The **Herfindahl-Hirschman index (HHI) of market concentration** accounts for both the number of firms and their market shares.

Measuring Market Power

- The **HHI** is given by

$$HHI = (s_1)^2 + (s_2)^2 + \dots (s_N)^2,$$

where s_1 represents the market share of firm 1 (in %), s_2 is that of firm 2, and similarly for all remaining N firms in the industry.

- In a monopoly, in which a single firm captures the entire market share, $s_1 = 100$,

$$HHI = (100)^2 = 10,000.$$

- In a duopoly, with two firms evenly sharing market power,

$$HHI = (50)^2 + (50)^2 = 5,000.$$

Measuring Market Power

- In an oligopoly, with 1,000 firms, each capturing $\frac{1}{1,000}$ of the market share,

$$\begin{aligned} HHI &= \left(\frac{1}{1,000}\right)^2 + \left(\frac{1}{1,000}\right)^2 + \dots + \left(\frac{1}{1,000}\right)^2 \\ &= 1,000 \left(\frac{1}{1,000}\right)^2 = 0.001. \end{aligned}$$

- Generally, in an industry with $N \geq 1$, with $s_i = \frac{1}{N}$,

$$\begin{aligned} HHI &= \left(\frac{1}{N}\right)^2 + \left(\frac{1}{N}\right)^2 + \dots + \left(\frac{1}{N}\right)^2 \\ &= N \left(\frac{1}{N}\right)^2 = \frac{1}{N}, \end{aligned}$$

which converges to zero when N is sufficiently large.

Measuring Market Power

- The HHI ranges from 10,000 to 0.
 - A high HHI arises in highly concentrated industries.
 - A low HHI emerges when market power is more evenly distributed.
- *Examples:*
 - US light bulb market, with around 57 firms,
 - $HHI = 2,757$. Some of these firms enjoy a large market share.
 - Glass container manufacturing, with 22 firms,
 - $HHI = 2,582$. Market shares are more evenly split among firms (i.e., the market is less concentrated).

Models of Imperfect Competition

Models of Imperfect Competition

- Consider a market with $N \geq 2$ firms, all of them selling a relatively homogeneous product (e.g., brands of unflavored water).
- In this scenario, we consider three models of firm competition:
 - (1) Cournot model of simultaneous quantity competition.
 - (2) Bertrand model of simultaneous price competition.
 - (3) Stackelberg model of sequential quantity competition.

Cournot Model– Simultaneous Quantity Competition

Cournot Model

- Consider an industry with $N = 2$ firms selling a homogeneous product.
- Every firm independently and simultaneously chooses its profit maximizing output (q_1 for firm 1 and q_2 for firm 2).
- The market price is determined by inserting q_1 and q_2 into the inverse demand function $p(q_1, q_2)$. Assume this function is linear, $p(q_1, q_2) = a - b(q_1 + q_2)$, where $a, b > 0$.
- Firm 1's total cost function is $TC_1(q_1) = cq_1$, where $c > 0$.
- Firm 2's total cost function is symmetric, $TC_2(q_2) = cq_2$.

Cournot Model

Firm 1. Its PMP is to choose q_1 to solve

$$\begin{aligned}\max_{q_1} \pi_1 &= TR_1 - TC_1 = \underbrace{p(q_1, q_2)q_1}_{TR_1} - \underbrace{cq_1}_{TC_1} \\ &= [a - b(q_1 + q_2)]q_1 - cq_1,\end{aligned}$$

where $TR_1 = p(q_1, q_2)q_1$ denotes total revenue (price per units sold), and $TC_1 = cq_1$ is its total cost.

- To maximize its profits, firm 1 differentiate this expression with respect to q_1 ,

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c = 0.$$

Rearranging and solving for q_1 ,

Cournot Model

$$\begin{aligned} a - c - bq_2 &= 2bq_1, \\ q_1(q_2) &= \frac{a - c}{2b} - \frac{1}{2}q_2, \end{aligned} \quad (BRF_1)$$

which is referred to as firm 1's “best response function.”

- The best response function describes the profit maximizing output that firm 1 chooses as a response to each of the output levels that firm 2 selects.

- If $a = 10$, $b = 1$, and $c = 2$, firm 1's best response function becomes

$$q_1(q_2) = \frac{10 - 2}{2 \times 1} - \frac{1}{2}q_2 = 4 - \frac{1}{2}q_2.$$

- If firm 2 produces $q_2 = 3$ units, firm 1 responds with

$$q_1(3) = 4 - \frac{1}{2}3 = 2.5 \text{ units.}$$

Cournot Model

- Firm 1's best response function, $q_1(q_2) = \frac{a-c}{2b} - \frac{1}{2}q_2$.
 - It originates at $\frac{a-c}{2b}$ on the vertical axis when firm 2 chooses $q_2 = 0$.
 - It decreases with a slope of $-1/2$ for every unit of q_2 .
 - When $q_1 \left(\frac{a-c}{b} \right) = \frac{a-c}{2b} - \frac{1}{2} \frac{a-c}{b} = 0$ units.

As firm 2 increases q_2 , firm 1 is left with a smaller residual demand to serve.

When $q_2 \geq \frac{a-c}{b}$, firm 1 shut down, producing $q_1 = 0$.

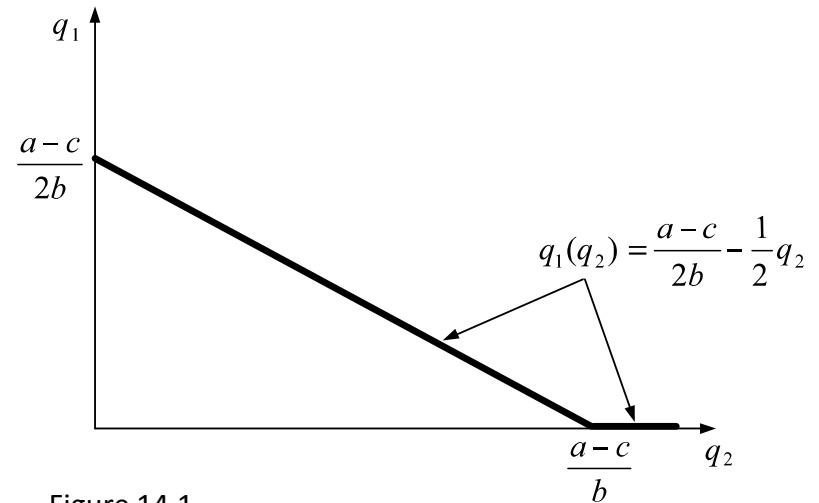


Figure 14.1

Cournot Model

Firm 2. A similar argument applies to firm 2, which solves

$$\begin{aligned}\max_{q_2} \pi_2 &= TR_2 - TC_2 = \underbrace{p(q_1, q_2)q_2}_{TR_2} - \underbrace{cq_2}_{TC_2} \\ &= [a - b(q_1 + q_2)]q_2 - cq_2.\end{aligned}$$

