Advanced Microeconomic Theory

Chapter 10: Contract Theory

Outline

- Moral Hazard
- Moral Hazard with a Continuum of Effort Levels—The First-Order Approach
- Moral Hazard with Multiple Signals
- Adverse Selection—The “Lemons” Problem
- Adverse Selection—The Principal–Agent Problem
- Application of Adverse Selection—Regulation

Moral Hazard

Moral Hazard

- **Moral hazard**: settings in which an agent does not observe the actions of the other individual(s).
  - Also referred to as "hidden action"
- Example:
  - A manager in a firm cannot observe the effort of employees in the firm even if the manager is perfectly informed about the worker's ability or productivity.
  - The worker might have incentives to *slack* from exerting a costly effort, thus giving rise to moral hazard problems.

Moral Hazard

- The manager can offer contracts that provide incentives to the worker to work hard
  - Paying a higher salary (bonus) if the worker's output is high but a low salary otherwise.
- Providing incentives to work hard is costly for the manager
- The manager only induces a high effort if the firm's expected profits are higher than those of inducing a low effort

Moral Hazard

- Consider a principal with benefit function
$$B(\pi - w)$$
where π is the profit that arises from the agent's effort and w is the salary that the principal pays to the agent.
- The benefit function satisfies $B' \geq 0$ and $B'' \leq 0$.
- The agent's (quasi-linear) utility function is
$$U(w, e) = u(w) - g(e)$$
where $u(w)$ is utility from the agent's salary, for $u' > 0$ and $u'' \leq 0$, and $g(e)$ is the agent's disutility from effort (e), for $g' > 0$ and $g'' \geq 0$.

Moral Hazard

- The agent's effort level e affects the probability that a certain profit occurs.
- For a given effort e, the conditional probability that a profit $\pi = \pi_i$ is

$$f(\pi_i|e) = \text{Prob}\{\pi = \pi_i|e\} \geq 0$$

  where $i = \{1,2, \dots, N\}$ is the profits that can emerge for a given effort e.
- Hence a high profit could arise even if the worker slacks
  - That is, a given profit level $\pi = \pi_i$ can arise from every effort level

Symmetric Information

- The principal **can observe** the agent's effort level e.
- The principal's maximization problem is

$$\max_{\{e, w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e) \cdot B(\pi_i - w(\pi_i))$$

$$\text{s.t.} \quad \sum_{i=1}^{N} f(\pi_i|e)\, u(w(\pi_i)) - g(e) \geq \bar{u}$$

- The principal seeks to maximize expected profits, subject to the agent participating in the contract.
  - The constraint guarantees the agent's voluntary participation in the contract.
  - Hence it is referred to as the participation constraint (PC) or the "individual rationality" condition.
- The constraint must be binding (holding with equality).

Symmetric Information

- The Lagrangian that solves the maximization problem is

$$\mathcal{L} = \sum_{i=1}^{N} f(\pi_i|e) \cdot B\big(\pi_i - w(\pi_i)\big) + \lambda \left[\sum_{i=1}^{N} f(\pi_i|e)\, u\big(w(\pi_i)\big) - g(e) - \bar{u}\right]$$

- Take FOC with respect to w to obtain

$$f(\pi_i|e) \cdot B'\big(\pi_i - w(\pi_i)\big) \cdot (-1) + \lambda f(\pi_i|e) u'\big(w(\pi_i)\big) = 0$$

where B' and u' is the derivative of $B(\cdot)$ and $u(\cdot)$ with respect to w.

Symmetric Information

- Rearranging

$$\lambda u'\big(w(\pi_i)\big) = B'\big(\pi_i - w(\pi_i)\big)$$

- Solving for λ

$$\lambda = \frac{B'\big(\pi_i - w(\pi_i)\big)}{u'\big(w(\pi_i)\big)} \qquad \textbf{(1)}$$

which is positive since $B'(\cdot) > 0$ and $u'(\cdot) > 0$.

- $\lambda > 0$ entails that the agent's participation constraint must bind (i.e., hold with equality)

$$\sum_{i=1}^{N} f(\pi_i|e)u\big(w(\pi_i)\big) - g(e) = \bar{u}$$

Symmetric Information

- Example 1:
  - Consider a risk-neutral principal hiring a risk-averse agent with utility function $u(w) = \sqrt{w}$, disutility of effort $g(e) = e$, and reservation utility $\bar{u} = 9$.
  - There are two effort levels $e_H = 5$ and $e_L = 0$.
  - When $e_H = 5$, the principal's sales are $0 with probability 0.1, $100 with probability 0.3, and $400 with probability 0.6.
  - When $e_L = 0$, the principal's sales are $0 with probability 0.6, $100 with probability 0.3, and $400 with probability 0.1.
  - In the case of $e_H = 5$, the expected profit is $270, while in the case of $e_L = 0$, the expected profit is $70.

Symmetric Information

- Example 1: (con't)
  - When effort is observable, the principal can induce an effort $e_H = 5$ by paying a wage w_e^* that solves

$$u(w_e^*) = \bar{u} + g(e)$$
$$\sqrt{w_e^*} = 9 + 5$$
$$w_e^* = 14^2 = 196$$

  - Similarly, the principal can induce a low effort $e_L = 0$ by offering a wage

$$\sqrt{w_e^*} = 9 + 0$$
$$w_e^* = 9^2 = 81$$

Symmetric Information

- Example 1: (con't)
  - Given these salaries, the profits that the principal obtains are

$$\$270 - \$196 = \$74 \text{ from } e_H = 5$$

$$\$70 - \$81 = -\$11 \text{ from } e_L = 0$$

  - Thus the principal prefers to induce e_H when effort is observable.

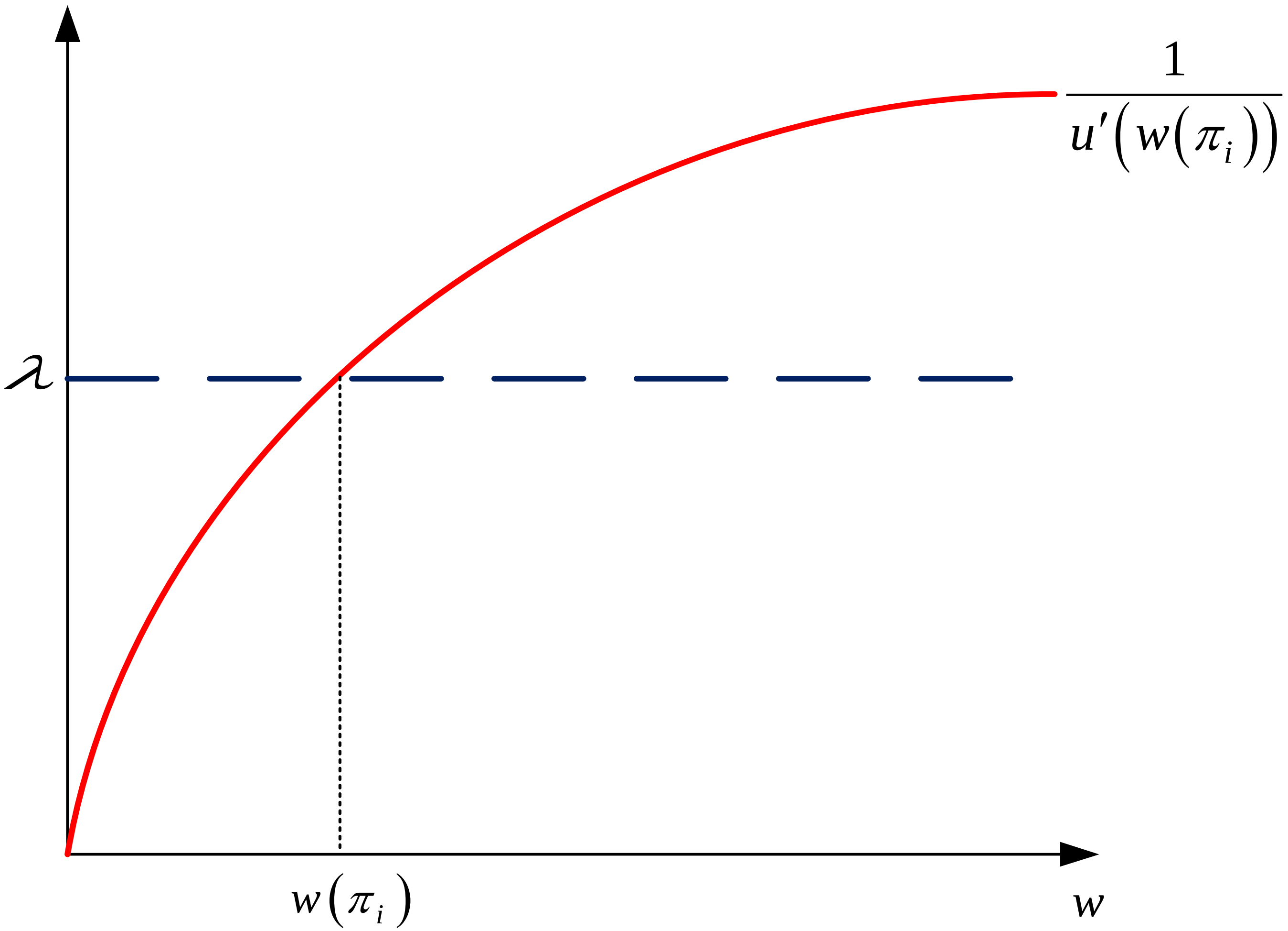
Risk Attitudes

- Continuing with the moral hazard setting under symmetric information, let us now consider the role of **risk aversion**.
- Three cases:
  1. The principal is risk neutral but the agent is risk averse
  2. The principal is risk averse but the agent is risk neutral
  3. Both the principal and the agent are risk averse

