EconS 503 - Microeconomic Theory II
Midterm Exam #2 - Answer key

1. **Prisoner's dilemma game with more attractive cheating.** Consider the following Prisoner's dilemma game, where parameter a satisfies $a > 4$.

		Player 2	
		Confess	Not confess
Player 1	Confess	$2, 2$	$a, 0$
	Not confess	$0, a$	$4, 4$

Players interact in an infinitely-repeated game and consider the following GTS with permanent punishments: in the first period, every player chooses NC; in all subsequent periods, every player chooses NC if (NC, NC) was the outcome in all previous periods. Otherwise, every player reverts to the NE of the stage game, (C, C), thereafter. Assume, for simplicity, that players can immediately observe deviations (perfect monitoring).

(a) Find the minimal discount factor sustaining the above GTS, $\underline{\delta}(a)$.

- *After a history of cooperation.* If player i observes (NC, NC) in all previous periods and behaves as prescribed by the GTS, choosing NC, his discounted stream of payoffs is

$$4 + \delta 4 + \delta^2 4 + ... = \frac{4}{1-\delta}$$

If, instead, player i unilaterally deviates to C, his payoff increases to a. Once the deviation is detected, every player reverts to the NE of the stage game, (C, C) thereafter, yielding a discounted stream of payoffs

$$\underbrace{a}_{\text{Deviation}} + \underbrace{\delta 2 + \delta^2 2 + ...}_{\text{Reversion to NE}} = a + \frac{2\delta}{1-\delta}$$

Therefore, after a history of cooperation, every player i keeps cooperating if

$$\frac{4}{1-\delta} \geq a + \frac{2\delta}{1-\delta}$$

or, multiplying by $(1 - \delta)$ on both sides of the inequality, we obtain $4 \geq a(1-\delta) + \delta 2$. Solving for δ, yields

$$\delta \geq \frac{a-4}{a-2} \equiv \underline{\delta}(a) \text{ , for all } a > 4.$$

where cutoff $\underline{\delta}(a)$ satisfies $\underline{\delta}(a) > 0$ since $a > 4$; and $\underline{\delta}(a) < 1$ because $a - 4 < a - 2$ simplifies to $4 > 2$.

- *After a deviation.* If player i observes a deviation in a previous period and behaves as prescribed by the GTS, choosing C forever after, his payoff stream is

$$2 + \delta 2 + \delta^2 2 + ... = \frac{2}{1-\delta}.$$

If, instead, player i unilaterally deviates to NC, while his opponent chooses C (as prescribed by the GTS), his payoff decreases to 0 in that period, followed by the punishment phase forever after, yielding

$$\underbrace{0}_{\text{Deviation to } NC} + \underbrace{\delta 2 + \delta^2 2 + ...}_{\text{Reversion to NE}} = \frac{2\delta}{1-\delta}.$$

Therefore, after a deviation in a previous period, every player i chooses C, as prescribed by the GTS, if

$$\frac{2}{1-\delta} \geq \frac{2\delta}{1-\delta}$$

which simplifies to $\delta \leq 1$.

- Therefore, the minimal discount factor to sustain this GTS is $\underline{\delta}(a) = \frac{a-4}{a-2}$.

(b) Does $\underline{\delta}(a)$ increase or decrese in a. Interpret.

- Cutoff $\underline{\delta}(a)$ increases in a. Intuitively, this means that, as the payoff of deviating from NC to C increases in that period, the range of discount factors sustaining this GTS as a SPE shrinks, so cooperation is harder to hold.

2. **First-price auction with asymmetrically distributed valuations.** Most applications generally assume that all bidders independently draw their valuation from a *common* distribution, $F(v_i)$. In this exercise, we analyze how our equilibrium results are affected by relaxing this assumption. Consider a first-price auction with two risk-neutral bidders, i and j, independently drawing their valuations for the object from the following cumulative distribution functions $F_i(v_i) = v_i^{\alpha}$ and $F_j(v_j) = v_j^{\gamma}$, respectively, where $\alpha \neq \gamma > 0$. For simplicity, assume that $v_i, v_j \in [0, 1]$.

(a) Find the equilibrium bidding function for bidder i and j.

- *Writing expected utility.* We can write bidder i's expected utility maximization problem (UMP) as follows:

$$\max_{b_i \geq 0} \ \Pr(win) \times (v_i - b_i),$$

which denotes the probability of winning the object times bidder i's net payoff from winning, $v_i - b_i$, because he values the object at v_i and pays his bid b_i for it.