- Differentiating with respect to q_2 ,

$$\frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - c = 0.$$

Rearranging and solving for q_2 , we find firm 2's best response function,

$$\begin{aligned}a - c - bq_1 &= 2bq_2, \\ q_2(q_1) &= \frac{a - c}{2b} - \frac{1}{2}q_1.\end{aligned}\tag{BRF_2}$$

Cournot Model

- Firm 2's best response function, $q_2(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1$, is symmetric to that of firm 1 because both face the same demand and costs.
 - It originates at $\frac{a-c}{2b}$ when firm 1 is inactive but it decreases at a rate of 1/2 as firm 1 increases its production.

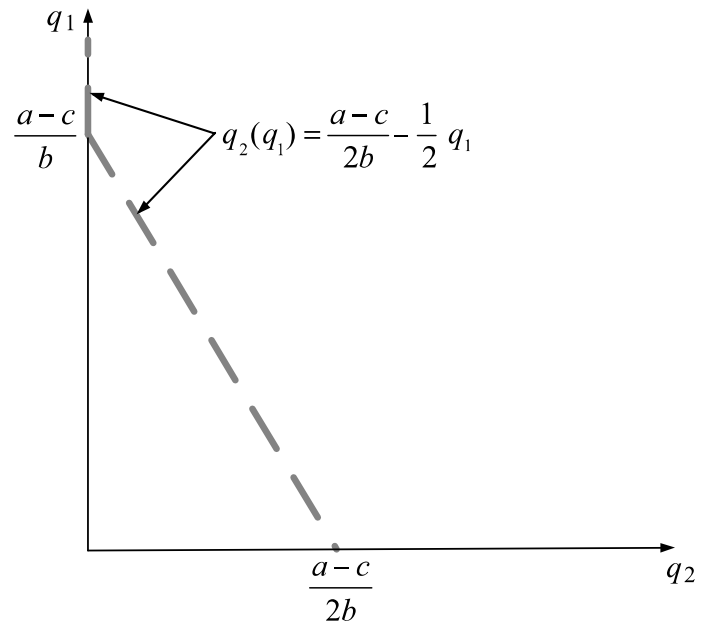


Figure 14.2

Cournot Model

- Superimposing firm 1's and firm 2's best response functions, we obtain their crossing point: Cournot Equilibrium.

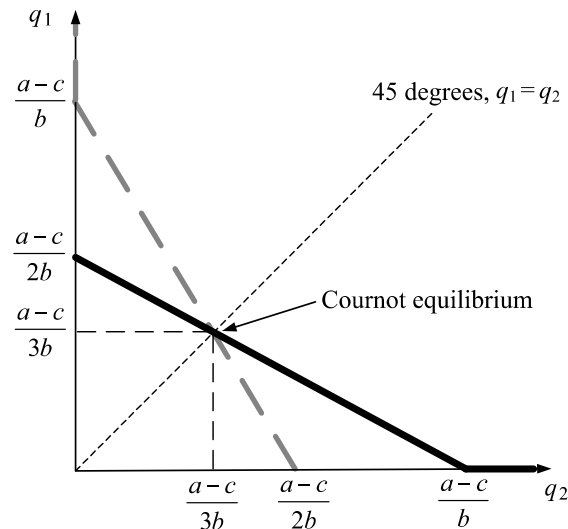


Figure 14.3

- Both firms are choosing output levels that are the best response to the output of its rival (i.e., firms are selecting *mutual* best responses, which is the Nash Equilibrium (NE) of a game).

Cournot Model

- To find the point where the best response functions cross each other, we can insert BRF_2 into BRF_1 ,

$$q_1 = \frac{a - c}{2b} - \frac{1}{2} \underbrace{\left(\frac{a - c}{2b} - \frac{1}{2} q_1 \right)}_{q_2},$$

- Rearranging and solving for q_1 , we find q_1^* ,

$$\begin{aligned} \frac{3}{4} q_1 &= \frac{a - c}{2b}, \\ q_1^* &= \frac{a - c}{3b}. \end{aligned}$$

Cournot Model

- Inserting this output level into BRF_1 , we find q_2^* ,

$$\begin{aligned}q_2 \left(\frac{a - c}{3b} \right) &= \frac{\overbrace{a - c}^{q_1^*}}{2b} - \frac{1}{2} \frac{a - c}{3b} \\ &= \frac{3(a - c) - (a - c)}{6b}, \\ q_2^* &= \frac{a - c}{3b}.\end{aligned}$$

Cournot Model

- The output pair $(q_1^*, q_2^*) = \left(\frac{a-c}{3b}, \frac{a-c}{3b}\right)$ is the Nash Equilibrium of the Cournot game.

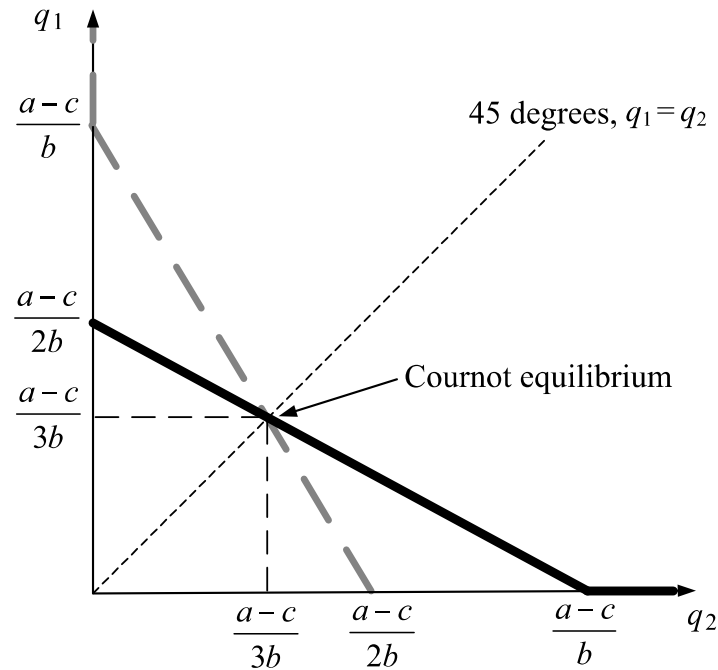


Figure 14.3

Cournot Model

- An alternative approach to solve for the equilibrium output is to **invoke symmetry**.
- Because firms are symmetric in their revenues and costs, we can claim that there must be a symmetric equilibrium where

$$q_1^* = q_2^* = q^*.$$

- Inserting this property into either firm's BRF,

$$q^* = \frac{a - c}{2b} - \frac{1}{2}q^*,$$
$$\frac{3}{2}q^* = \frac{a - c}{2b},$$
$$q^* = \frac{a - c}{3b}.$$