Risk Attitudes: Case 1

- The principal is risk neutral but the agent is risk averse.
- The principal's benefit function is
$$B\big(\pi_i - w(\pi_i)\big) = \pi_i - w(\pi_i)$$
- Hence,
$$B'\big(\pi_i - w(\pi_i)\big) = -1$$
- In this context, FOC in expression **(1)** becomes
$$\lambda = \frac{1}{u'(w(\pi_i))} \text{ for all } \pi_i \qquad \textbf{(2)}$$

Risk Attitudes: Case 1



Risk Attitudes: Case 1

- FOC in **(2)** entails that the principal pays a **fixed wage** level for all profit realizations.
- For any $\pi_i \neq \pi_j$,

$$\lambda = \frac{1}{u'(w(\pi_i))} = \frac{1}{u'\left(w(\pi_j)\right)}$$

$$u'(w(\pi_i)) = u'\left(w(\pi_j)\right)$$

$$w(\pi_i) = w(\pi_j) \text{ given } u' > 0$$

- This is a **standard risk-sharing result**
  - The risk-neutral principal offers a contract to the risk-averse agent that guarantees the latter a fixed salary of w^* regardless of the specific profit realization that emerges.
  - The risk-neutral principal bears all the risk.

Risk Attitudes: Case 1

- Since the agent's PC binds, we can express it

$$u(w_e^*) - g(e) = \bar{u}$$

- Rearranging the PC expression

$$u(w_e^*) = \bar{u} + g(e)$$

- Applying the inverse

$$w_e^* = u^{-1}(\bar{u} + g(e))$$

- This expression helps to identify the salary that the principal needs to offer in order to induce a specific effort level e from the agent.

Risk Attitudes: Case 1

- For two effort levels e_L and e_H, the disutility of effort function satisfies

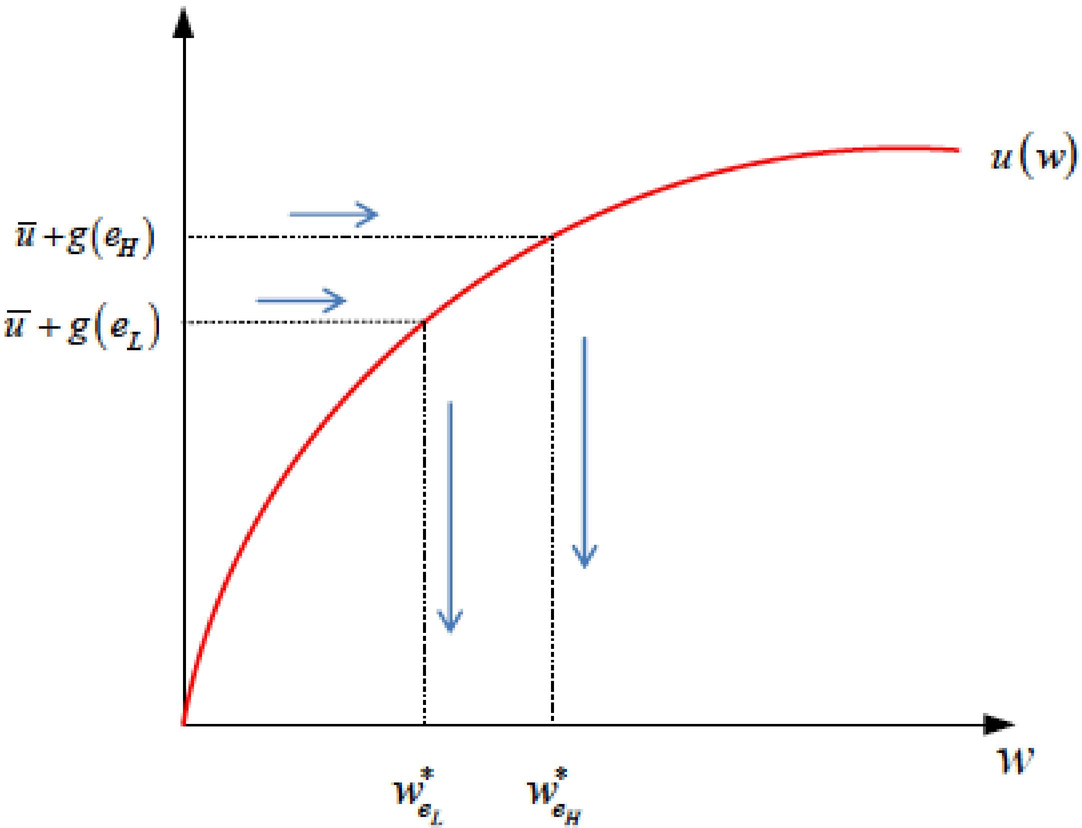
$$g(e_L) < g(e_H)$$

- This entails

$$w_{e_L}^* = u^{-1}\left(\bar{u} + g(e_L)\right) < u^{-1}\left(\bar{u} + g(e_H)\right) = w_{e_H}^*$$

- In order to induce e_H, we need to evaluate the utility function at a height of $\bar{u} + g(e_H)$.
- Inducing a higher effort implies offering a higher salary.

Risk Attitudes: Case 1



Risk Attitudes: Case 1

- We can plug a salary $w_e^* = u^{-1}\big(\bar{u} + g(e)\big)$ into the principal's objective function in order to find the effort level that maximizes the principal's expected profits

$$\max_{e} \sum_{i=1}^{N} f(\pi_i|e)\big(\pi_i - w(\pi_i)\big)$$

$$= \max_{e} \sum_{i=1}^{N} f(\pi_i|e) \cdot \pi_i - \underbrace{u^{-1}\big(\bar{u} + g(e)\big)}_{w_e^*}$$

where w_e^* does not depend on π_i.

- This helps to reduce the number of choice variables to only the effort level e.

Risk Attitudes: Case 1

- Taking FOC with respect to e yields

$$\sum_{i=1}^{N} f'(\pi_i|e) \cdot \pi_i - \frac{\partial u^{-1}\left(\bar{u} + g(e)\right)}{\partial e} g'(e) = 0$$

where $\frac{\partial u^{-1}(\bar{u}+g(e))}{\partial e}$ can be expressed as $(u^{-1})'\left(\bar{u} + g(e)\right)$.

- By the implicit function theorem,

$$(u^{-1})'\left(\bar{u} + g(e)\right) = (u')^{-1}\left(\bar{u} + g(e)\right)$$

- Hence the above FOC can be rewritten as

$$\sum_{i=1}^{N} f'(\pi_i|e) \cdot \pi_i = \frac{g'(e)}{u'\left(\bar{u} + g(e)\right)}$$

Risk Attitudes: Case 1

- Intuition:
  - Effort e is increased until the point at which its expected profit (left-hand side) coincides with its certain costs (in the right-hand side), which stems from a larger disutility of effort for the agent (numerator) that needs to be compensated with a more generous salary (denominator).
- See textbook for the second-order condition that guarantees concavity.

Risk Attitudes: Case 2

- The principal is risk averse but the agent is risk neutral.
- The principal's benefit function is $B\big(\pi_i - w(\pi_i)\big)$, with $B' > 0$ and $B'' < 0$.
- The agent's utility function is
$$u(w_i) - g(e) = w_i - g(e)$$
- In this context, FOC in expression **(1)** becomes
$$\lambda = B'\big(\pi_i - w(\pi_i)\big)$$
where $u'\big(w(\pi_i)\big) = 1$.