- *Finding the probability of winning.* At this point, we write the probability of winning as follows

$$\begin{aligned} \Pr(win) &= \Pr\{b_i > b_j\} \\ &= \Pr\{b_i > b_j(v_j)\} \end{aligned}$$

and inverting by $b_j^{-1}(\cdot)$, yields

$$\Pr\{b_j^{-1}(b_i) > b_j^{-1}(b_j(v_j))\} = \Pr\{b_j^{-1}(b_i) > v_j\} = \left(b_j^{-1}(b_i)\right)^{\gamma}$$

where the first equality uses $b_j^{-1}(b_j(v_j)) = v_j$, while the second equality considers that bidder j's valuation is distributed according to $F_j(v_j) = v_j^{\gamma}$. Therefore, the above expected utility maximization problem can be rewritten as

$$\max_{b_i \geq 0} \quad \left(b_j^{-1}(b_i)\right)^{\gamma} \times (v_i - b_i).$$

- *First order condition.* Differentiating with respect to b_i, yields

$$-\left(b_j^{-1}(b_i)\right)^{\gamma} + \gamma(v_i - b_i)\left(b_j^{-1}(b_i)\right)^{\gamma-1} \frac{\partial b_j^{-1}(b_i)}{\partial b_i} = 0.$$

Because $b_j(v_i) = b_i$, we can write that $b_j^{-1}(b_i) = v_i$. Therefore, $\frac{\partial b_j^{-1}(b_i)}{\partial b_i} = \frac{1}{b'\left(b_j^{-1}(b_i)\right)} = \frac{1}{b'(v_i)}$, implying that the above first-order condition simplifies to

$$-v_i^{\gamma} + \gamma(v_i - b_i)v_i^{\gamma-1}\frac{1}{b'(v_i)} = 0$$

or

$$\gamma v_i^{\gamma} = \gamma v_i^{\gamma-1} b_i + v_i^{\gamma} b'(v_i)$$

The right side is $\frac{\partial\left[v_i^{\gamma} b_i(v_i)\right]}{\partial v_i}$, which helps us rewrite this expression as

$$\gamma v_i^{\gamma} = \frac{\partial\left[v_i^{\gamma} b_i(v_i)\right]}{\partial v_i}.$$

Integrating both sides, yields

$$\int_0^{v_i} \gamma x^{\gamma} dx = v_i^{\gamma} b_i(v_i)$$

and solving for $b_i(v_i)$, we obtain the equilibrium bidding function, as follows

$$b_i(v_i) = \frac{1}{v_i^{\gamma}} \int_0^{v_i} \gamma x^{\gamma} dx.$$

Solving the integral, we can find a more precise expression for this bidding function, that is,

$$\begin{aligned} b_i(v_i) &= \frac{1}{v_i^{\gamma}} \int_0^{v_i} \gamma x^{\gamma} dx \\ &= \frac{1}{v_i^{\gamma}} \left[\frac{\gamma}{1+\gamma} x^{\gamma+1}\right]_0^{v_i} \\ &= \frac{1}{v_i^{\gamma}} \frac{\gamma}{1+\gamma} v_i^{\gamma+1} \\ &= \frac{\gamma}{1+\gamma} v_i \end{aligned}$$

where, intuitively, the term $0 < \frac{\gamma}{1+\gamma} < 1$ captures the extent of bid shading. Operating similarly for bidder j, we obtain that his equilibrium bidding function is

$$b_j(v_j) = \frac{\alpha}{1+\alpha} v_j.$$

- Finally, comparing $b_i(v_i)$ and $b_j(v_j)$, we claim that bidder i wins the auction if $\frac{\gamma}{1+\gamma} v_i > \frac{\alpha}{1+\alpha} v_j$, which holds if $v_i > \frac{\alpha(1+\gamma)}{\gamma(1+\alpha)} v_j$. Intuitively, this occurs if bidder i has a sufficiently higher valuation than bidder j. If both bidders have the same valuation $v_i = v_j$ but $\gamma > \alpha$, we have that $F_j(v_j) = v_j^{\gamma}$ first-order stochastically dominates $F_i(v_i) = v_i^{\alpha}$, thus bidder j assigning a larger probability weight on high valuations than bidder i.

(b) *Symmetrically distribution values.* Assume now that $\alpha = \gamma > 0$. How are your above results affected? How are equilibrium bids affected by a marginal increase in α? Interpret.

- When $\alpha = \gamma$, bidder i's bidding function becomes $b_i(v_i) = \frac{\alpha}{1+\alpha} v_i$, and that of bidder j is symmetric, that is, $b_j(v_j) = \frac{\alpha}{1+\alpha} v_j$. In that context, bidder i wins the auction if his valuation is higher than that of bidder j, $v_i > v_j$, and a marginal increase in α induces every bidder to submit (weakly) more aggressive bids because

$$\frac{\partial b_i(v_i)}{\partial \alpha} = \frac{1}{(1+\alpha)^2} v_i \geq 0.$$

Intuitively, an increase in α entails that the common cumulative distribution function assigns a larger probability weight on high valuations. As a consequence, for any given value v_i that bidder i privately observes, he knows that the probability that his rival draws a high valuation is, essentially, increasing in α, and thus responds submitting a higher bid b_i.