Cournot Model

- We find equilibrium price by evaluating the inverse demand function

$$p(q_1, q_2) = a - b(q_1 + q_2)$$

$$\text{at } q_1^* = q_2^* = \frac{a-c}{3b},$$

$$\begin{aligned} p^* \left(\frac{a-c}{3b}, \frac{a-c}{3b} \right) &= a - b \left(\frac{a-c}{3b} + \frac{a-c}{3b} \right) \\ &= a - \frac{2(a-c)}{3} = \frac{a+2c}{3}. \end{aligned}$$

Cournot Model

- Finally, equilibrium profits for every firm $i = \{1,2\}$ are

$$\begin{aligned}\pi_i^* &= p^* q_i^* - c q_i^* = \left(\frac{a+2c}{3}\right) \frac{a-c}{3b} - c \frac{a-c}{3b} \\ &= \frac{(a+2c)(a-c)}{9b} - \frac{3c(a-c)}{9b} \\ &= \frac{a^2 - 2ac + c^2}{9b},\end{aligned}$$

or, more compactly,

$$\pi_i^* = \frac{(a-c)^2}{9b}$$

because $(a-c)^2 = a^2 - 2ac - c^2$.

It can be alternatively expressed as $\pi_i^* = (q^*)^2$.

Cournot Model

- *Example 14.1: Cournot model with symmetric costs.*
 - Consider a duopoly with $p(q_1, q_2) = 12 - q_1 - q_2$, where every firm $i = \{1, 2\}$ faces a symmetric cost function $TC_i(q_i) = 4q_i$.
 - *Firm 1's best response function.* Firm 1 chooses q_1 to solve

$$\max_{q_1} \pi_1 = (12 - q_1 - q_2)q_1 - 4q_1.$$

Differentiating with respect to q_1 ,

$$\frac{\partial \pi_1}{\partial q_1} = 12 - 2q_1 - q_2 - 4 = 0.$$

Rearranging and solving for q_1 ,

$$\begin{aligned} 8 - q_2 &= 2q_1, \\ q_1(q_2) &= 4 - \frac{1}{2}q_2. \end{aligned} \quad (BRF_1)$$

Cournot Model

- *Example 14.1* (continued):

- *Firm 2's best response function.* Firm 2 chooses q_2 to solve

$$\max_{q_2} \pi_2 = (12 - q_1 - q_2)q_2 - 4q_2.$$

Differentiating with respect to q_2 ,

$$\frac{\partial \pi_2}{\partial q_2} = 12 - q_1 - 2q_2 - 4 = 0.$$

Rearranging and solving for q_1 ,

$$\begin{aligned} 8 - q_1 &= 2q_2, \\ q_2(q_1) &= 4 - \frac{1}{2}q_1, \end{aligned} \quad (BRF_2)$$

which is symmetric to that of firm 1.

Cournot Model

- *Example 14.1* (continued):
 - *Finding equilibrium output.*

We can invoke symmetry, and claim

$$q_1^* = q_2^* = q^*.$$

Inserting this property into either firm's best response function, and solving for q^* ,

$$\begin{aligned} q^* &= 4 - \frac{1}{2}q^*, \\ \frac{3}{2}q^* &= 4 \quad \Rightarrow \quad q^* = \frac{8}{3}. \end{aligned}$$

Cournot Model

- *Example 14.1* (continued):
 - *Finding equilibrium output* (cont.).

Equilibrium price is

$$p^* \left(\frac{8}{3}, \frac{8}{3} \right) = 12 - q^* - q^* = 12 - \frac{8}{3} - \frac{8}{3} = \frac{20}{3} \cong \$6.67,$$

producing for every firm $i = \{1,2\}$ equilibrium profits of

$$\pi_i^* = p^* q^* - c q^* = \left(\frac{20}{3} \right) \frac{8}{3} - 4 \frac{8}{3} = \frac{160}{9} - \frac{96}{9} = \frac{64}{9}.$$

Cournot Model

- *Example 14.2: Cournot model with asymmetric costs.*
 - Consider two firms competing á la Cournot, facing the same inverse demand as in example 14.1,

$$p(q_1, q_2) = 12 - q_1 - q_2,$$

but different cost functions

$$TC_1(q_1, q_2) = 4q_1,$$

$$TC_2(q_1, q_2) = 3q_2.$$

Cournot Model

- *Example 14.2* (continued):

- *Firm 1's best response.*

Firm 1's PMP is

$$\max_{q_1} \pi_1 = (12 - q_1 - q_2)q_1 - 4q_1.$$

This problem coincides with the one in example 14.1, yielding the same best response function,

$$q_1(q_2) = 4 - \frac{1}{2}q_2.$$

Cournot Model

- *Example 14.2* (continued):

- *Firm 2's best response.* Firm 2's PMP is

$$\max_{q_2} \pi_2 = (12 - q_1 - q_2)q_2 - 3q_2.$$

Differentiating with respect to q_2 ,

$$\frac{\partial \pi_2}{\partial q_2} = 12 - q_1 - 2q_2 - 3 = 0.$$

Rearranging and solving for q_2 , yields

$$9 - q_1 = 2q_2 \Rightarrow q_2(q_1) = \frac{9}{2} - \frac{1}{2}q_1.$$

This function has the same slope as that in example 14.1, but it originates at $9/2$ rather than at 4. This indicates that, for every output of firm 1, firm 2's output is now largest because its marginal cost is 3 rather than 4.