Risk Attitudes: Case 2

- FOC entails that it is now the principal who obtains a **fixed payoff** for all profit realizations.
- For any $\pi_i \neq \pi_j$,

$$\lambda = B'(\pi_i - w(\pi_i)) = B'\left(\pi_j - w(\pi_j)\right)$$

$$\pi_i - w(\pi_i) = \pi_j - w\left(\pi_j\right) = K \text{ given } B' > 0$$

- That is, the risk-averse principal receives the same payoff regardless of the profit realization π, whereas the risk-neutral agent now bears all the risk.

Risk Attitudes: Case 2

- The agent's salary is
$$w(\pi_i) = \pi_i - K$$
where K is found by making the agent indifferent between accepting and rejecting the franchise contract
- Fee K solves
$$\sum_{i=1}^{N} f(\pi_i|e)[\pi_i - K] - g(e) = \bar{u}$$
$$K = \sum_{i=1}^{N} f(\pi_i|e)\pi_i - \bar{u} - g(e)$$

Risk Attitudes: Case 2

- The principal's expected profit is

$$\sum_{i=1}^{N} f(\pi_i|e)B\big(\pi_i - w(\pi_i)\big)$$
$$= \sum_{i=1}^{N} f(\pi_i|e)B\big(\pi_i - (\pi_i - K)\big)$$
$$= \sum_{i=1}^{N} f(\pi_i|e)B(K) = B(K)$$

- The principal's problem can then be written as

$$\max_{e} \; B(K) = B\left(\sum_{i=1}^{N} f(\pi_i|e)\pi_i - \bar{u} - g(e)\right)$$

Risk Attitudes: Case 2

- Taking FOC with respect to e yields

$$B'\left(\sum_{i=1}^{N} f(\pi_i|e)\pi_i - \bar{u} - g(e)\right)\left(\sum_{i=1}^{N} f'(\pi_i|e)\pi_i - g'(e)\right) = 0$$

which simplifies to

$$\sum_{i=1}^{N} f'(\pi_i|e)\pi_i = g'(e)$$

- Intuition:
  - Effort e is increased until the point where marginal expected profit from having the agent exert more effort (left-hand side) coincides with his marginal disutility (right-hand side).

Risk Attitudes: Case 2

- The second-order condition is

$$\sum_{i=1}^{N} f''(\pi_i|e)\pi_i - g''(e) \leq 0$$

where $g''(e) \geq 0$.

Risk Attitudes: Case 3

- Both the principal and the agent are risk averse.
- Recall the FOCs with respect to w

$$B'\big(\pi_i - w(\pi_i)\big)\cdot(-1) + \lambda u'\big(w(\pi_i)\big) = 0 \qquad \textbf{(3)}$$

$$\lambda = \frac{B'\big(\pi_i - w(\pi_i)\big)}{u'\big(w(\pi_i)\big)} \qquad \textbf{(4)}$$

- To better understand how the profit-maximizing salary is affected by the profit realization π_i, differentiate **(3)** with respect to π_i

$$-B''\big(\pi_i - w(\pi_i)\big) + B''\big(\pi_i - w(\pi_i)\big)w'(\pi_i) + \lambda u''\big(w(\pi_i)\big)w'(\pi_i) = 0$$

Risk Attitudes: Case 3

- Plugging λ from **(4)** yields

$$-B''\big(\pi_i - w(\pi_i)\big) + B''\big(\pi_i - w(\pi_i)\big)w'(\pi_i) + \frac{B'\big(\pi_i - w(\pi_i)\big)}{u'\big(w(\pi_i)\big)}u''\big(w(\pi_i)\big)w'(\pi_i) = 0$$

- Factoring out $w'(\pi_i)$ yields

$$B''\big(\pi_i - w(\pi_i)\big) = \left[B''\big(\pi_i - w(\pi_i)\big) + B'\big(\pi_i - w(\pi_i)\big)\frac{u''\big(w(\pi_i)\big)}{u'\big(w(\pi_i)\big)}\right]w'(\pi_i)$$

Risk Attitudes: Case 3

- Solving for $w'(\pi_i)$ yields

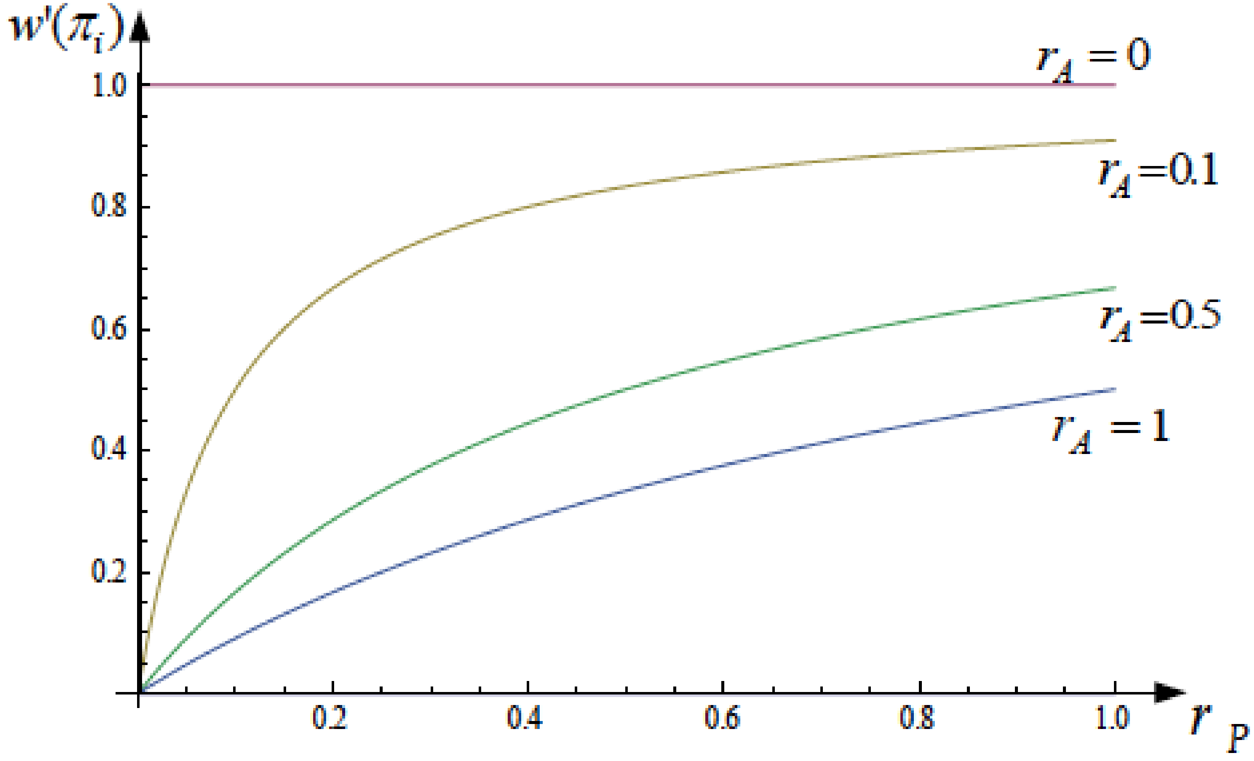
$$w'(\pi_i) = \frac{B''\big(\pi_i - w(\pi_i)\big)}{B''(\cdot) + \frac{u''\big(w(\pi_i)\big)}{u'\big(w(\pi_i)\big)} B'(\cdot)}$$

- Dividing numerator and denominator by $B'(\cdot)$ yields

$$w'(\pi_i) = \frac{B''(\cdot)/B'(\cdot)}{B''(\cdot)/B'(\cdot) + \frac{u''(\cdot)}{u'(\cdot)}} = \frac{r_P}{r_P + r_A} \qquad \textbf{(5)}$$

where r_P and r_A denote the the **Arrow–Pratt coefficient of absolute risk aversion** of the principal and the agent, respectively.

Risk Attitudes: Case 3



Risk Attitudes: Case 3

- Let us next evaluate the ratio in expression **(5)** at different values of r_P and r_A.
- **Risk neutral principal**: $r_P = 0$
  - The expression in **(5)** becomes $w'(\pi_i) = 0$.
  - This result holds regardless of the agent's coefficient of risk aversion $r_A > 0$.
- This setting coincides with that in Case 1, where the agent receives a fixed wage to insure him against the profit realization π_i, whereas the risk-neutral principal bears all the risk.