- For illustration purposes, figure 3.4 depicts the cumulative distribution function and figure 3.5 plots the corresponding equilibrium bidding function.

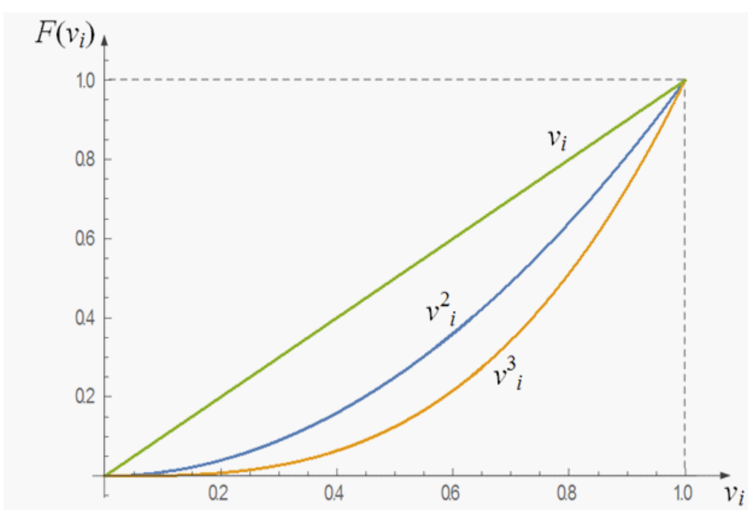

Figure 3.4. $F(v_i)$ evaluated at $\alpha = 2$ and at $\alpha = 3$.

If $\alpha = \gamma = 2$, equilibrium bidding functions become $b_i(v_i) = \frac{2}{3}v_i$ for every bidder i, which does not coincide with that in the standard first-price auction with two bidders independently drawing their valuation from a common, uniform, distribution. A similar argument applies if $\alpha = \beta = 3$, where the equilibrium bidding function becomes $b_i(v_i) = \frac{3}{4}v_i$, thus reducing bid shading. In the limit where $\alpha \to \infty$, bidders do not shade their bids since

$$\lim_{\alpha \to \infty} b_i(v_i) = \lim_{\alpha \to \infty} \left(1 - \frac{1}{1+\alpha}\right) v_i = v_i$$

where the common cumulative distribution function $F_i(v_i)$ assigns all probability weight to $v_i = 1$. In this context, every bidder, knowing for sure that the other bidder has a valuation of \$1 with certainty, can only have positive possibility to win the object if he does not shade his bid and submits a bid equal to his valuation at \$1, thus forming a tie with the other bidder.

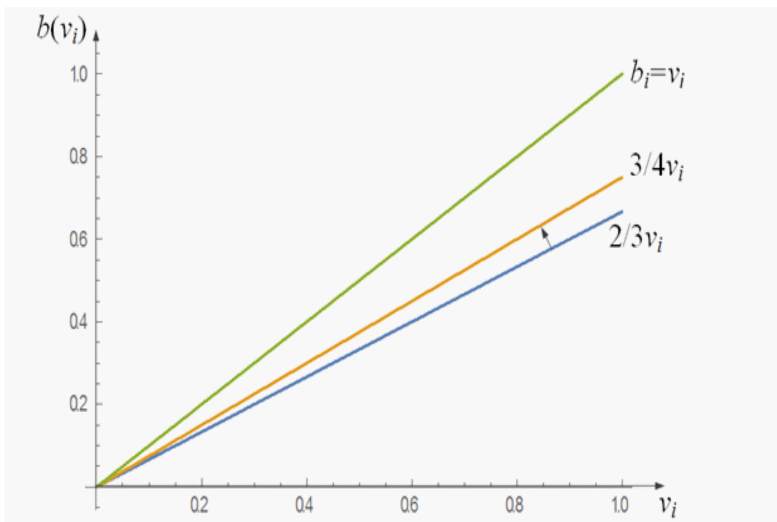

Figure 3.5. Equilibrium bidding function evaluated at $\alpha = 2$ and at $\alpha = 3$.

(c) *Uniformly distributed valuations.* How are the equilibrium results affected if $\alpha = \gamma = 1$?

- If $\alpha = \gamma = 1$, both bidder's valuations are distributed according to a uniform distribution, that is, $F_i(v_i) = v_i$, for every bidder i. In this context, equilibrium bidding functions simplify to

$$b_i(v_i) = \frac{1}{2}v_i,$$

as in the first-price auction with two bidders independently drawing their valuation from a common uniform distribution, as in previous exercises discussed in class.

