EconS 503 - Microeconomic Theory II
Final Exam, May 5th 2021 - Answer key

1. **Second-price auction with entry fees.** Consider a second-price auction where the seller announces an entry fee, E, that all bidders must pay in order to enter the auction room. Entry fees differ from reservation prices in the sense that they must be paid by all participating bidders, whether they win or not, whereas reservation prices is just the lowest bid that the seller considers from participating bidders. Every bidder i's valuation is independently distributed according to $F(v_i)$.

   (a) Find the optimal bidding function for bidder i, $b_i(v_i, E)$. How is the number of bidders participating in the auction affected by the entry fee E?

   - Every bidder i bidding according to his valuation, $b_i(v_i) = v_i$, is a weakly dominant strategy in a second-price auction, and constitutes the Bayesian Nash equilibrium of the game. This equilibrium result is unaffected by the entry fee, which only affects the type of bidders who participate in the auction. In particular, a bidder with valuation v_0 is indifferent between participating and not participating if

   $$v_0 F(v_0)^{N-1} - E = 0$$

   or

   $$v_0 F(v_0)^{N-1} = E.$$

   Therefore, every bidder i with valuation $v_i < v_0$ does not participate in this second-price auction, while all other bidders with valuations $v_i \geq v_0$ pay entry fee E to participate in the auction.

   - To investigate how participation in the auction is affected by a marginal increase in the entry fee, E, note that the left term in the above expression, $v_0 F(v_0)^{N-1}$, is unaffected by E, while the right term increases in E. Therefore, a marginal increase in E reduces the pool of bidders with valuations $v_i \geq v_0$, shrinking the number of participants in the auction.

   (b) Assume that valuations are uniformly distributed in $[0, 1]$. Which bidders participate in the auction, and what are their bidding functions?

   - In a setting where valuations are uniformly distributed, expression

   $$v_0 F(v_0)^{N-1} = E$$

   simplifies to $v_0^N = E$, so the cutoff valuation v_0 is

   $$v_0 = E^{\frac{1}{N}}.$$

   Differentiating v_0 with respect to E, we obtain

   $$\frac{\partial v_0}{\partial E} = \frac{1}{N} E^{-\frac{N-1}{N}} > 0$$

Therefore, an increase in the entry fee E increases cutoff valuation v_0, thus shrinking the pool of bidders participating in the auction, i.e., those with valuations $v_i \geq v_0$. These bidders submit a bid equal to their valuations, $b_i(v_i) = v_i$. Figure 1.3 illustrates this result. Assuming an entry fee $E = 0.7$ and $N = 2$ bidders, only those bidders with valuations $v_i \geq 0.84$ submit a bid equal to their valuations.

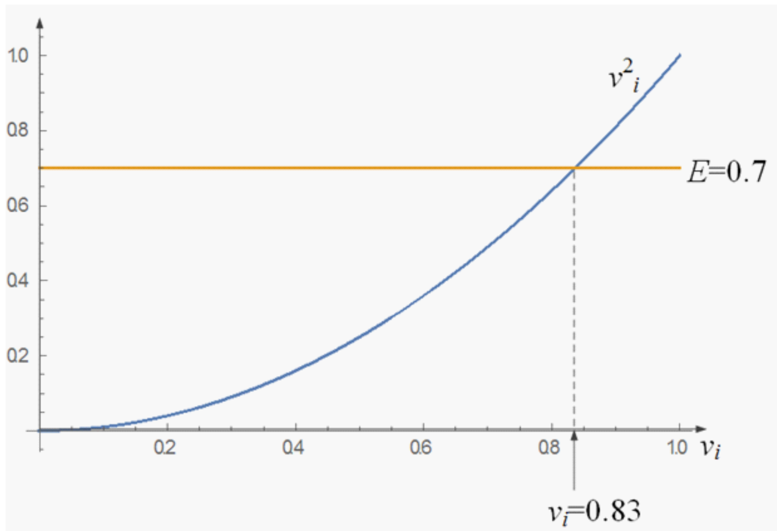

Figure 1.3. Entry fee and participating bidders.

Graphically, an increase in entry fee E shifts the horizontal line representing E in figure 1.3, producing a rightward shift in the crossing point between E and v_i^N, limiting the pool of bidders participating in the auction.