Cournot Model

- *Example 14.2* (continued):
 - *Finding equilibrium output.* We cannot invoke symmetry because firms face different production costs. Inserting BRF_2 into BRF_1 , and solving for q_1 ,

$$q_1 = 4 - \frac{1}{2} \left(\frac{9}{2} - \frac{1}{2} q_1 \right),$$
$$q_1 = 4 - \frac{9}{4} + \frac{1}{4} q_1,$$
$$\frac{3}{4} q_1 = \frac{7}{4} \implies q_1^* = \frac{7}{3} \cong 2.33.$$

Inserting this result into BRF_2 ,

$$q_2^* = \frac{9}{2} - \frac{1}{2} \frac{7}{3} = \frac{10}{3} \cong 3.33,$$

where $q_2^* > q_1^*$ because firm 2's marginal cost is lower.

Cournot Model

- *Example 14.2* (continued):

In this scenario, equilibrium price and equilibrium profits are

$$p^* = \frac{19}{3},$$
$$\pi_1^* = \frac{49}{9},$$
$$\pi_2^* = \frac{100}{9}.$$

Firm 2, which is benefiting from a cost advantage, earns a larger profit than firm 1 which suffers from a cost disadvantage.

Bertrand Model– Simultaneous Price Competition

Bertrand Model

- Two symmetric firms produce a homogeneous good and face common marginal cost, $c > 0$.
- They simultaneously and indecently set prices p_1 and p_2 .
 - If $p_1 < p_2$, firm 1 captures all the demand, while firm 2 captures none:
$$x_1(p_1, p_2, I) > 0,$$
$$x_2(p_1, p_2, I) = 0.$$
 - If $p_1 > p_2$, firm 2 captures all demand.
 - If $p_1 = p_2$, both firms equally share market demand:
$$\frac{1}{2}x_1(p_1, p_2, I) > 0,$$
$$\frac{1}{2}x_2(p_1, p_2, I) > 0.$$

Bertrand Model

- The Bertrand model of price competition claims that, in equilibrium:

$$p_1 = p_2 = c.$$

- To show this result, we next demonstrate that all possible price pairs (p_1, p_2) that are different from $(p_1, p_2) = (c, c)$, cannot be equilibria.

Bertrand Model

- We need to show that any price different than the marginal cost, c , is “unstable” in the sense that at least one firm has incentives to deviate to a different price.
- We examine:
 1. Asymmetric price profiles, where $p_1 \neq p_2$.
 2. Symmetric price profiles, where $p_1 = p_2$.

Bertrand Model

1. *Asymmetric price profiles.*

(a) Consider $p_1 > p_2 > c$.

- Firm 2 sets the lowest price and captures the entire market by making a positive margin because $p_2 > c$.
- This profile cannot be stable because firm 1 has incentives to deviate undercutting firm 2's prices by charging $p'_1 = p_2 - \varepsilon$, where ε indicates a small reduction in firm 2's price.

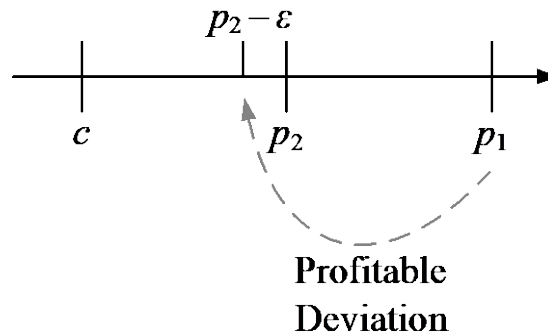


Figure 14.4

Bertrand Model

1. *Asymmetric price profiles* (cont.).

(b) Consider $p_1 > p_2 = c$.

- Firm 2 captures the entire market, but it makes no profit per unit.
- Firm 1 would not have incentives to undercut firm 2's price that would entail charging a price below c , incurring in a per unit cost.
- Instead, firm 2 would have incentives to deviate by increasing its price from $p_2 = c$ to slightly below its rival's price, $p'_2 = p_1 - \varepsilon$, and make a higher profit.

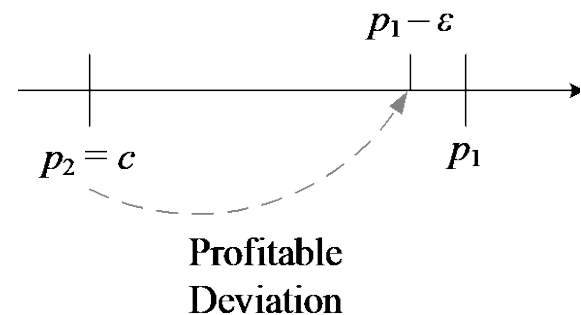


Figure 14.5

Bertrand Model

2. Symmetric price profiles.

(a) Consider $p_1 = p_2 > c$.

- Both firms evenly share the market because their prices are the same.
- Every firm i has the incentive to deviate by undercutting its rival's price p by a small amount ε , by charging $p'_i = p - \varepsilon$.

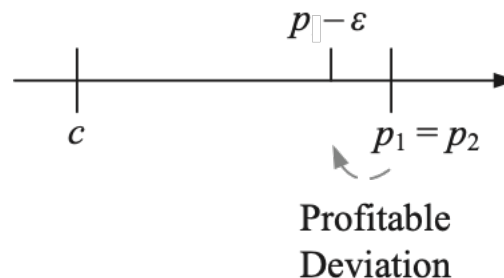


Figure 14.6

Bertrand Model

2. *Symmetric price profiles (cont.)*

(b) Consider $p_1 = p_2 = c$.

- Prices coincide, leading firms to evenly share the market.
- These prices leave no positive margin per unit because $p_i = c$ for every firm i .
- No firm can strictly increase its payoff by unilaterally deviating:
 - A lower price would attract all consumers, but at a lower per unit loss.
 - A higher price would reduce the deviating firm's sales to zero.

We can claim that setting $p_i = c$ is a weakly dominant strategy in the Bertrand model of price competition because no firm can strictly increase its profit by deviating from such a price.

Bertrand Model

- *Example 14.3: Bertrand model.*

- Consider the inverse demand function in example 14.1,
 $p(q_1, q_2) = 12 - q_1 - q_2$.
- Because $Q \equiv q_1 + q_2$ denotes the aggregate output in the industry, the inverse demand can be expressed as

$$p(Q) = 12 - Q.$$

- In the Bertrand model of price competition, all firms in the industry lower their prices until

$$p = c \implies 12 - Q = c.$$

Solving for Q , $Q = 12 - c$.

- If $c = 4$, $Q = 12 - 4 = 8$ units, each of which sold at a price of \$4.

Reconciling the Cournot and Bertrand models

- *Why are the results in the Cournot model and Bertrand model so dramatically different?*
 - In the Cournot model,
 - firms set a price above marginal cost, making a positive profit.
 - In the Bertrand model,
 - firms set $p = c$, earning no economic profits.