3. **Selten's horse.** Consider the "Selten's Horse" game depicted in Figure 1. Player 1 is the first mover in the game, choosing between C and D. If he chooses C, player 2 is

called on to move between C' and D'. If player 2 selects C' the game is over. If player 1 chooses D or player 2 chooses D', then player 3 is called on to move without being informed whether player 1 chose D before him or whether it was player 2 who chose D'. Player 3 can choose between L and R, and then the game ends.

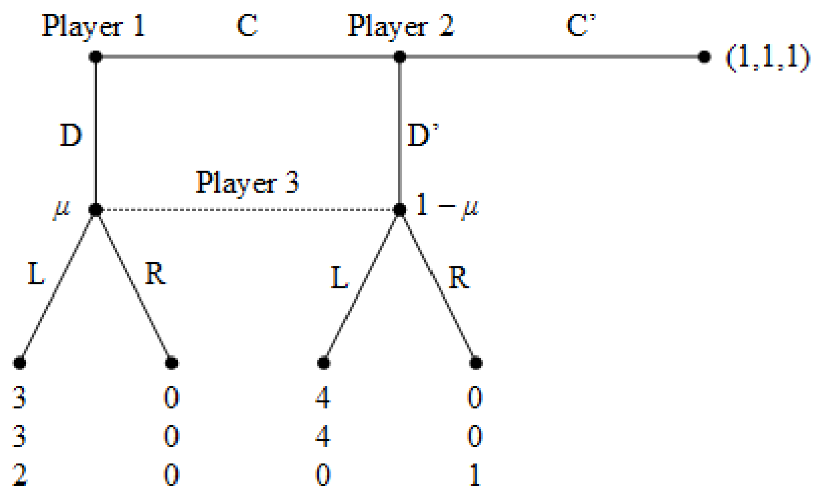

Figure 1. Selten's horse.

(a) Define the strategy spaces for each player. Then find all pure strategy Nash equilibria (psNE) of the game. [*Hint*: This is a three-player game, so you can consider that player 1 chooses rows, player 2 columns, and player 3 chooses matrices.]

- The strategy spaces of the players are as follows:

$$S_1 = \{C, D\}$$
$$S_2 = \{C', D'\}$$
$$S_3 = \{L, R\}$$

In Figure 2, we represent the strategies and payoffs of the three players in the following normal form representation of the game, where Player 1 chooses between the rows, Player 2 chooses between the columns, and Player 3 chooses between the matrixes.

Player 3 choosing L

Player 1 \ Player 2	C'	D'
C	1, 1, 1	4, 4, 0
D	3, 3, 2	3, 3, 2

Player 3 choosing R

Player 1 \ Player 2	C'	D'
C	1, 1, 1	0, 0, 1
D	0, 0, 0	0, 0, 0

Figure 2. Selten's horse - Matrix representation.

- We next underline the best responses of the three players in Figure 3, and identify that (C, C', R), and (D, C', L) are the pure strategy Nash equilibria of this game.

Player 3 choosing L:

Player 1 \ Player 2	C'	D'
C	1, 1, $\underline{1}$	$\underline{4}$, $\underline{4}$, 0
D	$\underline{3}$, $\underline{3}$, $\underline{2}$	3, $\underline{3}$, $\underline{2}$

Player 3 choosing R:

Player 1 \ Player 2	C'	D'
C	$\underline{1}$, $\underline{1}$, $\underline{1}$	$\underline{0}$, 0, $\underline{1}$
D	0, $\underline{0}$, 0	$\underline{0}$, $\underline{0}$, 0

Figure 3. Selten's horse - Underlining best response payoffs.

(b) Argue that one of the two psNEs you found in part (a) is not sequentially rational. A short verbal explanation suffices.

- (D, C', L) is not sequentially rational. If Player 1 chooses D, then Player 3's belief is $\mu = 1$, responding with L (see left-hand side at the bottom of the tree). Anticipating that Player 3 choosing L, Player 2 compares his payoff from C', 1, against that from D' (which is followed by Player 3 responding with L), 4, and thus chooses D'. Therefore, Player 2 choosing C' is not sequentially rational.

(c) Show that strategy profile $\{C, C', R\}$ can be sustained as a PBE of the game. (You don't need to show that this is actually the unique PBE we can sustain in this game.) Discuss that this strategy profile is based on credible beliefs by player 3.

- We check the pooling strategy profile, C, C', where Player 1 chooses C and Player 2 selects C'.
  As depicted in Figure 4, since player 1 chooses C (as illustrated by the blue horizontal arrow) and player 2 chooses C' (as illustrated by the green horizontal arrow), messages D and D' are off-the-equilibrium path, leaving the beliefs of Player 3 unrestricted, that is, $\mu \in [0, 1]$. In other words, Player 3's information set should never be reached in this strategy profile.