- Next, differentiating v_0 with respect to N, we obtain

$$\frac{\partial v_0}{\partial N} = \underbrace{\frac{-\log E}{N^2}}_{>0} \underbrace{E^{\frac{1}{N}}}_{>0} > 0$$

Figure 1.4 examines how the crossing point between E and v_i^N is affected by the number of bidders. As N increases from $N = 2$ to $N = 4$, for a given entry fee E, the auction becomes more competitive, as it is more likely that other bidders have a higher valuation for the object, and fewer bidders choose to participate.

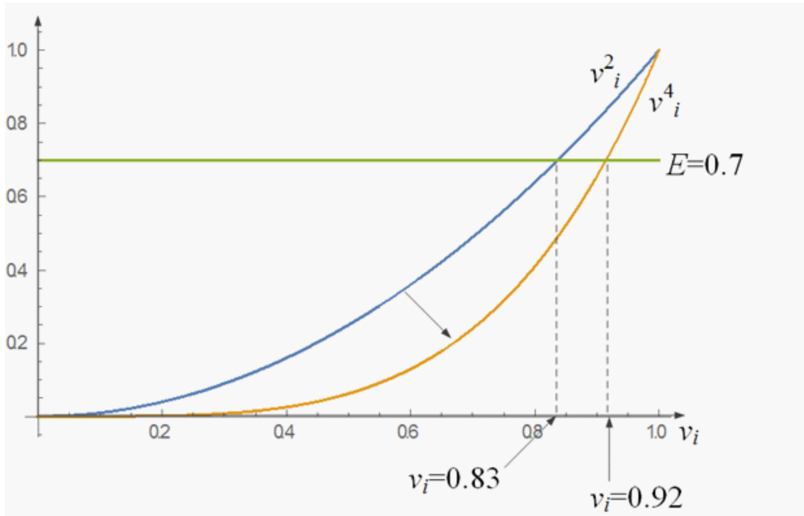

Figure 1.4. Entry fee and participating bidders - More bidders.

- This bidding behavior does not always coincide with that observed in controlled experiments where subjects often decrease their bids when an entry fee is imposed, or when the entry fee increases. Once a bidder has paid the entry fee, however, this cost is sunk, implying that his bidding behavior should not be affected by the entry fee. For this reason, this bidding behavior is commonly cited as an example of the "sunk cost fallacy." For more details, see Augenblick (2016), and references therein.

2. **Moral hazard in the farm.** Consider a setting between a land owner and a tenant farmer, who does not own the land but works in it, and suppose that both of them are risk neutral. The farmer chooses an effort level of either $e = 0$ (at zero cost for the farmer) or $e = 1$ (at a cost $c > 0$ for the farmer). When the farmer exerts a positive effort, $e = 1$, the harvest is low, x_L, with probability p_1, and high, x_H, with probability $1 - p_1$. Similarly, when the farmer exerts a zero effort, $e = 0$, the harvest is low, x_L, with probability p_0, and high, x_H, with probability $1 - p_0$; where $p_0 > p_1$. Assume that the farmer's reservation utility from rejecting the contract is zero, and that $w_L, w_H \geq 0$ (limited liability).

   (a) *Symmetric information.* As a benchmark, let us first solve the principal's problem when he can perfectly observe the effort level that the farmer exerts. Find the contract (w_H, w_L), specifying the salary to the farmer when the harvest is high and low. Show that the land owner's expected profits are higher when he induces effort $e = 1$ than when he induces $e = 0$.

   - The principal (owner of land) observes whether the farmer exerts effort $e = 0$ or $e = 1$. We next separately analyze expected profits when inducing an effort level $e = 1$ and when inducing $e = 0$.

- *Positive effort.* If the owner of the land seeks to induce an effort level $e = 1$, he solves the following expected profit maximization problem

$$\max_{w_H \geq 0} \; p_1(x_L - w_H) + (1 - p_1)(x_H - w_H)$$

$$\text{subject to} \;\; w_H - c \geq 0. \qquad \text{(PC)}$$

The participation constraint, PC, represents that the farmer is better off accepting the contract than rejecting it. Intuitively, the owner pays a salary w_H to the farmer with certainty, as the owner observes whether the farmer exerted a high effort level, but the harvest can be low (with probability p_1) or high otherwise.