Risk Attitudes: Case 3

- **Risk neutral agent**: $r_A = 0$
  - The expression in **(5)** becomes $w'(\pi_i) = 1$.
  - This holds regardless of principal's coefficient of risk aversion $r_P > 0$.
  - This setting coincides with that in Case 2, where the risk-neutral agent bears all the risk while the principal receives a fixed payment K that insures him against different profit realization π_i.

Risk Attitudes: Case 3

- **Agent is more risk averse than principal**: $r_A > r_P > 0$
  - The expression in **(5)** becomes $w'(\pi_i) < 1/2$.
  - It is optimal for the agent's salary $w'(\pi_i)$ to exhibit small variations in the profit realization π_i.
  - The more risk-averse agent bears less payoff volatility.

Risk Attitudes: Case 3

- **Principal is more risk averse than agent**: $r_P > r_A > 0$
  - The expression in **(5)** becomes $w'(\pi_i) > 1/2$.
  - The less risk-averse agent bears more payoff volatility.

Risk Attitudes: Case 3

- **Same degree of risk aversion**: $r_A = r_P = r > 0$
  - The expression in **(5)** becomes $w'(\pi_i) = 1/2$.
  - Both the agent and the principal bear the same risk in the contract.

Asymmetric Information

- The principal **cannot observe** the agent's effort level e.
- The principal needs to offer to the agent enough incentives to exert the profit-maximizing effort level.
- How can the principal achieve this objective?
  - Make the salary an increasing function of the realized profit.
  - This is optimal even if the agent is risk averse.

Asymmetric Information

- The principal's problem is

$$\max_{\{e, w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e) \cdot B(\pi_i - w(\pi_i))$$

s.t. $\sum_{i=1}^{N} f(\pi_i|e)\,[u(w(\pi_i)) - g(e)] \geq \bar{u}$

$$e^* \in \arg\max_{e} \sum_{i=1}^{N} f(\pi_i|e)\,[u(w(\pi_i)) - g(e)]$$

- The principal seeks to maximize its expected profits subject to:
  1. The voluntary participation of the agent (PC condition);
  2. The effort that he anticipates the agent will optimally choose in order to maximize his expected utility after receiving the contract from the principal (incentive compatibility, IC, condition).

Asymmetric Information

- Assume there are only two different effort levels available to the agent (e_L and e_H, where $e_H > e_L$).
- The agent can choose to work a positive number of hours or completely slack from the job ($e_H = e > 0$ and $e_L = 0$).
- Consider that the principal seeks to induce the high effort level e_H and that the principal is **risk neutral** while the agent is **risk averse**.

Asymmetric Information

- The principal's problem reduces to

$$\max_{\{e,w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e_H) \cdot [\pi_i - w(\pi_i)]$$

s.t. $\sum_{i=1}^{N} f(\pi_i|e_H)\,[u(w(\pi_i)) - g(e_H)] \geq \bar{u}$ (PC)

$\sum_{i=1}^{N} f(\pi_i|e_H)\,\big[u\big(w(\pi_i)\big) - g(e_H)\big] \geq$
$\sum_{i=1}^{N} f(\pi_i|e_L)\,[u(w(\pi_i)) - g(e_L)]$ (IC)

where the IC condition induces the agent to choose effort level e_H as such effort yields a higher expected utility than e_L for the agent.

Asymmetric Information

- The Lagrangian becomes

$$\mathcal{L} = \sum_{i=1}^{N} f(\pi_i|e_H) \cdot [\pi_i - w(\pi_i)]$$
$$+ \lambda \left[\sum_{i=1}^{N} f(\pi_i|e_H) \left[u\big(w(\pi_i)\big) - g(e_H) \right] - \bar{u} \right]$$
$$+ \mu \left\{ \sum_{i=1}^{N} f(\pi_i|e_H) \left[u\big(w(\pi_i)\big) - g(e_H) \right] \right.$$
$$\left. - \left[\sum_{i=1}^{N} f(\pi_i|e_L) \left[u\big(w(\pi_i)\big) - g(e_L) \right] \right] \right\}$$

Asymmetric Information

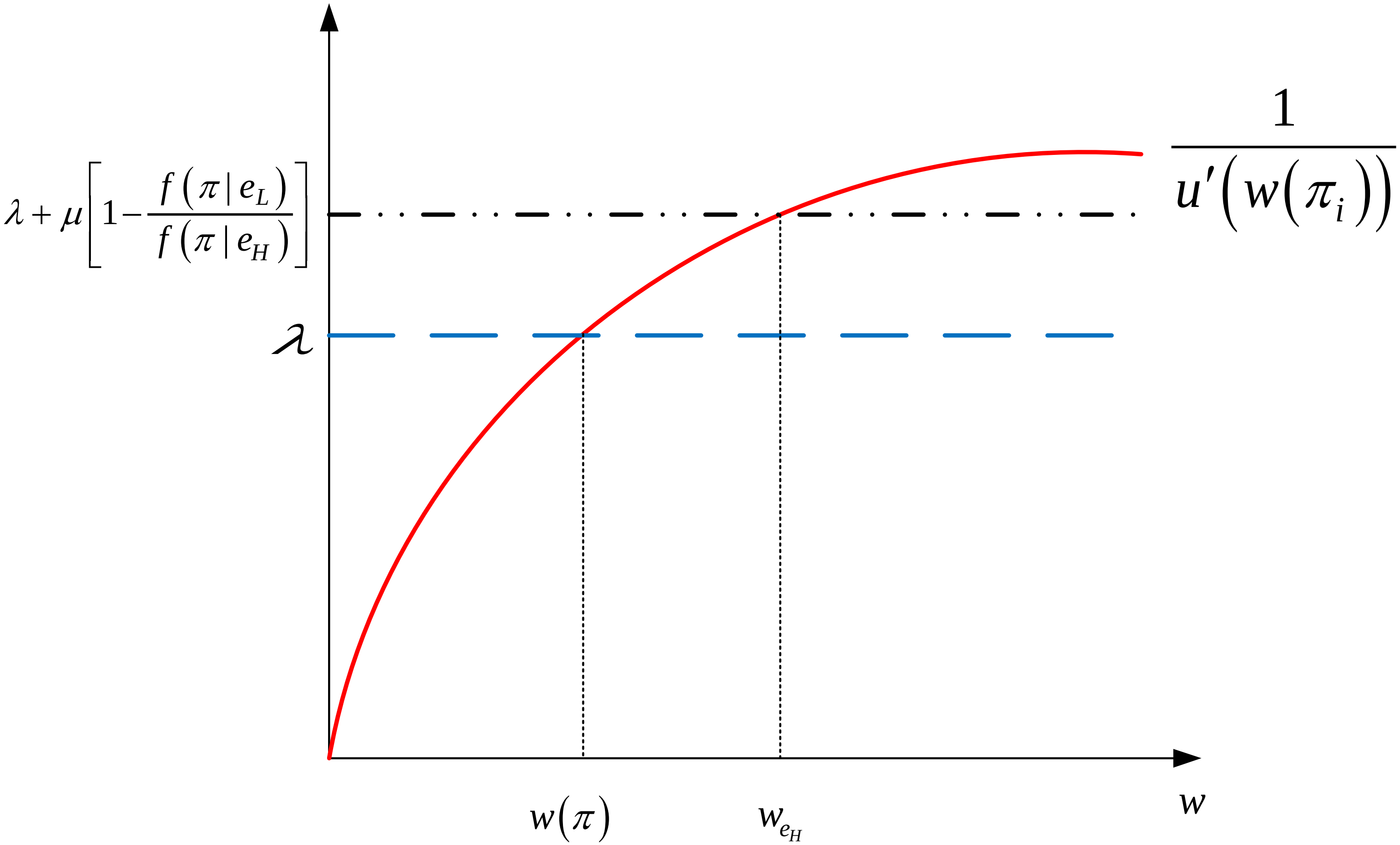
- Taking FOC with respect to w yields
  $$-f(\pi_i|e_H) + \lambda f(\pi_i|e_H)u'\big(w(\pi_i)\big) + \mu\big[f(\pi_i|e_H)u'\big(w(\pi_i)\big) - f(\pi_i|e_L)u'\big(w(\pi_i)\big)\big] = 0$$
- Rearranging
  $$\lambda + \underbrace{\mu\left[1 - \frac{f(\pi_i|e_L)}{f(\pi_i|e_H)}\right]}_{\text{New}} = \frac{1}{u'\big(w(\pi_i)\big)} \qquad \textbf{(6)}$$
- Compare **(6)** with expression **(2)** in case 1, where the principal was risk neutral but the agent was risk averse.

