EconS 594 - Cournot competition with $n \geq 2$ asymmetric firms[1]

1. Consider an industry of $n \geq 2$ firms competing a la Cournot. Firms face an inverse demand curve $p(Q) = a - Q$, where $Q \geq 0$ denotes aggregate output. Every firm i has a marginal cost of production c_i, where $a > c_i \geq 0$.

   (a) Set up firm i's profit-maximization problem and find its first-order condition.

   - Every firm i chooses its output q_i to solve

   $$\max_{q_i \geq 0} \; [a - (q_i + Q_{-i})]\, q_i - c_i q_i$$

   where Q_{-i} denotes the aggregate output of firm i's rivals. Differentiating with respect to q_i, yields

   $$a - 2q_i - Q_{-i} - c_i = 0$$

   which we can rearrange as

   $$a - c_i = 2q_i + Q_{-i}.$$

   (b) Find equilibrium output. [*Hint*: Find the aggregate output Q from the first-order condition that you found in part (a). Then sum over all n firms, and finally insert it into firm i's first-order condition from part (a).]

   - From the first-order condition found in part (a), we have that

   $$a - c_i = 2q_i + Q_{-i},$$

   which can be rewritten as

   $$a - c_i = q_i + Q$$

   since $Q = q_i + Q_{-i}$ by definition. Therefore, summing over all n firms, yields

   $$na - \sum_{i=1}^{n} c_i = \sum_{i=1}^{n} q_i + nQ$$

   which simplifies to

   $$na - \sum_{i=1}^{n} c_i = (1+n)Q$$

   since $Q = \sum_{i=1}^{n} q_i$ by definition. Denoting, for compactness, $C = \sum_{i=1}^{n} c_i$ for the aggregate costs, and solving for Q, we find an expression for aggregate output in equilibrium, as follows

   $$Q = \frac{na - C}{1+n}.$$

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Returning to firm i's first-order condition again, $a - c_i = q_i + Q$, we can rewrite it as $q_i = a - c_i - Q$. Inserting the above expression of aggregate output in equilibrium, yields the equilibrium output of firm i, as follows

$$
\begin{aligned}
q_i^* &= a - c_i - \frac{\overbrace{na - C}^{Q}}{1+n} \\
&= \frac{a - (1+n)c_i + C}{1+n},
\end{aligned}
$$

which can also be expressed, alternatively, as

$$q_i^* = \frac{a - nc_i + \sum_{j \neq i} c_j}{1+n}.$$

(c) *First example.* Consider a setting with $n = 2$ firms (firm 1 and 2) facing inverse demand function $p(Q) = 1 - Q$, and marginal production costs c_1 and c_2, where $1 > c_i \geq 0$ for every firm $i = \{1, 2\}$. Evaluate your results from part (b) to find the equilibrium output for each firm, aggregate output, and profits. Then evaluate your results in the case that marginal production costs coincide, $c_1 = c_2 = c$, where $1 > c \geq 0$.

- *Asymmetric costs, $c_1 \neq c_2$.* In this context, the sum of marginal costs is $C = c_1 + c_2$, and demand parameters are $a = b = 1$. Therefore, aggregate output becomes

$$Q = \frac{2 - (c_1 + c_2)}{2+1} = \frac{2 - (c_1 + c_2)}{3}$$

individual output is

$$q_i = \frac{1 - 2c_i + c_j}{3}$$

and profits become

$$
\begin{aligned}
\pi_i &= \left(1 - \frac{1 - 2c_i + c_j}{3} - \frac{1 - 2c_j + c_i}{3} - c_i\right) \frac{1 - 2c_i + c_j}{3} \\
&= \frac{(1 - 2c_i + c_j)^2}{9}
\end{aligned}
$$

- *Symmetric costs, $c_1 = c_2 = c$.* In this setting, the above results become

$$Q = \frac{2(1-c)}{3}$$

individual output is

$$q_i = \frac{Q}{2} = \frac{1-c}{3}$$

and profits become

$$\pi_i = \frac{(1-c)^2}{9}$$

(d) *Second example.* Consider a setting with $n \geq 2$ firms facing inverse demand function $p(Q) = 1 - Q$, and symmetric marginal production cost c, where $1 > c \geq 0$. Assuming that k firms merge, benefiting from a lower marginal cost $c - x$, while the $n - k$ unmerged firms still face marginal cost c. Find the aggregate output in equilibrium when k firms merge, and compare it against aggregate output before the merger. For which parameter values the merger produces an increase in aggregate output?

- *Before the merger.* With n firms in the industry, all facing marginal cost c, the sum of marginal costs is $C = nc$. Therefore, expression $Q = \frac{na-C}{n+1}$, we can then write aggregate output in this setting as

$$Q^{NM} = \frac{n - nc}{n+1} = \frac{n(1-c)}{n+1}$$

since $a = 1$, where superscript NM denotes "no merger."

- *After the merger.* If k out of n firms merge, leaving $n - k$ firms unmerged, then there are $(n - k) + 1$ firms in the industry. In this context, the sum of marginal costs is

$$C = \underbrace{(c - x)}_{\text{Merged firm}} + \underbrace{(n - k)c}_{\text{Unmerged firms}} = (n - k + 1)c - x.$$

Using expression $Q = \frac{na-C}{n+1}$, we can then write aggregate output in this setting as

$$\begin{aligned} Q^M &= \frac{[(n-k)+1] - [(n-k+1)c - x]}{[(n-k)+1]+1} \\ &= \frac{(n-k+1)(1-c) + x}{n-k+2} \end{aligned}$$

since $a = 1$, where superscript M denotes "merger."

- *Output comparison.* Aggregate output after the merger increases if $Q^M \geq Q^{NM}$, which entails

$$\frac{(n-k+1)(1-c) + x}{n-k+2} \geq \frac{n(1-c)}{n+1}.$$

Rearranging, we obtain

$$\theta \equiv \frac{x}{1-c} \geq \frac{k-1}{n+1}.$$

Intuitively, the merger increases aggregate output (and, as a consequence, consumer surplus) if the cost-reduction effect relative to firms' margin (left-hand side, θ) is sufficiently large.

As an illustration, we can fix the total number of firms at $n = 10$, and evaluate cutoff $\frac{k-1}{n+1}$ at $k = 2$, obtaining that

$$\frac{2-1}{10+1} = 0.09.$$

Intuitively, the cost-reduction effect, relative to per-unit margins (as measured by θ), must be larger than 9% for the merger to increase consumer surplus. Mergers between more firms (higher k) produce an even larger ratio $\frac{k-1}{n+1}$, thus increasing the minimum cost-reduction effect, θ, required for the merger to increase consumer surplus.