Reconciling the Cournot and Bertrand models

- These differences are driven by the absence of capacity constraints in the Bertrand model:
 - If a firm charges 1 cent less than its rival, it captures the market demand, regardless of its size.
- This assumption might be reasonable for goods such as online movie streaming
 - but difficult to justify with others (e.g., smartphones) with a world demand that cannot be served by a single firm.

Cartels and Collusion

Cartels and Collusion

- Firms competing in quantities can earn profits below those under monopoly, which is emphasized when firms compete in prices.
- *What if, rather than competing, firms were to coordinate their production decisions?*
- We analyze how **collusion** can help firms increase their profits, and under which conditions cooperation holds.
- **Cartels** seek to coordinate production decisions to raise profits and profits for participants.
 - In a cartel firms seek to maximize their *joint* rather than their individual profits.
 - *Example: OPEC.*

Cartels and Collusion

- *Example 14.4: Collusion when firms compete in quantities.*

- Consider the industry in example 14.2, where

$$p(q_1, q_2) = 12 - q_1 - q_2,$$

$$TC_i(q_i) = 4q_i \text{ for every firm } i.$$

- If firms join a cartel, they choose q_1 and q_2 to maximize their joint profits, $\pi = \pi_1 + \pi_2$ as follows:

$$\max_{q_1, q_2} \pi = \underbrace{(12 - q_1 - q_2)q_1 - 4q_1}_{\pi_1} + \underbrace{(12 - q_1 - q_2)q_2 - 4q_2}_{\pi_2}.$$

Cartels and Collusion

- *Example 14.4* (continued):

- The previous expression can be simplified as

$$\max_{q_1, q_2} (12 - q_1 - q_2)(q_1 + q_2) - 4(q_1 + q_2),$$

$$\max_{q_1, q_2} [12 - (q_1 + q_2)](q_1 + q_2) - 4(q_1 + q_2).$$

Cartels and Collusion

- *Example 14.4* (continued):

- Because $Q = q_1 + q_2$ denotes aggregate output, we can rewrite the cartel's PMP as it were a single monopolist,

$$\max_{q_1, q_2} [12 - Q]Q - 4Q.$$

- Differentiating with respect to Q ,

$$12 - 2Q - 4 = 0.$$

Cartels and Collusion

- *Example 14.4* (continued):

- Solving for Q ,

$$Q^* = \frac{8}{2} = 4 \text{ units.}$$

- Because firms are symmetric, each produces $q_i = \frac{Q^*}{2} = 2$ units.
- In contrast, under Cournot competition, every firm produces $q = \frac{8}{3} \cong 2.66$ units.

Cartels and Collusion

- *Example 14.4* (continued):

- Under cartel, every firm limits its own production to increase market price and profits.
- We confirm this result by finding that the cartel price is

$$p(2,2) = 12 - 2 - 2 = \$8,$$

which is higher than under Cournot competition (\$6.67).

- The cartel profits for every firm i are

$$\pi_i^* = (12 - q_1 - q_2)q_i - 4q_i = (12 - 2 - 2)2 - (4 \times 2) = \$8,$$

while under Cournot competition, $\pi_i^* = \frac{64}{9} \cong \7.11 .

Cartels and Collusion

- *Why are cartel profits larger than under Cournot competition?*
 - Under Cournot, when every firm increases its output, it considers the effect of such additional production has in its own profits, but it ignores the effect on its rival's profit.
 - Under the cartel, firms take into account each other's benefits. Firms produce less but elevate market prices and increase profits.
- We next identify the conditions to sustain collusion over time.
 - If firms interact only once, cooperation cannot be sustained in equilibrium.
 - If firms interact infinitely (there is a probability that firms will be in the industry tomorrow), cooperation can be sustained.

Cartels and Collusion

- *Example 14.5: Sustaining cooperation within the cartel.*
 - Assume that firms play an infinitely repeated Cournot game, and they seek to coordinate their production decisions through the following Grim-Trigger Strategy (GTS):
 1. In $t = 1$, every firm starts cooperating (producing 2 units).
 2. In $t > 1$,
 - (a) Every firm continues cooperating, so long as all firms cooperated in all previous periods.
 - (b) If, instead, it observes some past cheating (deviating this GTS), then it produces the Cournot output $q^* = \frac{8}{3}$ hereafter.

Cartels and Collusion

- *Example 14.5* (continued):

- We only need to check if every firm has incentives to deviate from the GTS: (1) after observing a history of cooperation; and (2) after observing that some firm/s cheated.
- *Cooperation.* If firm i continues cooperating (producing under cartel $q = 2$), it obtains profit of \$8. Then, its stream of discounted payoffs from cooperating is

$$\begin{aligned} 8 + \delta 8 + \delta^2 8 + \dots &= 8(1 + \delta + \delta^2 + \dots) \\ &= \frac{8}{1 - \delta}, \end{aligned}$$

where δ denotes the discount factor weighting future payoffs.

Cartels and Collusion

- *Example 14.5* (continued):

- *Best deviation.* If firm i deviates from $q = 2$ while its rival sticks to the cartel agreement, its profits could increase.

What is firm i 's best deviation? We need to evaluate its profits when its rival produces the cartel output, $q_j = 2$,

$$(12 - q_i - 2)q_i - 4q_i = (10 - q_i)q_i - 4q_i.$$

- Differentiating with respect to q_i ,

$$10 - 2q_i - 4 = 0 \quad \Rightarrow \quad q_i = 3 \text{ units.}$$

- Inserting this “best deviation” into firm i 's profits, deviation profits are

$$\pi^{Dev} = (10 - 3)3 - (4 \times 3) = \$9 > \text{cartel profit of } \$8.$$

Cartels and Collusion

- *Example 14.5* (continued):

- If firm i deviates, its stream of discounted payoffs becomes

$$\begin{aligned} \underbrace{9}_{\text{Deviation}} + \underbrace{\delta \frac{64}{9} + \delta^2 \frac{64}{9} + \dots}_{\text{Punishment}} &= 9 + \frac{64}{9} (\delta + \delta^2 + \dots) \\ &= 9 + \frac{64}{9} \delta (1 + \delta + \dots) \\ &= 9 + \frac{64}{9} \frac{\delta}{1 - \delta}. \end{aligned}$$

- The deviating firm increases its profits for from \$8 to \$9 one period.
- Its defection is detected by its cartel partner, which triggers an infinite punishment in which both firms produce the Cournot output, yielding a profit of $\frac{64}{9}$ thereafter.