Asymmetric Information

- Because $\lambda > 0$, $\mu > 0$, and $\frac{f(\pi_i|e_L)}{f(\pi_i|e_H)} < 1$, then

$$\underbrace{\lambda + \mu\left[1 - \frac{f(\pi_i|e_L)}{f(\pi_i|e_H)}\right]}_{\text{asymmetric info.}} > \underbrace{\lambda}_{\text{symmetric info.}}$$

Asymmetric Information



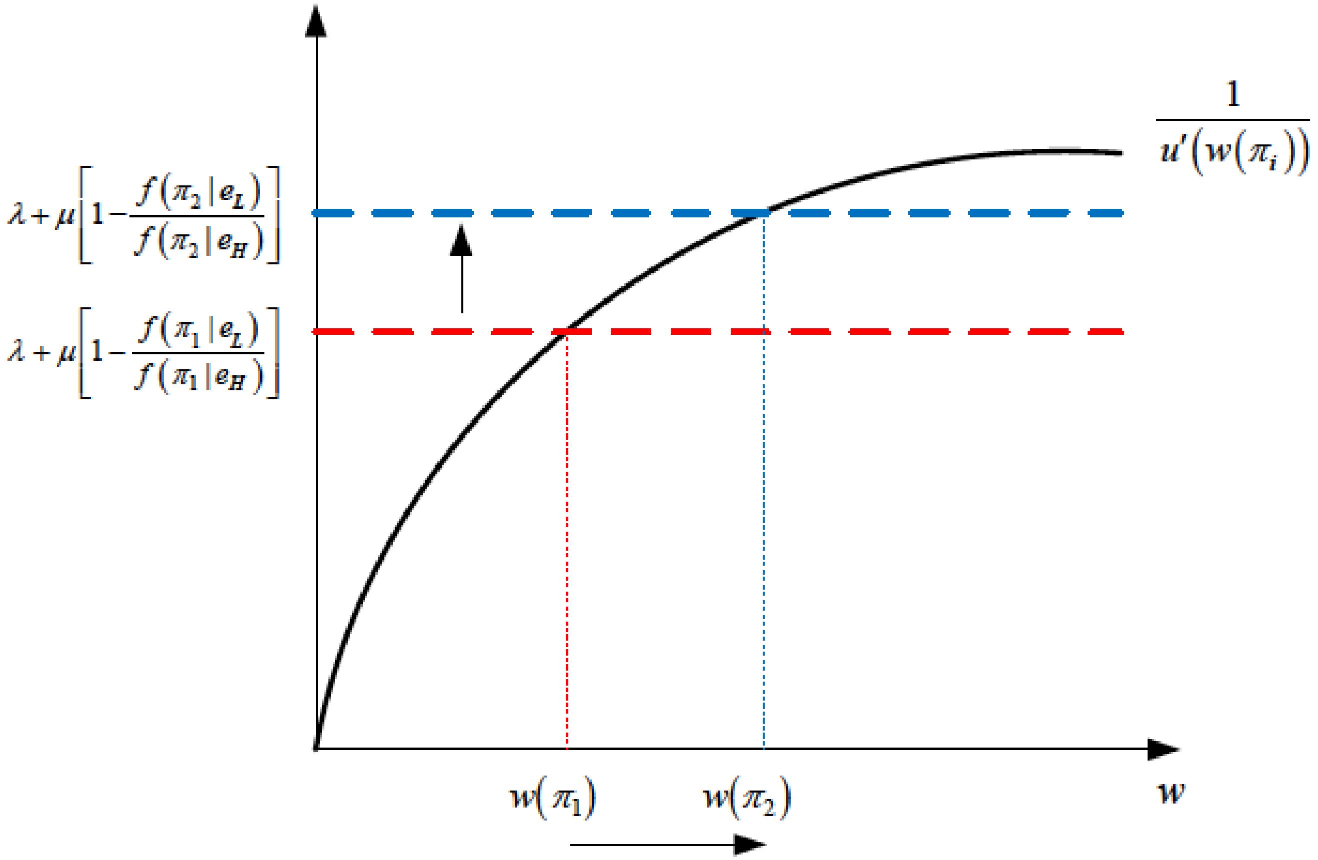
Asymmetric Information

- When deciding which effort to implement, the principal compares the effects of inducing a high effort level e_H.
  - Effort e_H yields a **positive effect** on profits since it increases the likelihood of higher profits.
  - This positive effect emerges under both symmetric and asymmetric information.
  - Effort e_H also entails a **negative effect** on profits since the salary that induces such effort is higher under asymmetric than under symmetric information $w_{e_H} > w(\pi)$.
  - Hence the principal is less willing to induce e_H when the agent's effort is unobservable than when it is observable.

Comparative Statics

- How does the salary above change as a function of the profit realization?
  - For that to happen, the left-hand side of **(6)** needs to increase in π.
  - This occurs if the likelihood ratio $\frac{f(\pi_i|e_L)}{f(\pi_i|e_H)}$ decreases in π.
  - Intuitively, as profits increase, the likelihood of obtaining a profit level of π from effort e_H increases faster than the probability of obtaining such a profit level from e_L.
  - This probability is commonly known as the **monotone likelihood ratio property**, MLRP.

Comparative Statics



Comparative Statics

- Example 2:
  - Consider Example 1, but assuming that the principal cannot observe the agent's effort.
  - In this incomplete information setting, the principal must offer a salary that increases in profit if he seeks to induce $e_H = 5$.
  - The principal's maximization problem becomes

$$\max_{\{w(\pi_i)\}_{i=1}^{3}} \quad 270 - [0.1w(\pi_1) + 0.3w(\pi_2) + 0.6w(\pi_3)]$$

s.t. $0.1\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} - 5 \geq 9$ (PC)

$0.1\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} - 5 \geq$
$0.6\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.1\sqrt{w(\pi_3)}$ (IC)

Comparative Statics

- Example 2: (con't)
  - Since the principal's revenue is a constant (\$270), he can alternatively minimize its expected costs

$$\min_{\{w(\pi_i)\}_{i=1}^{3}} \quad 0.1w(\pi_1) + 0.3w(\pi_2) + 0.6w(\pi_3)$$

s.t. $0.1\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} - 14 \geq 0$ (PC)

$-0.5\sqrt{w(\pi_1)} + 0.5\sqrt{w(\pi_3)} - 5 \geq 0$ (IC)

where the IC constraint has been simplified.

Comparative Statics

- Example 2: (con't)
  - The associated Lagrangian is

$$\mathcal{L} = \quad 0.1w(\pi_1) + 0.3w(\pi_2) + 0.6w(\pi_3)$$
$$-\lambda\left[0.1\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} - 14\right]$$
$$-\mu\left[-0.5\sqrt{w(\pi_1)} + 0.5\sqrt{w(\pi_3)} - 5\right]$$

  - Taking FOC with respect to $w(\pi_1)$, $w(\pi_2)$, and $w(\pi_3)$ yields

$$\frac{\partial\mathcal{L}}{\partial w(\pi_1)} = 0.1 - \frac{0.1\lambda}{2\sqrt{w(\pi_1)}} + \frac{0.5\mu}{2\sqrt{w(\pi_1)}} = 0 \qquad \textbf{(7)}$$

$$\frac{\partial\mathcal{L}}{\partial w(\pi_2)} = 0.3 - \frac{0.3\lambda}{2\sqrt{w(\pi_2)}} = 0 \qquad \textbf{(8)}$$

$$\frac{\partial\mathcal{L}}{\partial w(\pi_3)} = 0.6 - \frac{0.6\lambda}{2\sqrt{w(\pi_3)}} - \frac{0.5\mu}{2\sqrt{w(\pi_3)}} = 0 \qquad \textbf{(9)}$$

Comparative Statics

- Example 2: (con't)
  - Rearranging **(7)** and **(8)**

$$\lambda = 2\sqrt{w(\pi_2)}$$
$$\mu = 0.4\sqrt{w(\pi_2)} - 0.4\sqrt{w(\pi_1)}$$

  - Plugging these values into **(9)** and rearranging

$$0.1\sqrt{w(\pi_1)} - 0.7\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} = 0 \quad \textbf{(10)}$$

  - Combining equation **(10)** with the (PC) and (IC) equations, we have three equations and three unknowns $w(\pi_1)$, $w(\pi_2)$, and $w(\pi_3)$.

