EconS 503 - Microeconomic Theory II
Homework #6 - Due date: Monday, April 5th.

1. **Exercises from textbooks:**

   (a) MWG, Chapter 9: Exercise 9.C.7.

   (b) Bolton and Dewatripont, Exercise 6 (see page 650 since all exercises are at the end of the book).

2. **Entry deterrence with a sequence of potential entrants.** The following entry model is inspired on the original paper of Kreps and Wilson (*JET*, 1982). Consider an incumbent monopolist building a reputation as a tough competitor who does not allow entry without a fight. The entrant first decides whether to enter the market, and, if he does, the monopolist chooses whether to fight or acquiesce. If the entrant stays out, the monopolist obtains a profit of $a > 1$, and the entrant gets 0. If the entrant enters, the monopolist gets 0 from fighting and -1 from acquiescing if he is a "tough" monopolist, and -1 from fighting and 0 from acquiescing if he is a "normal" monopolist. The entrant obtains a profit of b if the monopolist acquiesces and $b - 1$ if he fights, where $0 < b < 1$. Suppose the entrant believes the monopolist to be tough (normal) with probability p ($1 - p$, respectively), while the monopolist observes his own type.

   (a) Depict a game tree representing this incomplete information game.

   (b) Solve for the PBE of this game.

   (c) Suppose the monopolist faces two entrants in sequence, and the second entrant observes the outcome of the first game (there is no discounting). Depict the game tree, and solve for the PBE. [*Hint:* you can use backward induction to reduce the game tree as much as possible before checking for the existence of separating or pooling PBEs. For simplicity, focus on the case in which prior beliefs satisfy $p \leq b$.]

3. **Signaling when the expert receives imprecise signals.** Consider the following signaling model between an expert (E), such a special interest group, and a decision maker (DM), such as a politician. For simplicity, assume that the state of the world is discrete, either $\theta = 1$ or $\theta = 0$ with prior probability $p \in (0, 1)$ and $1 - p$, respectively. The expert privately observes an informative but noisy signal s, which also takes two discrete values $s \in \{0, 1\}$. The precision of the signal is given by the conditional probability

$$\text{prob}(s = k|\theta = k) = q,$$

   where $k = \{0, 1\}$, and $q > \frac{1}{2}$. In words, the probability that the signal s coincides with the true state of the world θ is q (precise signal), while the probability of an imprecise signal where $s \neq \theta$ is $1 - q$. The time structure of the game is as follows:

1) Nature chooses θ according to the prior p.

2) Expert observes signal s and reports a message $m \in \{0, 1\}$

3) Decision maker observes m and responds with $x \in \{0, 1\}$

4) θ is observed and payoffs are realized

The payoff function for the decision maker is

$$u(x, \theta) = \left(\theta - \frac{1}{2}\right) x$$

while that of the expert is

$$v(m, \theta) = \begin{cases} 1, & \theta = m \\ 0, & \theta \neq m \end{cases}$$

which, in words, indicates that the expert's payoff is 1 when the message she sends coincides with the true realization of the state of the world, but becomes zero otherwise. Importantly, her payoff is unaffected by the signal, which she only uses to infer the actual realization of parameter θ. Intuitively, $v(m, \theta)$ is often understood as a "reputation function" since it provides the expert with a payoff of 1 only when his message was an accurate representation of the true state of the world (which in this model he does not precisely observe).

(a) Is there a Perfect Bayesian equilibrium (PBE) in which the expert reports his signal truthfully?