Cartels and Collusion

- *Example 14.5* (continued):

- *Comparing profits.* Every firm i prefers to cooperate if

$$\frac{8}{1-\delta} \geq 9 + \frac{64}{9} \frac{\delta}{1-\delta},$$
$$(1-\delta) \frac{8}{1-\delta} \geq \left[9 + \frac{64}{9} \frac{\delta}{1-\delta} \right] (1-\delta),$$
$$8 \geq 9(1-\delta) \frac{64}{9} \delta.$$

- The cartel output can be sustained with this GTS if

$$\delta \geq \frac{9}{17} \cong 0.53.$$

That is, if firms assign sufficiently importance to their profits. If $\delta < 0.53$, the cartel agreement cannot be sustained.

Stackelberg Model— Sequential Quantity Competition

Stackelberg Model

- We modify the Cournot model by considering that firms *sequentially* compete in quantities.
- The structure of the game is:
 1. Firm 1 chooses its output q_1 .
 2. Firm 2 observes q_1 and responds with its own output q_2 .
- This timing may be due to industry or legal reasons that provide firm 1 with an advantage.
 - *Example*: Firm 1 is the first to develop a new product, allowing it to choose its output before firm 2.
- This is a sequential-move game in which Firm 1 is the leader and firm 2 is the follower. We solve it by applying backward induction.

Stackelberg Model

- **Firm 2 (follower).**

- Firm 2 takes the leader's output q_1 as given, because it is already chosen by the time firm 2 gets to move. Its PMP is

$$\max_{q_2} [a - b(q_1 + q_2)]q_2 - cq_2.$$

- Differentiating with respect to q_2 ,

$$a - bq_1 - 2bq_2 - c = 0,$$

and solving for q_2 ,

$$q_2(q_1) = \frac{a - c}{2b} - \frac{1}{2}q_1. \quad (BRF_2)$$

- This BRF coincides with that of the Cournot model. In both scenarios firm 2 treats firm 1's output q_1 as given, because firm 2 cannot alter it (Cournot) or because q_1 is already produced (Stackelberg).

Stackelberg Model

- **Firm 1 (leader).**

- Firm 1 chooses its output q_1 to maximize its profits,

$$\max_{q_1} [a - b(q_1 + q_2)]q_1 - cq_1.$$

- Firm 1 can anticipate that firm 2 will respond with

$$BRF_2 = q_2(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1,$$

as this maximizes the follower's profits.

Stackelberg Model

- **Firm 1 (leader)** (cont.)

- Inserting BRF_2 into the leader's PMP,

$$\max_{q_1} [a - b(q_1 + \underbrace{q_2(q_1)}_{BRF_2})]q_1 - cq_1$$
$$\max_{q_1} \left[a - b \left(q_1 + \underbrace{\left(\frac{a-c}{2b} - \frac{1}{2}q_1 \right)}_{q_2(q_1) \text{ from } BRF_2} \right) \right] q_1 - cq_1.$$

- After simplifying,

$$\max_{q_1} \frac{1}{2} (a + c - bq_1)q_1 - cq_1.$$

Stackelberg Model

- **Firm 1 (leader)** (cont.)

- Differentiating with respect to q_1 , and solving for q_1 ,

$$\frac{1}{2}(a - c - 2bq_1) = 0,$$

$$q_1^* = \frac{a - c}{2b}.$$

- We find the follower's equilibrium output by inserting q_1^* into BRF_2 ,

$$q_2 \left(\frac{a - c}{2b} \right) = \frac{a - c}{2b} - \frac{1}{2} \underbrace{\left(\frac{a - c}{2b} \right)}_{q_1^*} = \frac{2(a - c)}{4b} - \frac{a - c}{4b} = \frac{a - c}{4b},$$

which is half of leader output $q_2^* = \frac{1}{2} q_1^*$.

Stackelberg Model

- More generally, the subgame perfect equilibrium (SPE) of the game is described as

$$q_1^* = \frac{a - c}{2b},$$
$$q_2(q_1) = \frac{a - c}{2b} - \frac{1}{2}q_1,$$

because the follower's BRF allows firm 2 to optimally respond to the leader's output, both:

- in equilibrium, $q_1^* = \frac{a - c}{2b}$,
- and off the equilibrium $q_1^* \neq \frac{a - c}{2b}$.

Stackelberg Model

- If instead, the follower chooses $q_2^* = \frac{a-c}{4b}$ in the SPE of the game, we would provide no information about how the follower responds if the leader “made a mistake” by deviating from q_1^* .

- The leader produces more in the Stackelberg model than in Cournot,

$$\frac{a-c}{2b} > \frac{a-c}{4b},$$

whereas the follower produces less,

$$\frac{a-c}{4b} < \frac{a-c}{3b}.$$

Stackelberg Model

- In this scenario, equilibrium price is

$$\begin{aligned} p^* &= a - b \left(\frac{a - c}{2b} + \frac{a - c}{4b} \right) \\ &= a - \frac{2(a - c)}{4} - \frac{a - c}{4} \\ &= \frac{3a + c}{4}. \end{aligned}$$

- The equilibrium profits for the leader are

$$\pi_1^* = \left(\frac{3a + c}{4} - c \right) \frac{a - c}{2b} = \frac{3(a - c)^2}{8b}.$$

- And equilibrium profits for the follower are

$$\pi_2^* = \left(\frac{3a + c}{4} - c \right) \frac{a - c}{4b} = \frac{3(a - c)^2}{16b},$$

that is exactly half of the leader's profits, $\pi_2^* = \frac{1}{2} \pi_1^*$.

Stackelberg Model

- *Example 14.6: Stackelberg model.*

- Consider the same inverse demand function as in example 14.1, $p(q_1, q_2) = 12 - q_1 - q_2$, and marginal cost $c = 4$.