This constraint must hold with equality. Otherwise, the land owner could reduce salaries and still induce the farmer's participation. Therefore, we must have that

$$w_H = c$$

Inserting this result into the principal's objective function, we obtain the the principal's expected profits from inducing effort $e = 1$ to be

$$\begin{aligned} \pi_H &= p_1(x_L - c) + (1 - p_1)(x_H - c) \\ &= p_1 x_L + (1 - p_1) x_H - c \end{aligned}$$

- *Zero effort.* If the owner of the land seeks to induce an effort level $e = 0$, he solves the following expected profit maximization problem

$$\max_{w_L \geq 0} \; p_0(x_L - w_L) + (1 - p_0)(x_H - w_L)$$

$$\text{subject to} \;\; w_L \geq 0 \qquad \text{(PC)}$$

Similarly, this participation constraint must hold with equality. Thus, we have

$$w_L = 0$$

Inserting this result into the principal's objective function, we obtain the principal's expected profits from inducing effort $e = 0$ to be

$$\begin{aligned} \pi_L &= p_0(x_L - 0) + (1 - p_0)(x_H - 0) \\ &= p_0 x_L + (1 - p_0) x_H. \end{aligned}$$

- *Profit comparison.* Comparing equilibrium profits, we find that the owner seeks to induce effort $e = 1$ rather than $e = 0$ if

$$p_1 x_L + (1 - p_1) x_H - c > p_0 x_L + (1 - p_0) x_H$$

which simplifies to

$$\underbrace{\underbrace{(p_0 - p_1)}_{+}\underbrace{(x_L - x_H)}_{-}}_{-} < c$$

which holds since $p_0 > p_1$ and $x_L < x_H$ by assumption. As a consequence, the owner's expected profits under symmetric information are higher when he induces effort $e = 1$ than $e = 0$.

(b) *Asymmetric information.* Find the contract (w_H, w_L), specifying the salary to the farmer when the harvest is high and low, respectively, where $w_L \geq 0$ by assumption (limited liability). Recall that the land owner prefers to induce effort $e = 1$.

- The principal (owner of land) solves the following expected profit maximization problem

$$\max_{w_H, w_L} \; p_1(x_L - w_L) + (1 - p_1)(x_H - w_H)$$

subject to

$$p_1 w_L + (1 - p_1) w_H - c \geq p_0 w_L + (1 - p_0) w_H \qquad \text{(IC)}$$

$$p_1 w_L + (1 - p_1) w_H - c \geq 0 \qquad \text{(PC)}$$

The incentive compatibility condition, IC, says that the farmer's expected utility from exerting an effort $e = 1$ is higher than that from exerting a zero effort level. The participation constraint, PC, represents that the farmer is better off accepting the contract than rejecting it.
Since $w_L \geq 0$ from the limited liability assumption, the PC condition must hold. In addition, $w_L = 0$, as otherwise the principal could reduce the salary in the low harvest contingency and still increase its profits. Inserting this salary in the binding IC condition, we obtain

$$p_1 \underbrace{w_L}_{0} + (1 - p_1) w_H - c = p_0 \underbrace{w_L}_{0} + (1 - p_0) w_H$$

which simplifies to

$$(1 - p_1) w_H - c = (1 - p_0) w_H.$$

Solving for w_L, yields

$$w_H = \frac{c}{p_0 - p_1}.$$

Let $\Delta p \equiv p_0 - p_1$ denote the probability differential of obtaining a low harvest when the effort is $e = 0$ than when it is $e = 1$, where $\Delta p > 0$ since $p_0 > p_1$. We can then summarize the contract that the principal offers to the tenant farmer as follows

$$(w_H, w_L) = \left(0, \frac{c}{\Delta p}\right).$$

(c) *Crop sharing.* Real-life contracts to tenant farmers are, however, different from that found in part (b). In particular, most contracts specify that the tenant farmer keeps a fixed proportion of the harvest (i.e., a harvest share). Find the contract that the principal offers if he must provide a harvest share to the farmer.

- The principal in this setting anticipates an expected harvest $p_1 x_L + (1 - p_1) x_H$ when he induces the agent to exert an effort level $e = 1$, and offers a share α of this expected harvest, keeping the remaining $(1 - \alpha)$. Formally, he then solves

$$\max_{\alpha \in [0,1]} \; (1 - \alpha) \overbrace{[p_1 x_L + (1 - p_1) x_H]}^{\text{Expected harvest}} = (1 - \alpha) \left[x_L + (1 - p_1) \Delta p \right]$$

subject to

$$\alpha\left[p_1 x_L + (1-p_1)x_H\right] - c \geq \alpha\left[p_0 x_L + (1-p_0)x_H\right] \quad \text{(IC)}$$