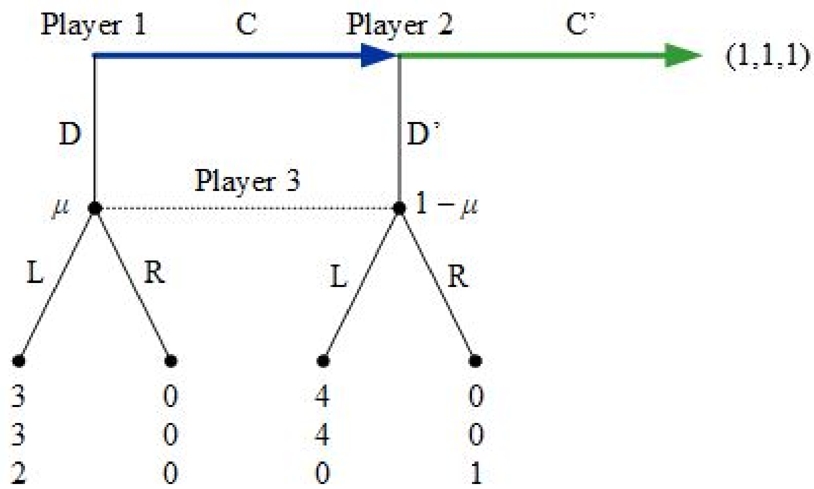

Figure 4. Pooling Strategy Profile C, C'

Therefore, if Player 3 is ever called out to move, he compares the expected payoff from responding with L and R, as follows:

$$EU_3\left(L\right) = 2 \times \mu + 0 \times \left(1 - \mu\right) = 2\mu$$
$$EU_3\left(R\right) = 0 \times \mu + 1 \times \left(1 - \mu\right) = 1 - \mu$$

Player 3 then responds with L if $2\mu > 1 - \mu$, which simplifies to $\mu > \frac{1}{3}$. Otherwise, he responds with R. This gives rise to two cases (one in which $\mu > \frac{1}{3}$, and Player 3 responds with L; and another in which $\mu \leq \frac{1}{3}$ and Player 3 responds with R), which we separately analyze below.
- *Case 1,* $\mu > \frac{1}{3}$. As depicted in Figure 5a, Player 3 responds with L (as illustrated by the red arrows) since $\mu > \frac{1}{3}$. In this context, Player 2 can improve his payoff by deviating from C', which yields a payoff of 1, to D', which yields a payoff of 4. Therefore, the pooling strategy profile C, C' cannot be supported as a PBE of this game when Player 3's beliefs satisfy $\mu > \frac{1}{3}$.

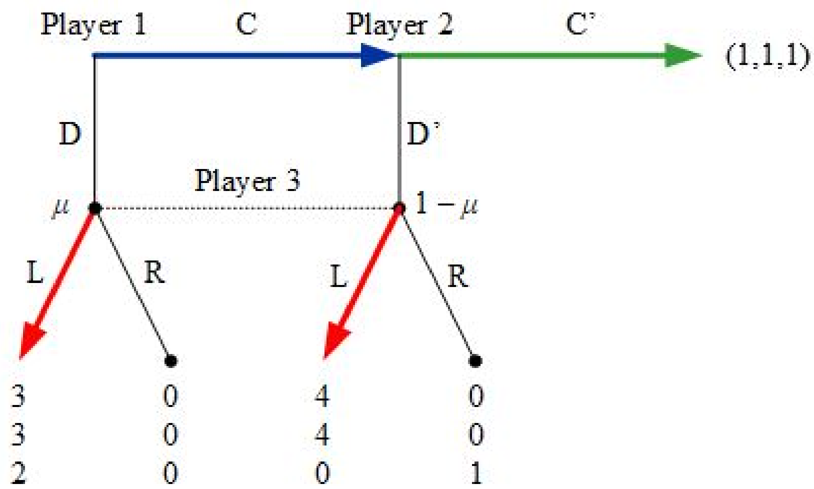

Figure 5a. Pooling Strategy Profile C, C' when $\mu > \frac{1}{3}$.

- *Case 2,* $\mu \leq \frac{1}{3}$. As depicted in Figure 5b, Player 3 responds with R (as illustrated by the red arrows) given that his beliefs are $\mu \leq \frac{1}{3}$. In this context, Player 2 does not deviate because his prescribed strategy, C', gives him a payoff of 1, while deviating to D' would give him a payoff of 0. Similarly, Player 1 does not deviate because his prescribed strategy, C, gives him a payoff of 1, exceeds his payoff from deviating to D, zero. Therefore, strategy profile C, C' can be supported as a PBE of this game when Player 3's beliefs satisfy $\mu \leq \frac{1}{3}$.