- Inserting the follower's BRF found in example 14.1, $q_2(q_1) = 4 - \frac{1}{2}q_1$, into the leader's PMP,

$$\max_{q_1} \left[12 - \left(q_1 + \left(4 - \frac{1}{2}q_1 \right) \right) \right] q_1 - 4q_1,$$

$$\max_{q_1} \frac{1}{2} (16 - q_1)q_1 - 4q_1.$$

- Differentiating with respect to q_1 ,

$$8 - q_1 - 4 = 0.$$

Stackelberg Model

- *Example 14.6* (continued):
 - Solving for q_1 we find the profit-maximizing output for the leader, $q_1^* = 4$ units.
 - Then, $q_2 = 2$ units.
 - In this scenario, equilibrium price is $p^* = \$6$.
 - And equilibrium profits become

$$\pi_1^* = (6 \times 4) - (4 \times 4) = \$8,$$

$$\pi_2^* = (6 \times 2) - (4 \times 2) = \$4.$$

Product Differentiation

Product Differentiation

- Most goods are differentiated from those of their rivals:
 - Coke and Pepsi, in the soda industry.
 - Dell and Lenovo, the the computer industry.
 - iPhone and Samsung Galaxy, in the smartphone market.
- *Demand for product differentiation.*
 - Consider two firms, A and B , with inverse demand functions,
$$p_A(q_A, q_B) = a - bq_A - dq_B,$$
$$p_B(q_A, q_B) = a - bq_B - dq_A,$$
where $b, d \geq 0$ and $b \geq d$
 - These demands are symmetric. Let us focus on good A .
 - An increase in q_A or q_B reduces p_A , but the effect of q_A is larger because $b > d$. The “own-price effects” dominate “cross-price” effect.

Product Differentiation

- When $d = 0$, the inverse demand function for good A collapses to

$$p_A(q_A, q_B) = a - bq_A,$$

indicating that every firm's price is unaffected by its rival's output, as in two separate monopolies.

- When $d = b$, the inverse demand function for good A becomes

$$p_A(q_A, q_B) = a - bq_A - bq_B = a - b(q_A + q_B),$$

reflecting that p_A is symmetrically affected by an increase in either q_A or q_B as in the Cournot model with homogeneous goods.

Product Differentiation

- *Best responses with product differentiation.*

- Assume every firm $i = \{A, B\}$ faces a cost function $TC(q_i) = cq_i$, where $c > 0$.

- The PMP of firm A is

$$\max_{q_A} [a - bq_A - dq_B]q_A - cq_A.$$

- Differentiating with respect to q_A ,

$$a - c - 2bq_A - dq_B = 0.$$

- Rearranging and solving for q_A ,

$$a - c - dq_B = 2bq_A,$$
$$q_A(q_B) = \frac{a - c}{2b} - \frac{d}{2b}q_B.$$

Product Differentiation

- Figure 14.7 depicts $BRF_A, q_A(q_B) = \frac{a-c}{2b} - \frac{d}{2b} q_B$.

- $q_A = \frac{a-c}{2b}$ when $q_B = 0$, but it decrease at rate of $\frac{d}{2b}$.
- $q_A = 0$ when $q_B \geq \frac{a-c}{d}$.
- When $d = 0$, BRF_A reduces to $q_A(q_B) = \frac{a-c}{2b}$ (monopolist).

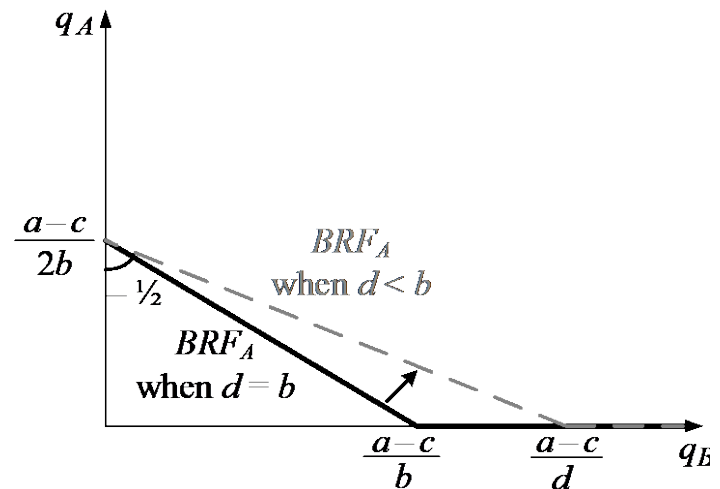


Figure 14.7

Product Differentiation

- Figure 14.7 depicts $BRF_A, q_A(q_B) = \frac{a-c}{2b} - \frac{d}{2b} q_B$.
 - When $d = b$, $q_A(q_B) = \frac{a-c}{b} - \frac{1}{2} q_B$ (Cournot), with a slope of $-1/2$.
 - When $d < b$, the slope becomes smaller than $-1/2$. Competition is ameliorated, because every firm i is induce to reduce its output when products are differentiated.

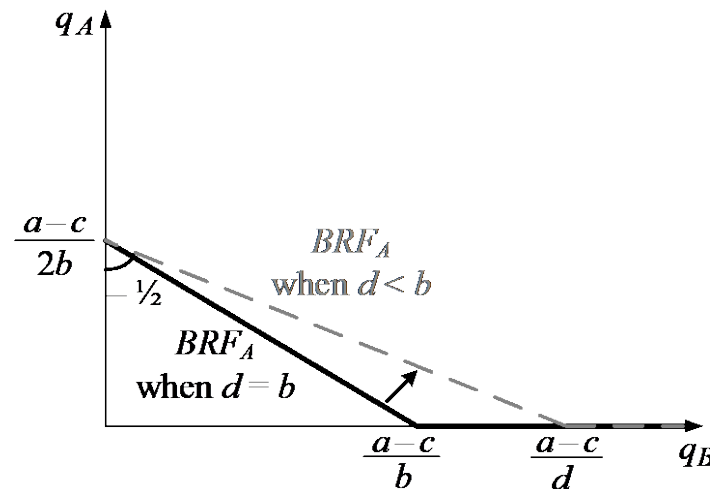


Figure 14.7

Product Differentiation

- We can invoke symmetry in equilibrium output $q_i^* = q_j^* = q^*$,

$$q = \frac{a - c}{2b} - \frac{d}{2b}q,$$
$$\frac{(2b + d)q}{2b} = \frac{a - c}{2b},$$
$$q^* = \frac{a - c}{2b + d}.$$

- When products are completely differentiated ($d = 0$), this output becomes $q^* = \frac{a - c}{2b}$, as in monopoly.
- When products are homogeneous ($d = b$), $q^* = \frac{a - c}{2b + b} = \frac{a - c}{3b}$, as in the Cournot model.