Comparative Statics

- Example 2: (con't)
  - The (IC) equation yields
$$\sqrt{w(\pi_3)} = 10 + \sqrt{w(\pi_1)}$$
  - Substituting this into the (PC) equation
$$3\sqrt{w(\pi_2)} = 80 - 7\sqrt{w(\pi_1)}$$
  - Last, substituting the values of $w(\pi_2)$ and $w(\pi_3)$ in equation **(10)**
$$w(\pi_1) = \$29.47, w(\pi_2) = \$196, w(\pi_2) = \$238.04$$
  - The principal's expected profit is then
$$270 - [0.1 \cdot 29.47 + 0.3 \cdot 196 + 0.6 \cdot 238.04] = \$65.43$$
which is lower than its profit when effort is observable (\$74).

Moral Hazard with a Continuum of Effort Levels—The First-Order Approach

Continuum of Effort Levels

- So far we assumed that a worker could only have a discrete number of effort levels.
- Let us now consider a continuum of effort levels.
- The principal seeks to maximize its expected profits by anticipating the effort level that the agent selects in the second stage of the game:

$$\max_{\{e, w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e) \cdot [\pi_i - w(\pi_i)]$$

s.t. $\sum_{i=1}^{N} f(\pi_i|e)\, u(w(\pi_i)) - g(e) \geq \bar{u}$ (PC)

$e^* \in \arg\max_{e} \sum_{i=1}^{N} f(\pi_i|e) \left[u\big(w(\pi_i)\big) - g(e)\right]$ (IC)

Continuum of Effort Levels

- Difference/similarities between discrete and continuum effort levels
  - The objective function of the principal and the PC condition for the agent coincide.
  - The agent's IC condition, however, differs as it now allows him to choose among a continuum of effort levels.
  - Intuitively, the IC condition represents the agent's UMP where, for a given salary $w(\pi_i)$, the agent selects an effort level e that maximizes his expected utility.

Continuum of Effort Levels

- Differentiating the agent's expected utility with respect to e yields

$$\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e) = 0$$

- The agent's FOC above can be used as the IC condition in the principal's problem.
- This approach is known as the **first-order approach**.

Continuum of Effort Levels

- The principal's problem, using a "first-order approach," is then

$$\max_{\{e,w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e) \cdot [\pi_i - w(\pi_i)]$$

s.t. $\sum_{i=1}^{N} f(\pi_i|e)\, u(w(\pi_i)) - g(e) \geq \bar{u}$ (PC)

$\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e) = 0$ (IC)

Continuum of Effort Levels

- The the Lagrangian becomes

$$\mathcal{L} = \sum_{i=1}^{N} f(\pi_i|e) \cdot [\pi_i - w(\pi_i)] + \lambda\left[\sum_{i=1}^{N} f(\pi_i|e)\, u\big(w(\pi_i)\big) - g(e) - \bar{u}\right] + \mu\left[\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e)\right] = 0$$

- Taking FOC with respect to w yields

$$\frac{\partial \mathcal{L}}{\partial w} = -f(\pi_i|e) + \lambda f(\pi_i|e) u'\big(w(\pi_i)\big) + \mu f'(\pi_i|e) u'\big(w(\pi_i)\big) = 0$$

Continuum of Effort Levels

- Dividing both sides by $f(\pi_i|e)$

$$-1 + \lambda u'\big(w(\pi_i)\big) + \mu \frac{f'(\pi_i|e)}{f(\pi_i|e)} u'\big(w(\pi_i)\big) = 0$$

- Factoring out $u'\big(w(\pi_i)\big)$ on the left-hand side and rearranging

$$\lambda + \mu \frac{f'(\pi_i|e)}{f(\pi_i|e)} = \frac{1}{u'\big(w(\pi_i)\big)}$$

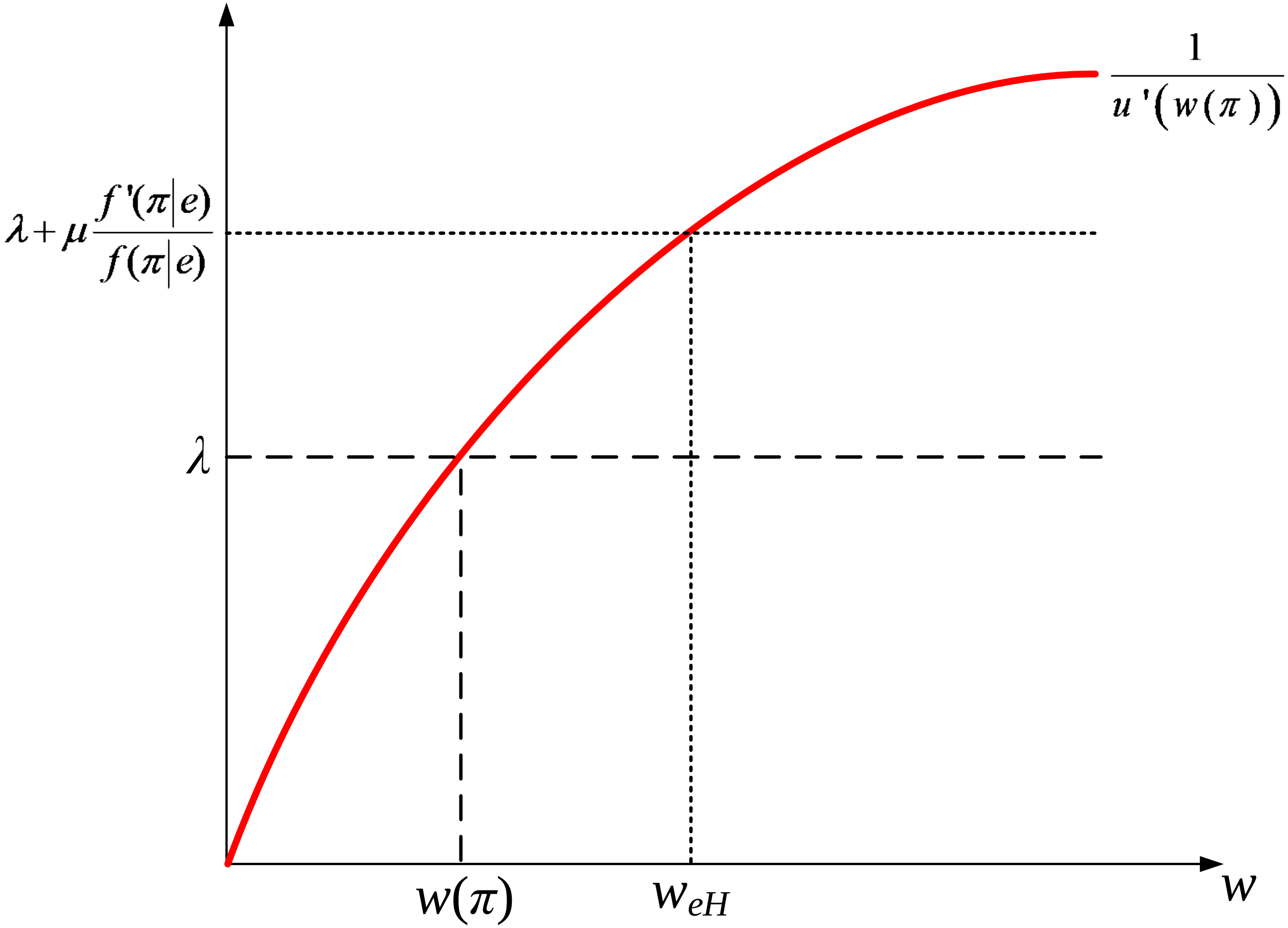
- This result is similar to that in previous sections.
- Because $\lambda > 0$ and $\mu > 0$ (since PC and IC bind), the left-hand side satisfies

$$\lambda + \mu \frac{f'(\pi_i|e)}{f(\pi_i|e)} > \lambda$$

Continuum of Effort Levels

- Since u' is decreasing in w (by concavity), its inverse, $1/u'$, is increasing in w.
- Hence the principal offers a larger salary under asymmetric information than symmetric information.
- $\frac{f'(\pi_i|e)}{f(\pi_i|e)}$ is the likelihood ratio, which measures how a marginally higher effort entails a larger probability of obtaining a given profit level π_i relative to an initial effort level.