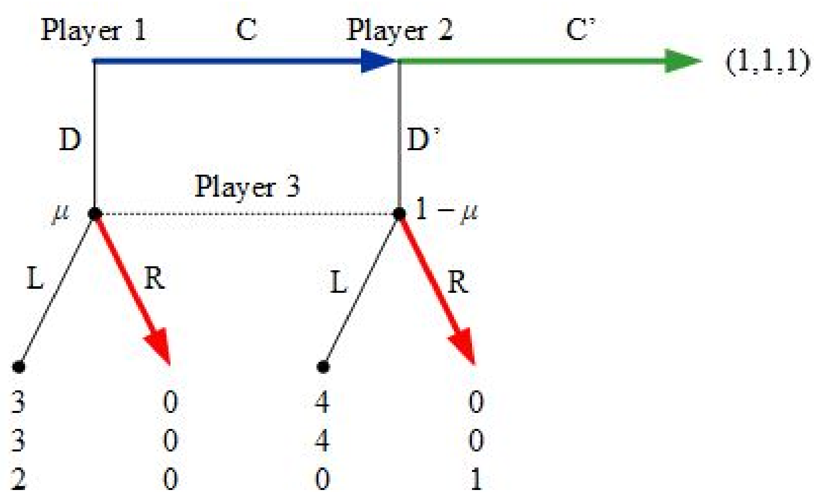

Figure 5b. Pooling Strategy Profile C, C' when $\mu \leq \frac{1}{3}$.

4. **Signaling and Limit pricing.** Consider a market with inverse demand function $p(Q) = 1 - Q$, where $Q = q_1 + q_2$ denotes aggregate output. Let us analyze an entry game with an incumbent monopolist (Firm 1) and an entrant (Firm 2) who analyzes whether or not to join the market. The incumbent's marginal costs are either high

H or low L, i.e., $c_1^H = \frac{1}{2} > c_1^L = \frac{1}{3}$. To make the entry decision interesting, assume that when the incumbent's costs are low, entry is unprofitable; whereas when the incumbent's costs are high, entry is profitable. (Otherwise, the entrant would enter regardless of the incumbent's cost, or stay out regardless of the incumbent's cost.) For simplicity, assume no discounting of future payoffs throughout all the exercise.

(a) *Complete information.* Let us first examine the case in which entrant and incumbent are informed about each others' marginal costs. Consider a two-stage game where:

1. In the first stage, the incumbent has monopoly power and selects its output level.
2. In the second stage, a potential entrant decides whether or not to enter. If entry occurs, firms compete as Cournot duopolists, simultaneously and independently selecting production levels. If entry does not occur, the incumbent maintains its monopoly power and selects its monopoly output again.

Find the subgame perfect equilibrium (SPNE) of this complete information game.

- We next apply backward induction, starting from the second-period game.
  - *Second period.* When no entry occurs, the incumbent selects output $x_1^{K,m} = \frac{1-c_1^K}{2}$ for every incumbent type $K = \{H, L\}$. If entry occurs, incumbent and entrant choose $x_1^{K,d} = \frac{1+c_2-2c_1^K}{3}$ and $x_2^{K,d} = \frac{1-2c_2+c_1^K}{3}$.
  - *First period.* Regardless of the entrant's entry decision during the second period, the incumbent selects $q^{K,Info} = \frac{1-c_1^K}{2}$ in the first period. This is because the incumbent's output choice does not affect the entrant's entry decision.

(b) *Incomplete information.* In this section we investigate the case where the incumbent is privately informed about its marginal costs, while the entrant only observes the incumbent's first-period output which the entrant uses as a signal to infer the incumbent's cost. The time structure of this signaling game is as follows:

1. Nature decides the realization of the incumbent's marginal costs, either high or low, with probabilities $p \in (0, 1)$ and $1 - p$, respectively. The incumbent privately observes this realization but the entrant does not.
2. The incumbent chooses its first-period output level, q.
3. Observing the incumbent's output decision, the entrant forms beliefs about the incumbent's initial marginal costs. Let $\mu(c_1^H|q)$ denote the entrant's posterior belief about the initial costs being high after observing a particular first-period output from the incumbent q.
4. Given the above beliefs, the entrant decides whether or not to enter the industry.
5. If entry does not occur, the incumbent maintains its monopoly power; whereas if entry occurs, both agents compete as Cournot duopolists and the entrant observes the incumbent's type.

Write down the incentive compatibility conditions that must hold for a separating Perfect Bayesian Equilibrium (PBE) to be sustained. Then find the set of separating PBEs.

- In a separating equilibrium in which the high-cost firm selects q^H while the low-cost firm chooses q^L information about the incumbent's type is conveyed to the potential entrant, who responds entering after observing the incumbent producing q^H, and does not enter after observing q^L. For simplicity, we assume that all other output levels $q \neq q^H \neq q^L$ (i.e., off-the-equilibrium outputs) also lead the entrant to enter the industry; as depicted in figure 1.