$$\alpha\left[p_1 x_L + (1-p_1)x_H\right] - c \geq 0 \quad \text{(PC)}$$

As usual, the incentive compatibility condition, IC, says that the farmer's expected utility from exerting an effort $e = 1$ is higher than that from exerting a zero effort level. The participation constraint, PC, represents that the farmer is better off accepting the contract than rejecting it.
From the binding IC, we obtain that

$$\alpha\left[p_1 x_L + (1-p_1)x_H\right] - c = \alpha\left[p_0 x_L + (1-p_0)x_H\right]$$

or, after rearranging,

$$\alpha\left[p_1 x_L + (1-p_1)x_H - p_0 x_L - (1-p_0)x_H\right] = c.$$

Solving for the harvest share, α, yields

$$\alpha = \frac{c}{\Delta p\,(x_H - x_L)}$$

where $\Delta p\,(x_H - x_L) > 0$ since $\Delta p > 0$ and $x_H - x_L > 0$. Finally, this harvest share is bounded between zero and one if $c < \Delta p\,(x_2 - x_1)$, and this condition holds when the principal finds it optimal to induce effort level $e = 1$ under symmetric information.

(d) Is the harvest share contract of part (c) better for the farmer than the optimal contract found in part (b)?

- The agent's expected utility in the harvest share contract of part (b) is

$$\begin{aligned} EU^{Share} &= \frac{c}{\Delta p\,(x_H - x_L)}\left[p_1 x_L + (1-p_1)x_H\right] - c \\ &= \left(1 - \frac{c}{\Delta p\,(x_H - x_L)}\right)\left[p_1 x_L + (1-p_1)x_H\right], \end{aligned}$$

while that in the optimal contract of part (a) is

$$\begin{aligned} EU^{Optimal} &= p_1 w_L + (1-p_1) w_H \\ &= p_1 0 + (1-p_1)\frac{c}{\Delta p}. \end{aligned}$$

Therefore, the farmer is better off with the harvest share than the optimal contract if

$$\left(1 - \frac{c}{\Delta p\,(x_H - x_L)}\right)\left[p_1 x_L + (1-p_1)x_H\right] > (1-p_1)\frac{c}{\Delta p}.$$

Intuitively, the principal in the harvest share contract of part (c) faces a new constraint relative to the optimal contract of part (b), $\alpha x_L > 0$, implying that he finds it more difficult to induce the farmer to exert effort level $e = 1$ and, at the same time, lower the farmer's expected utility close to his zero reservation utility.

3. **Emission fees and mechanisms, Duggan and Roberts (2003).**[1] Consider an industry with $N \geq 2$ polluting firms producing a homogenous good. Let the profit function of firm i be $\pi_i(q_i) = \ln q_i$, which is increasing and concave in its pollutants q_i. The social cost from pollution is

$$C(q_1, \cdots, q_n) = \sum_{i=1}^{n} \frac{\gamma_i}{2} q_i^2,$$

which is also increasing but convex in the pollutants q_i emitted by firm i. Finally, a regulator (e.g., government agency) considers the following welfare function

$$W(q_1, \cdots, q_n) = \sum_{i=1}^{n} \pi_i(q_i) - C(q_1, \cdots, q_n)$$

(a) *Complete information.* Assume that the regulator can observe pollution levels and sets an emission fee t_i per unit of emissions. Find the following: (i) firm i's profit-maximizing pollution level as a function of fee t_i, $q_i(t_i)$; (ii) the socially optimal pollution from firm i, q_i^{SO}; and (iii) the emission fee t_i that induces firm i to produce q_i^{SO}, i.e., the fee t_i that solves $q_i(t_i) = q_i^{SO}$.

- *Equilibrium pollution.* Firm i solves

$$\max_{q_i \geq 0} \pi_i(q_i) - t_i q_i = \ln q_i - t_i q_i$$

Differentiating with respect to q_i, we find

$$\frac{1}{q_i} = t_i$$

which, after solving for q_i, yields

$$q_i(t_i) = \frac{1}{t_i}.$$

- *Socially optimal pollution.* Differentiating with respect to q_i in the social welfare function, yields

$$\frac{1}{q_i} = \gamma_i q_i$$

which solving for q_i yields a socially optimal pollution of

$$q_i^{SO} = \frac{1}{\sqrt{\gamma_i}}.$$

- *Emission fee.* Hence, the emission fee t_i should be set to induce every firm i to produce the socially optimal amount of pollution, that is, $q_i(t_i) = q_i^{SO}$

$$\frac{1}{t_i} = \frac{1}{\sqrt{\gamma_i}}$$

[1] Duggan J. and Roberts J. (2003). Implementing the Efficient Allocation of Pollution. *American Economic Review,* 92(4), pp. 1070-78.

which yields an emission fee

$$t_i = \sqrt{\gamma_i}.$$

Intuitively, the emission fee is set to make firm i's marginal profit from one more unit of pollution, t_i, to coincide with its marginal social cost, $\sqrt{\gamma_i}$.

