EconS 503 - Microeconomic Theory II
Final Exam, May 5th 2021

Instructions:

- This exam has three (3) exercises for a total of 100 points.
- You must answer all exercises.
- Please read all exercises carefully.
- Answer each exercise in a formal and concise manner, but include all your steps. This will allow you to obtain partial credit.

Good luck!!

1. **Second-price auction with entry fees.** Consider a second-price auction where the seller announces an entry fee, E, that all bidders must pay in order to enter the auction room. Entry fees differ from reservation prices in the sense that they must be paid by all participating bidders, whether they win or not, whereas reservation prices is just the lowest bid that the seller considers from participating bidders. Every bidder i's valuation is independently distributed according to $F(v_i)$.

   (a) Find the optimal bidding function for bidder i, $b_i(v_i, E)$. How is the number of bidders participating in the auction affected by the entry fee E?

   (b) *Uniformly distributed values.* Assume that valuations are uniformly distributed in $[0, 1]$. Which bidders participate in the auction, and what are their bidding functions?

2. **Moral hazard in the farm.** Consider a setting between a land owner and a tenant farmer, who does not own the land but works in it, and suppose that both of them are risk neutral. The farmer chooses an effort level of either $e = 0$ (at zero cost for the farmer) or $e = 1$ (at a cost $c > 0$ for the farmer). When the farmer exerts a positive effort, $e = 1$, the harvest is low, x_L, with probability p_1, and high, x_H, with probability $1 - p_1$. Similarly, when the farmer exerts a zero effort, $e = 0$, the harvest is low, x_L, with probability p_0, and high, x_H, with probability $1 - p_0$; where $p_0 > p_1$. Assume that the farmer's reservation utility from rejecting the contract is zero, and that $w_L, w_H \geq 0$ (limited liability).

   (a) *Symmetric information.* As a benchmark, let us first solve the principal's problem when he can perfectly observe the effort level that the farmer exerts. Find the contract (w_H, w_L), specifying the salary to the farmer when the harvest is high and low. Show that the land owner's expected profits are higher when he induces effort $e = 1$ than when he induces $e = 0$.

   (b) *Asymmetric information.* Find the contract (w_H, w_L), specifying the salary to the farmer when the harvest is high and low, respectively, where $w_L \geq 0$ by assumption (limited liability). Recall that the land owner prefers to induce effort $e = 1$.

   (c) *Crop sharing.* Real-life contracts to tenant farmers are, however, different from that found in part (b). In particular, most contracts specify that the tenant farmer keeps a fixed proportion of the harvest (i.e., a harvest share). Find the contract that the principal offers if he must provide a harvest share to the farmer.

   (d) Is the harvest share contract of part (c) better for the farmer than the optimal contract found in part (b)?

3. **Emission fees and mechanisms.** Consider an industry with $N \geq 2$ polluting firms producing a homogenous good. Let the profit function of firm i be $\pi_i(q_i) = \ln q_i$, which is increasing and concave in its pollutants q_i. The social cost from pollution is

$$C(q_1, \cdots, q_n) = \sum_{i=1}^{n} \frac{\gamma_i}{2} q_i^2,$$

which is also increasing but convex in the pollutants q_i emitted by firm i. Finally, a regulator (e.g., government agency) considers the following welfare function

$$W(q_1, \cdots, q_n) = \sum_{i=1}^{n} \pi_i(q_i) - C(q_1, \cdots, q_n)$$

(a) *Complete information.* Assume that the regulator can observe pollution levels and sets an emission fee t_i per unit of emissions. Find the following: (i) firm i's profit-maximizing pollution level as a function of fee t_i, $q_i(t_i)$; (ii) the socially optimal pollution from firm i, q_i^{SO}; and (iii) the emission fee t_i that induces firm i to produce q_i^{SO}, i.e., the fee t_i that solves $q_i(t_i) = q_i^{SO}$.

(b) *Incomplete information.* Assume that the level of pollution is unobservable to the regulator but observable among all firms. Then, the regulator can devise a circular monitoring mechanism, in which firm i reports the observed pollution level of firm $i-1$, $\overline{q}_{i-1}$, firm $i-1$ reports the observed pollution of firm $i-2$, $\overline{q}_{i-2}$, and firm 1 reports that of firm n, $\overline{q}_n$. This allows the regulator to set an emission fee per unit of pollution

$$t_i = \frac{\partial C(\overline{q}_i, q_{-i})}{\partial q_i},$$

where $\overline{q}_i$ denotes firm i's pollution (reported by firm $i+1$), and q_{-i} represents the true pollution level of all other firms. In addition, firm i faces a penalty of $\left(\overline{q}_{i-1} - q_{i-1}\right)^2$ for misporting his neighbor's pollution level not at q_{i-1}.

1. Will firm i misreport the output of firm $i-1$? Why or why not?
2. Write down firm i's profit-maximization problem and solve for its optimal output.
3. Find the tax revenue generated by the mechanism, and the social cost of pollution.

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