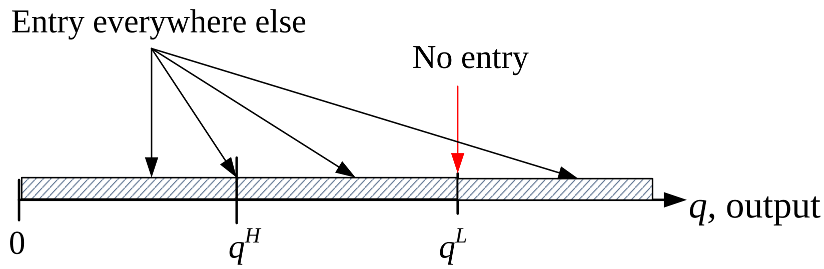

Fig 1. Output choices and entry in the separating PBE.

Let us next separately analyze each type of incumbent.

- *High-cost incumbent.* Since, by selecting q^H this type of incumbent attract entry, this firm selects the output that maximizes its first-period (monopoly) profits, that is, q^H coincides with its output under complete information $q^{H,Info} = \frac{1-c_1^H}{2}$. If, instead, the incumbent deviates towards the low-cost incumbent's output q^L, it conceals its type from the entrant and deters entry. Hence, the high-cost incumbent selects its equilibrium output q^H rather than deviating if $M_1^H(q^{H,Info}) + \delta D_1^H \geq M_1^H(q^L) + \delta \overline{M}_1^H$, where

$$M_1^H(q) = (1-q)q - c^H q \quad \text{for every output } q$$

denotes the incumbent's first-period monopoly profits, D_1^H represents second-period duopoly profits when the incumbent's costs are high, and $\overline{M}_1^H$ indicates the second-period monopoly profits for the incumbent (in the case of no entry) when its costs are high. We can now rewrite the above incentive compatibility condition as follows

$$M_1^H(q^{H,Info}) - M_1^H(q^L) \geq \delta \left[\overline{M}_1^H - D_1^H \right] \qquad (IC_H)$$

(where we grouped first-period profits on the left-hand side, and discounted second-period profits on the right-hand side). For our parameter values, we obtain profits of $M_1^H(q^{H,Info}) = \overline{M}_1^H = \frac{1}{16}$ since $c_1^H = 1/2$, and $D_1^H = \frac{1}{36}$ given that $c_1^H = c_2 = 1/2$. Hence, condition IC_H reduces to

$$\frac{1}{16} - \left[(1-q^L)q^L - \frac{1}{2}q^L \right] \geq \delta \left[\frac{1}{16} - \frac{1}{36} \right]$$

Figure 2 depicts IC_H. Specifically, the curve depicting the difference in first-period profits, $M_1^H(q^{H,Info}) - M_1^H(q^L)$, becomes zero at $q^L = q^{H,Info}$ since at that point $M_1^H(q^{H,Info}) = M_1^H(q^L)$, but otherwise is positive since $M_1^H(q^{H,Info}) > M_1^H(q^L)$ for all $q^L \neq q^{H,Info}$. In contrast, the difference in discounted second-period profits, $\delta\left[\overline{M}_1^H - D_1^H\right]$, is constant in first-period output q^L. Hence, IC_H holds if output q^L lies in the range depicted in the horizontal axis of figure 2.

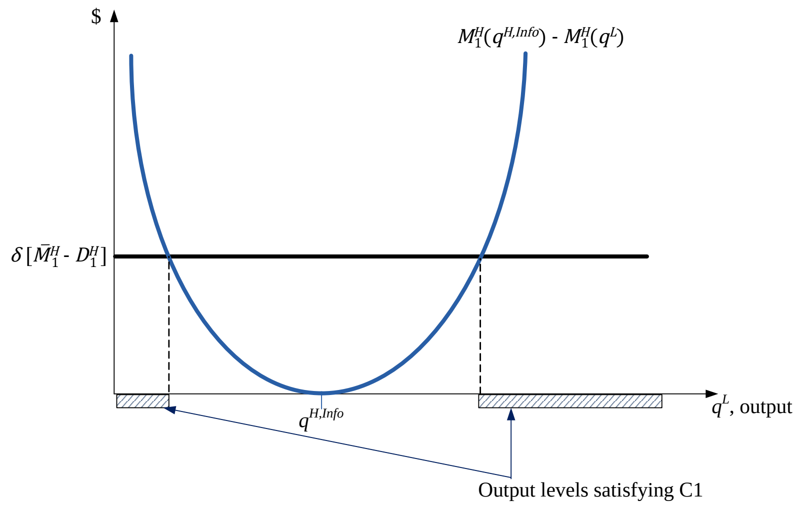

Fig 2. Incentive compatibility condition IC_H.