Continuum of Effort Levels



Continuum of Effort Levels

- Taking FOC with respect to e yields

$$\frac{\partial \mathcal{L}}{\partial e} = \sum_{i=1}^{N} f'(\pi_i|e) \cdot [\pi_i - w(\pi_i)] + \mu \left[\sum_{i=1}^{N} f''(\pi_i|e)\, u\big(w(\pi_i)\big) - g''(e)\right]$$

$$+ \lambda \left[\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e)\right] = 0$$

- Rearranging

$$\sum_{i=1}^{N} f'(\pi_i|e)\, \pi_i = \sum_{i=1}^{N} f'(\pi_i|e)\, w(\pi_i)$$
$$-\mu \left[\sum_{i=1}^{N} f''(\pi_i|e)\, u\big(w(\pi_i)\big) - g''(e)\right]$$
$$-\lambda \left[\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e)\right]$$

(11)

- Intuitively, effort is increased until the point where its expected profits (left-hand side) coincide with its associated costs (right-hand side).

Continuum of Effort Levels

- The cost of inducing a higher effort originates from two sources:
  1. A higher effort increases the probability of obtaining a higher profit, and thus the salary that the principal pays the agent once the profit is realized (first term on the right-hand side).
  2. The principal must provide more incentives (higher salary) in order for the agent to exert the effort level that the principal intended (second term on the right-hand side).

Continuum of Effort Levels

- Example 3:
  - Moral hazard with continuous effort but only two possible outcomes.
  - Consider a setting in which the conditional probability satisfies

$$f(\pi_i|e) = ef_H(\pi_i) + (1-e)f_L(\pi_i),\ e \in [0,1]$$

  - When effort is relatively high $e \to 1$, the probability of obtaining a profit level π_i is $f_H(\pi_i)$, where $f_H(\pi_i) > f_L(\pi_i)$.
  - When $e \to 0$, the probability of obtaining a profit level π_i is $f_L(\pi_i)$.

Continuum of Effort Levels

- Example 3: (con't)
  - The agent's expected utility is

$$EU(e) = \sum_{i=1}^{N} [ef_H(\pi_i) + (1-e)f_L(\pi_i)]u(w(\pi_i)) - g(e)$$

  - Since

$$ef_H(\pi_i) + (1-e)f_L(\pi_i) = e[f_H(\pi_i) - f_L(\pi_i)] + f_L(\pi_i)$$

  - Then

$$EU(e) = \sum_{i=1}^{N} e[f_H(\pi_i) - f_L(\pi_i)]u(w(\pi_i)) + \sum_{i=1}^{N} f_L(\pi_i)u(w(\pi_i)) - g(e)$$

  - Differencing $EU(e)$ twice with respect to effort e, yields $-g''(e)$, which is negative by definition.
  - So we can use the first-order approach.

Continuum of Effort Levels

- Example 3: (con't)
  - The agent's FOC with respect to e is
    $$\sum_{i=1}^{N}[f_H(\pi_i) - f_L(\pi_i)]u(w(\pi_i)) = g'(e)$$
  - Plugging this FOC into the principal's problem
    $$\max_{\{e,w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N}[ef_H(\pi_i) + (1-e)f_L(\pi_i)] \cdot [\pi_i - w(\pi_i)]$$

s.t. $\sum_{i=1}^{N}[ef_H(\pi_i) + (1-e)f_L(\pi_i)]\, u\big(w(\pi_i)\big) - g(e) \geq \bar{u}$ (PC)

$\sum_{i=1}^{N}[f_H(\pi_i) - f_L(\pi_i)]u(w(\pi_i)) = g'(e)$ (IC)

Continuum of Effort Levels

- Example 3: (con't)
  - The Lagrangian of this program is

$$\mathcal{L} = \sum_{i=1}^{N} [ef_H(\pi_i) + (1-e)f_L(\pi_i)] \cdot [\pi_i - w(\pi_i)]$$
$$+ \lambda \left[\sum_{i=1}^{N} [ef_H(\pi_i) + (1-e)f_L(\pi_i)]\, u\big(w(\pi_i)\big) - g(e) - \bar{u} \right]$$
$$+ \mu \left[\sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)] u\big(w(\pi_i)\big) - g'(e) \right]$$

  - Taking FOC with respect to w and rearranging

$$\lambda + \mu \frac{f_H(\pi_i) - f_L(\pi_i)}{ef_H(\pi_i) + (1-e)f_L(\pi_i)} = \frac{1}{u'\big(w(\pi_i)\big)}$$

Continuum of Effort Levels

- Example 3: (con't)
  - Taking FOC with respect to e and rearranging

$$\sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\, \pi_i$$
$$= \sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\, w(\pi_i) + \mu g''(e)$$
$$- \lambda \left[\sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\, u\big(w(\pi_i)\big) - g'(e) \right]$$

Continuum of Effort Levels

- Example 3: (con't)
  - From the binding (IC), we can further simplify and obtain

$$\sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\,\pi_i$$
$$= \sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\,w(\pi_i) + \mu g''(e)$$

  - The expected profit to the principal (left-hand side) is exactly balanced by the expected cost of inducing effort e from the agent (right-hand side).

Continuum of Effort Levels

- Example 4:
  - Moral hazard using the first-order approach
  - Assume the expected utility function of the agent is

$$u(w,e) = E(w) - \frac{1}{2}\rho Var(w) - c(e)$$

    where:

    - ρ is the Arrow–Pratt coefficient of absolute risk aversion for utility function $u(w) = -\mathrm{e}^{-\rho w}$,
    - $e \in [0,1]$ is the agent's effort, and
    - $c(e) = 0.5e^2$ is the cost of effort.
  - The outcome of the project, x, is stochastic and given by

$$x = f(e,x) = e + \varepsilon, \text{ where } \varepsilon \sim N(0,\sigma^2)$$

Continuum of Effort Levels

- Example 4: (con't)
  - The agent's reservation utility is $\bar{u} = \frac{1}{2}$.
  - The principal offers a linear contract to the agent
  $$w(x) = a + bx$$
  - where $a > 0$ is a fixed payment, and $b \in [0,1]$ is the share of profits that the agent receives (bonus).
  - The principal's expected profits are
  $$E(\pi) = E(x - w) = E(x) - E(w)$$
  $$= E(x) - [a + bE(x)] = (1 - b)e - a$$

Continuum of Effort Levels

- Example 4: (con't)
  - Since $E(x) = e$, the expected utility of the agent when he exerts effort level e is

$$E[u(w,e)] = E(w) - \frac{1}{2}\rho Var(w) - c(e)$$
$$= a + be - \frac{1}{2}\rho b^2\sigma^2 - \frac{1}{2}e^2$$

where $E(w) = a + be, Var(w) = b^2\sigma^2$, and $c(e) = \frac{1}{2}e^2$.

Continuum of Effort Levels

- Example 4: (con't)
  - Taking FOC with respect to e, we can find the effort that the agent chooses

$$\frac{\partial E[u(w,e)]}{\partial e} = b - e = 0$$

$$e = b$$

  - The principal's problem is to choose the fixed payment, a, and the bonus, b, to solve

$$\max_{e,a,b} \ (1-b)e - a$$

s.t. $$a + be - \frac{1}{2}\rho b^2\sigma^2 - \frac{1}{2}e^2 \geq \frac{1}{2} \qquad \text{(PC)}$$

$$e = b \qquad \text{(IC)}$$

Continuum of Effort Levels

- Example 4: (con't)
  - Plugging $e = b$ into the program and simplifying

$$\max_{a,b} (1-b)b - a$$

$$\text{s.t.} \qquad a + \frac{1}{2}b^2(1-\rho\sigma^2) \geq \frac{1}{2} \qquad \text{(PC)}$$

  - The Lagrangian is

$$\mathcal{L} = (1-b)b - a + \lambda\left[a + \frac{1}{2}b^2(1-\rho\sigma^2) - \frac{1}{2}\right]$$

Continuum of Effort Levels

- Example 4: (con't)
  - The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial a} = -1 + \lambda = 0 \rightarrow \lambda = 1 \qquad \textbf{(12)}$$

$$\frac{\partial \mathcal{L}}{\partial b} = 1 - 2b + \lambda b(1 - \rho\sigma^2) = 0 \qquad \textbf{(13)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = a + \frac{1}{2} b^2(1 - \rho\sigma^2) - \frac{1}{2} = 0 \qquad \textbf{(14)}$$

  - Plugging **(12)** into **(13)** yields

$$1 - 2b + b(1 - \rho\sigma^2) = 0$$

$$b = \frac{1}{1 + \rho\sigma^2}$$

Continuum of Effort Levels

- Example 4: (con't)
  - Plugging $b = \frac{1}{1+\rho\sigma^2}$ into the binding (PC) constraint yields

$$a + \frac{1 - \rho\sigma^2}{2(1 + \rho\sigma^2)^2} = \frac{1}{2}$$

  - Solving for the fixed payment a

$$a = \frac{1}{2}\left[1 - \frac{1 - \rho\sigma^2}{(1 + \rho\sigma^2)^2}\right]$$

Continuum of Effort Levels

- Example 4: (con't)
  - If $\sigma^2 = 0$, effort e is deterministic (a perfect predictor of profits)

$$x = f(e) = e$$

  - Then,

$$b = \frac{1}{1 + \rho \cdot 0} = 1$$

$$a = \frac{1}{2}\left[1 - \frac{1 - \rho \cdot 0}{(1 + \rho \cdot 0)^2}\right] = 0$$

  - Intuitively, the principal does not offer a fixed payment, and the agent is benefited from high-powered incentives.