Product Differentiation

- Equilibrium price is given by

$$\begin{aligned} p_i^* &= a - bq_i^* + dq_j^* = a - b \underbrace{\frac{a-c}{2b+d}}_{q_i^*} - d \underbrace{\frac{a-c}{2b+d}}_{q_j^*} \\ &= \frac{ab + c(b+d)}{2b+d}. \end{aligned}$$

- Equilibrium profits for every firm i are

$$\begin{aligned} \pi_i^* &= (p^* - c)q^* = \left(\frac{ab + c(b+d)}{2b+d} - c \right) \frac{a-c}{2b+d} \\ &= \frac{(a-c)^2 b}{(2b+d)^2}. \end{aligned}$$

Product Differentiation

- When products are completely differentiated ($d = 0$),

$$\pi_i^* = \frac{(a - c)^2}{4b},$$

as in monopoly.

- When products are homogeneous ($d = b$),

$$\pi_i^* = \frac{(a - c)^2 b}{(2b + b)^2} = \frac{(a - c)^2}{9b},$$

as in Cournot model.

Stackelberg Model

- *Example 14.7: Output competition with product differentiation.*

- Consider two firms, A and B , facing the demand curves

$$p_A(q_A, q_B) = 100 - 5q_A - 2q_B,$$

$$p_B(q_A, q_B) = 100 - 5q_B - 2q_A.$$

- Parameters are $a = 100$, $b = 5$, and $d = 2$, which indicates that own-price effects are larger than cross-price effect (i.e., $b > d$).
- Both firms have symmetric marginal cost of $c = 3$.
- Inserting these parameters in in the previous equilibrium results, equilibrium output is

$$q^* = \frac{100-3}{(2 \times 5)+2} = \frac{97}{12} \cong 8.08 \text{ units.}$$

Stackelberg Model

- *Example 14.7* (continued):

- The equilibrium price is

$$p_i^* = \frac{(100 \times 5) + 3(5 + 2)}{(2 \times 5) + 2} = \frac{521}{12} \cong \$43.41.$$

- And profits become

$$\pi_i^* = \frac{(100 - 3)^2 5}{[(2 \times 5) + 2]^2} \cong \$326.7.$$

Appendix.

Cournot Model with N Firms

Cournot Model with N Firms

- The inverse demand function is

$$p(Q) = a - bQ.$$

- $Q = q_i + Q_{-i}$ denotes the aggregate output by all firms.

- q_i is the output that firm i produces.

- Q_{-i} represents the production of all the firms different than firm i ,

$$Q_{-i} = q_1 + q_2 + \dots + q_{i-1} + q_{i+1} + \dots + q_N.$$

- If $N = 4$, and $i = 2$, then $Q_{-2} = q_1 + q_3 + q_4$.

- We can rewrite the inverse demand function as

$$p(q_i, Q_{-i}) = a - b \underbrace{(q_i + Q_{-i})}_Q.$$

Cournot Model with N Firms

- If all N firms face the same marginal cost c , where $a > c > 0$, every firm i solves the following PMP:

$$\max_{q_i} [a - b(q_i + Q_{-i})]q_i - cq_i. \quad (PMP_i)$$

- Differentiating with respect to firm i 's output,

$$a - 2bq_i - bQ_{-i} - c = 0.$$

- Rearranging and solving for q_i , we find firm i 's best response function,

$$\begin{aligned} a - c - bQ_{-i} &= 2bq_i, \\ q_i(Q_{-i}) &= \frac{a - c}{2b} - \frac{1}{2}Q_{-i}. \end{aligned} \quad (BRF_i)$$

Cournot Model with N Firms

- $BRF_i \equiv q_i(Q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}Q_{-i}$ informs about this firm's profit maximizing output q_i , as a function of its rivals' output Q_{-i} .
 - It originates at $\frac{a-c}{2b}$ and decreases in Q_{-i} at a rate of $1/2$.
 - This function captures the Cournot model with 2 firms as a special case.
 - If we consider only two firms, i and j , then firm i has a single rival (firm j), and $Q_{-i} = q_j$.

Cournot Model with N Firms

- Because all firms are symmetric, they all solve a problem similar to PMP_i , obtaining $BRF_i \equiv q_i(Q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}Q_{-i}$.

- We can invoke symmetry in equilibrium output,

$$q_1 = q_2 = \dots = q_N = q.$$

- Then,

$$Q = Nq,$$

$$Q_{-i} = (N - 1)q.$$

Cournot Model with N Firms

- Inserting this result into BRF_i ,

$$q_i(Q_{-i}) = \frac{a - c}{2b} - \frac{1}{2} \underbrace{(N - 1)q}_{Q^*}.$$

- Rearranging and solving for q ,

$$\frac{2q + (N - 1)q}{2} = \frac{a - c}{2b},$$

$$q[2 + (N - 1)] = \frac{a - c}{b},$$

$$q^* = \frac{1}{N + 1} \frac{a - c}{b}.$$

which is decreasing in the number of firms operating in the market, N . As more firms compete, the individual production of each firm decreases.

Cournot Model with N Firms

- The aggregate output becomes

$$Q^* = Nq^* = N \left(\frac{1}{N+1} \frac{a-c}{b} \right),$$

which increases as more firms enter the industry.

- The equilibrium price is

$$\begin{aligned} p(Q^*) &= a - bQ^* = a - b \left[N \left(\frac{1}{N+1} \frac{a-c}{b} \right) \right] \\ &= \frac{a + Nc}{N+1}, \end{aligned}$$

which is decreasing in N .

Cournot Model with N Firms

- *Monopoly* ($N = 1$):

$$q^* = \frac{1}{1+1} \frac{a-c}{b} = \frac{a-c}{2b},$$

$$Q^* = Nq^* = \frac{a-c}{2b},$$

$$p^* = \frac{a+c}{1+1} = \frac{a+c}{2}.$$

Cournot Model with N Firms

- Duopoly ($N = 2$):

$$q^* = \frac{1}{2+1} \frac{a-c}{b} = \frac{a-c}{3b},$$

$$Q^* = Nq^* = 2q^* = 2 \frac{a-c}{3b},$$

$$p^* = \frac{a+2c}{2+1} = \frac{a+2c}{3}.$$

Cournot Model with N Firms

- *Perfect competition* ($N \rightarrow +\infty$):

$$\lim_{N \rightarrow +\infty} q^* = \lim_{N \rightarrow +\infty} \frac{1}{N+1} \frac{a-c}{b} = 0,$$

$$\lim_{N \rightarrow +\infty} Q^* = \lim_{N \rightarrow +\infty} N \left(\frac{1}{N+1} \frac{a-c}{b} \right) = \frac{a-c}{b},$$

$$\lim_{N \rightarrow +\infty} p^* = \lim_{N \rightarrow +\infty} \frac{a + Nc}{N+1} = c.$$