- *Low-cost incumbent.* If the low-cost incumbent chooses the equilibrium output q^L, it deters entry. If instead the incumbent deviates towards the high-cost incumbent's output, q^H, it attracts entry. Conditional on attracting entry, the low-cost incumbent would select output $q^{L,Info}$, since such output maximizes its first-period profits, yielding $M_1^L(q^{L,Info}) + \delta D_1^L$. Thus, the low-cost incumbent selects its equilibrium output of q^L if $M_1^L(q^{L,Info}) + \delta D_1^L \leq M_1^L(q^L) + \delta\overline{M}_1^L$, or equivalently,

$$M_1^L(q^{L,Info}) - M_1^L(q^L) \leq \delta\left[\overline{M}_1^L - D_1^L\right] \qquad (IC_L)$$

which, for our parameter values, yields $M_1^L(q^{L,Info}) = \overline{M}_1^L = 1/9$ and $D_1^L = \frac{25}{324}$ given that $c_1^L = 1/3$ and $c_2 = 1/2$. Hence, condition IC_L reduces to

$$\frac{1}{9} - \left[(1-q^L)q^L - \frac{1}{3}q^L\right] \leq \delta\left[\frac{1}{9} - \frac{25}{324}\right]$$

A similar argument as for IC_H applies to the graphical representation of IC_L. As figure 3 illustrates, the curve depicting the difference in first-period profits, $M_1^L(q^{L,Info}) - M_1^L(q^L)$, becomes zero at $q^L = q^{L,Info}$ since at that point $M_1^L(q^{L,Info}) = M_1^L(q^L)$, but otherwise is positive since $M_1^L(q^{L,Info}) > M_1^L(q^L)$ for all $q^L \neq q^{L,Info}$. In contrast, the difference in discounted second-period profits, $\delta\left[\overline{M}_1^L - D_1^L\right]$, is constant in first-period output q^L. Hence, IC_L holds if output q^L lies in the range depicted in the horizontal axis of figure 3.

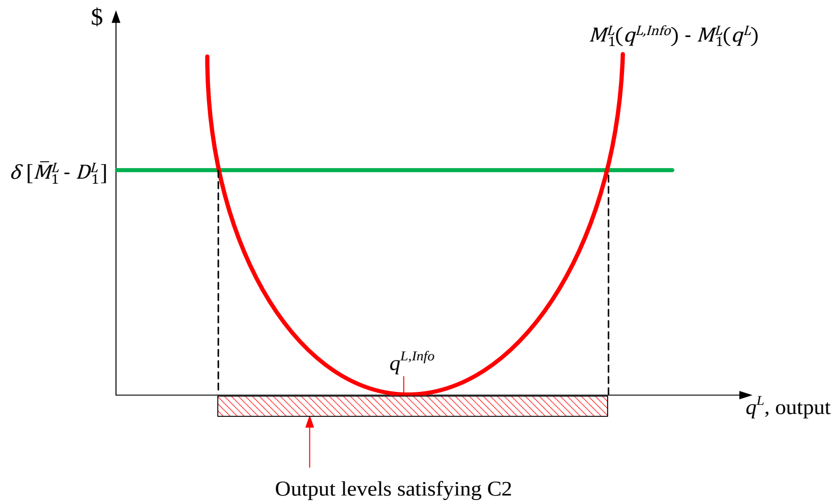

Fig 3. Incentive compatibility condition IC_L.

- *Combining both ICs.* Superimposing figures 2 and 3, we can examine the set of output levels that simultaneously satisfy condition IC_H and IC_L, as depicted in figure 4. In particular, the overlap between the range of outputs identified in figures 2 and 3 provides us with the set of output levels that constitute a separating PBE of the signaling game. Intuitively, the high-cost incumbent does not have incentives to mimic the output level chosen by the low-cost firm, i.e., $q^L \in [q^A, q^B]$. The low-cost firm, by contrast, has incentives to choose an output level in the interval $q^L \in [q^A, q^B]$, which is above its first-period output under complete information, $q^{L,Info} = \frac{1-c_1^L}{2}$. Thus, the low-cost incumbent increases its first-period output in order to communicate its efficient costs to the potential entrant, deterring entry as a result.

In particular, the lower-bound output q^A solves condition IC_H with equality, and the upper-bound output q^B solves IC_L with equality. Rearranging condition IC_H, and assuming that there is no discounting, $\delta = 1$, we obtain

$$1 - 1 + \frac{1}{36} = (1 - q^L)q^L - \frac{1}{2}q^L$$

or

$$36\left(q^L\right)^2 - 18q^L + 1 = 0$$

and solving for output q^L yields two roots for the lower bound q^A, $q^A = 0.06$ and $q^A = 0.43$. Similarly operating with condition IC_L in order to obtain the upper bound q^B, and since there is no discounting, $\delta = 1$, IC_L simplifies to

$$324\left(q^L\right)^2 - 216q^L + 25 = 0.$$

Solving for output q^L yields two roots for the upper bound q^B, $q^B = 0.14$ and $q^B = 0.51$. Hence, the set of separating output levels for the low-cost firm must lie on the interval $q^L \in [0.43, 0.51]$.