EconS 503 - Microeconomic Theory II
Midterm Exam #2, April 7th 2021

Instructions:

- This exam has four exercises for a total of 100 points.
- You can choose between exercise 1 and 2, but you must answer exercises 3 and 4.
- Please read all exercises carefully.
- Answer each exercise in a formal and concise manner, but include all your steps. This will allow you to obtain partial credit.

Good luck!!

1. **Prisoner's dilemma game with more attractive cheating.** Consider the following Prisoner's dilemma game, where parameter a satisfies $a > 4$.

		Player 2	
		Confess	Not confess
Player 1	Confess	$2, 2$	$a, 0$
	Not confess	$0, a$	$4, 4$

Players interact in an infinitely-repeated game and consider the following GTS with permanent punishments: in the first period, every player chooses NC; in all subsequent periods, every player chooses NC if (NC, NC) was the outcome in all previous periods. Otherwise, every player reverts to the NE of the stage game, (C, C), thereafter. Assume, for simplicity, that players can immediately observe deviations (perfect monitoring).

(a) Find the minimal discount factor sustaining the above GTS, $\underline{\delta}(a)$.

(b) Does $\underline{\delta}(a)$ increase or decrese in a. Interpret.

2. **First-price auction with asymmetrically distributed valuations.** Most applications generally assume that all bidders independently draw their valuation from a *common* distribution, $F(v_i)$. In this exercise, we analyze how our equilibrium results are affected by relaxing this assumption. Consider a first-price auction with two risk-neutral bidders, i and j, independently drawing their valuations for the object from the following cumulative distribution functions $F_i(v_i) = v_i^\alpha$ and $F_j(v_j) = v_j^\gamma$, respectively, where $\alpha \neq \gamma > 0$. For simplicity, assume that $v_i, v_j \in [0, 1]$.

   (a) Find the equilibrium bidding function for bidder i and j.

   (b) *Symmetrically distribution values.* Assume now that $\alpha = \gamma > 0$. How are your above results affected? How are equilibrium bids affected by a marginal increase in α? Interpret.

   (c) *Uniformly distributed valuations.* How are the equilibrium results affected if $\alpha = \gamma = 1$? Interpret.

3. **Selten's horse.** Consider the "Selten's Horse" game depicted in Figure 1. Player 1 is the first mover in the game, choosing between C and D. If he chooses C, player 2 is called on to move between C' and D'. If player 2 selects C' the game is over. If player 1 chooses D or player 2 chooses D', then player 3 is called on to move without being informed whether player 1 chose D before him or whether it was player 2 who chose D'. Player 3 can choose between L and R, and then the game ends.

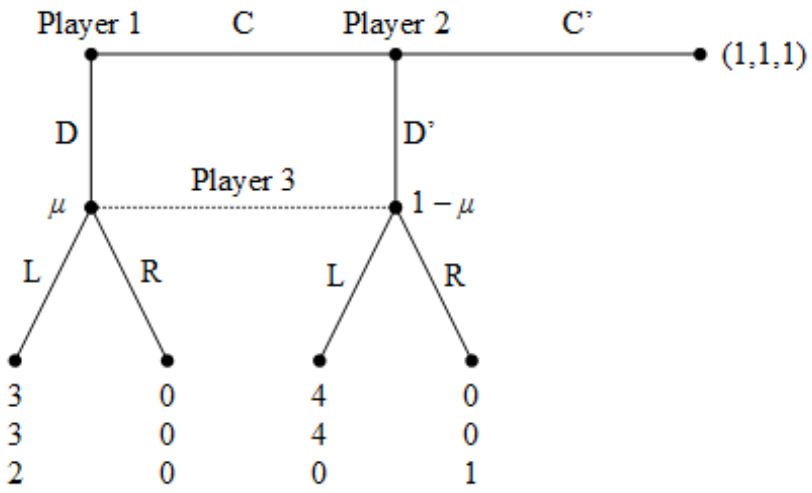

Figure 1. Selten's horse.

(a) Define the strategy spaces for each player. Then find all pure strategy Nash equilibria (psNE) of the game.

(b) Argue that one of the three psNEs you found in part (a) is not sequentially rational. A short verbal explanation suffices.

(c) Show that strategy profile $\{C, C', R\}$ can be sustained as a PBE of the game. (You don't need to show that this is actually the unique PBE we can sustain in this game.)

4. **Signaling and Limit pricing.** Consider a market with inverse demand function $p(Q) = 1 - Q$, where $Q = q_1 + q_2$ denotes aggregate output. Let us analyze an entry game with an incumbent monopolist (Firm 1) and an entrant (Firm 2) who analyzes whether or not to join the market. The incumbent's marginal costs are either high H or low L, i.e., $c_1^H = \frac{1}{2} > c_1^L = \frac{1}{3}$. To make the entry decision interesting, assume that when the incumbent's costs are low, entry is unprofitable; whereas when the incumbent's costs are high, entry is profitable. (Otherwise, the entrant would enter regardless of the incumbent's cost, or stay out regardless of the incumbent's cost.) For simplicity, assume no discounting of future payoffs throughout all the exercise.

   (a) *Complete information.* Let us first examine the case in which entrant and incumbent are informed about each others' marginal costs. Consider a two-stage game where:

      1. In the first stage, the incumbent has monopoly power and selects its output level.
      2. In the second stage, a potential entrant decides whether or not to enter. If entry occurs, firms compete as Cournot duopolists, simultaneously and independently selecting production levels. If entry does not occur, the incumbent maintains its monopoly power and selects its monopoly output again.

      Find the subgame perfect equilibrium (SPNE) of this complete information game.

   (b) *Incomplete information.* In this section we investigate the case where the incumbent is privately informed about its marginal costs, while the entrant only observes the incumbent's first-period output which the entrant uses as a signal to infer the incumbent's cost. The time structure of this signaling game is as follows:

      1. Nature decides the realization of the incumbent's marginal costs, either high or low, with probabilities $p \in (0, 1)$ and $1 - p$, respectively. The incumbent privately observes this realization but the entrant does not.
      2. The incumbent chooses its first-period output level, q.
      3. Observing the incumbent's output decision, the entrant forms beliefs about the incumbent's initial marginal costs. Let $\mu(c_1^H|q)$ denote the entrant's posterior belief about the initial costs being high after observing a particular first-period output from the incumbent q.
      4. Given the above beliefs, the entrant decides whether or not to enter the industry.
      5. If entry does not occur, the incumbent maintains its monopoly power; whereas if entry occurs, both agents compete as Cournot duopolists and the entrant observes the incumbent's type.

      Write down the incentive compatibility conditions that must hold for a separating Perfect Bayesian Equilibrium (PBE) to be sustained. Then find the set of separating PBEs.

      [*Hint:* You need to identify one incentive compatibility condition for the high-cost incumbent to choose the same production level as under complete information

rather than mimicking the output level of the low-cost incumbent; and another incentive compatibility condition for the low-cost incumbent to select a higher output than under complete information (thus exerting a "separating effort."]

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