(b) *Incomplete information.* Assume that the level of pollution is unobservable to the regulator but observable among all firms. Then, the regulator can devise a circular monitoring mechanism, in which firm i reports the observed pollution level of firm $i-1$, $\overline{q}_{i-1}$, firm $i-1$ reports the observed pollution of firm $i-2$, $\overline{q}_{i-2}$, and firm 1 reports that of firm n, $\overline{q}_n$. This allows the regulator to set an emission fee per unit of pollution

$$t_i = \frac{\partial C\left(\overline{q}_i, q_{-i}\right)}{\partial q_i},$$

where $\overline{q}_i$ denotes firm i's pollution (reported by firm $i+1$), and q_{-i} represents the true pollution level of all other firms. In addition, firm i faces a penalty of $\left(\overline{q}_{i-1} - q_{i-1}\right)^2$ for misporting his neighbor's pollution level not at q_{i-1}.

1. Will firm i misreport the output of firm $i-1$? Why or why not?
   - No, because firm i will face a penalty proportional to his misreporting, which is given by $\left(\overline{q}_{i-1} - q_{i-1}\right)^2$. As a consequence, firm i truly reports what it has observed from firm $i-1$ in order to avoid penalties. This applies to every firm $i \in \{1, \cdots, n\}$.
2. Write down firm i's profit-maximization problem and solve for its optimal output.
   - To find the pollution level that maximizes firm i's profit, we fix every firm j's report at truthtelling, $\overline{q}_j = q_j$, since firms have no incentives to misreport (see part a). Firm i chooses q_i to solve

     $$\max_{q_i \geq 0} \quad \pi_i\left(q_i\right) - t_i q_i - \left(\overline{q}_{i-1} - q_{i-1}\right)^2$$

     where the first term denotes firm i's profits, the second represents the emission fee payments, and the third captures the penalty from misreporting firm $i-1$'s pollution.
     Since fee t_i is, by definition, $t_i = \frac{\partial C(\overline{q}_i, q_{-i})}{\partial q_i}$, and every firm i has no incentives to misreport (see part a), we can fix firms' reports at truthtelling, $\overline{q}_i = q_i$, the above problem becomes

     $$\max_{q_i \geq 0} \quad \pi_i\left(q_i\right) - \frac{\partial C\left(q_i, q_{-i}\right)}{\partial q_i} q_i - \left(q_{i-1} - q_{i-1}\right)^2 = \ln q_i - \gamma_i q_i^2$$

     Differentiating with respect to q_i, yields

     $$\frac{1}{q_i} = 2\gamma_i q_i$$

     Therefore, every firm i chooses a pollution level $q_i = \frac{1}{\sqrt{2\gamma_i}}$, which is lower than that under complete information, $q_i^{SO} = \frac{1}{\sqrt{\gamma_i}}$, as found in part (a).

As a consequence, the difference

$$\frac{1}{\sqrt{\gamma_i}} - \frac{1}{\sqrt{2\gamma_i}} = \frac{2-\sqrt{2}}{2\sqrt{\gamma_i}} \simeq \frac{0.29}{\sqrt{\gamma_i}}$$

can be interpreted as the output inefficiency that arises due to regulator's incomplete information.

3. Find the tax revenue generated by the mechanism, and the social cost of pollution.
   - Total tax revenue is

$$\begin{aligned}\sum_{i=1}^{n} t_i q_i &= \sum_{i=1}^{n} \frac{\sqrt{\gamma_i}}{\sqrt{2}} \frac{1}{\sqrt{2\gamma_i}} \\ &= \sum_{i=1}^{n} \frac{1}{2} \\ &= \frac{n}{2}.\end{aligned}$$

   - The social cost of pollution, evaluated at the equilibrium output, $(q_1, \cdots, q_n)$, is

$$\begin{aligned}C(q_1, \cdots, q_n) &= \sum_{i=1}^{n} \frac{\gamma_i}{2} \left(\frac{1}{\sqrt{2\gamma_i}} \right)^2 \\ &= \sum_{i=1}^{n} \frac{1}{4} \\ &= \frac{n}{4}.\end{aligned}$$