Continuum of Effort Levels

- Example 4: (con't)
  - If $\sigma^2 = 1$, effort e is imprecise predictor of outcomes.
  - Then,

$$b = \frac{1}{1+\rho}$$

$$a = \frac{\rho(\rho+3)}{2(1+\rho)^2}$$

Continuum of Effort Levels

- Example 4: (con't)
  - When the agent becomes more risk averse (ρ increases), the agent is offered a higher fixed payment but a lower bonus, since

$$\frac{\partial b}{\partial \rho} = -\frac{1}{(1+\rho)^2} < 0$$

$$\frac{\partial a}{\partial \rho} = \frac{3-\rho}{2(1+\rho)^3} > 0$$

Moral Hazard with Multiple Signals

Multiple Signals

- Consider a setting in which the principal, still not observing effort e, observes:
  - the profits π of the firm;
  - a signal s, based on a middle management report about the agent's performance.
- Signal s provides no intrinsic economic value but it provides information about effort e.
- Hence the probability density function has two observables, π and s.
- Then, similar to equation **(6)**, we have

$$\frac{1}{u'(w)} = \gamma + \mu\left[1 - \frac{f(\pi, s|e_L)}{f(\pi, s|e_H)}\right]$$

Multiple Signals

- Hence variations in s affect wages only if
$$f(\pi, s|e) \neq f(\pi|e)$$
- That is, if π is not a sufficient statistic of e.
- Intuitively, the pair (π, s) contains more information about the agent's exerted effort e than π alone.
- Signal s is uninformative (provides no more information than π alone), if
$$f(\pi, s|e) = f(\pi|e)$$
- We can examine under which conditions w increases in signal s.

Multiple Signals

- For two signals s_1 and s_2, where $s_2 > s_1$, if salary increases in the signal, $w(\pi, s_2) > w(\pi, s_1)$, then $u'(w)$ decreases and its inverse, $1/u'(w)$, increases.
- Therefore,

$$\gamma + \mu\left[1 - \frac{f(\pi, s_2|e_L)}{f(\pi, s_2|e_H)}\right] > \gamma + \mu\left[1 - \frac{f(\pi, s_1|e_L)}{f(\pi, s_1|e_H)}\right]$$

- Simplifying this inequality to express it in terms of the likelihood ratio, $\frac{f(\pi,s|e_L)}{f(\pi,s|e_H)}$, we obtain

$$\frac{f(\pi, s_2|e_L)}{f(\pi, s_2|e_H)} < \frac{f(\pi, s_1|e_L)}{f(\pi, s_1|e_H)}$$

- In words, this condition says that, for the salary to increase in the intermediate signal s that the principal receives, we need such a signal to have a *decreasing* likelihood ratio.

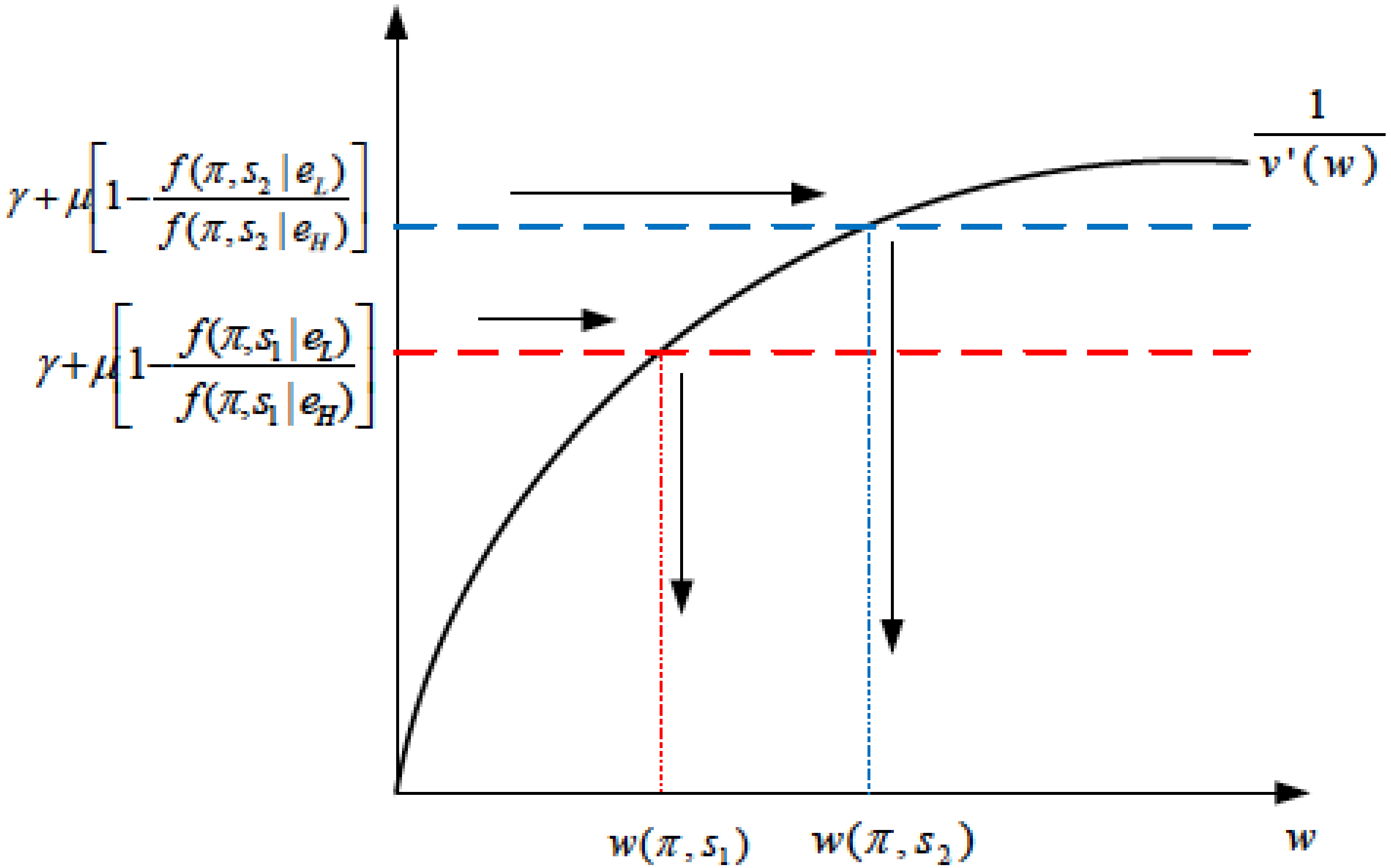
Multiple Signals

- Alternatively, we can rearrange expression $\frac{f(\pi,s_2|e_L)}{f(\pi,s_2|e_H)} < \frac{f(\pi,s_1|e_L)}{f(\pi,s_1|e_H)}$ as follows

$$\frac{f(\pi, s_1|e_H)}{f(\pi, s_1|e_L)} < \frac{f(\pi, s_2|e_H)}{f(\pi, s_2|e_L)}$$

- Intuitively, signal s_2 is more likely to originate from the high than the low effort, relative to signal s_1.

Multiple Signals



Adverse Selection
The "Lemons" Problem

Adverse Selection

- **Adverse selection**: settings in which an agent does not observe the payoff of the other individual.
  - Also referred to as “hidden information”
- Example:
  - A manager in a firm might not observe the worker’s ability
  - The manager could err in its selection of candidates for a job if he does not observe their ability, thus giving rise to adverse selection
- Under symmetric information markets often work well.
- Under asymmetric information, however, markets do not necessarily work well.

Adverse Selection

- Akerloff's (1970) model:
  - Consider a market of used cars, whose quality is denoted by q, where $q \in U[0, Q]$ and $Q \in (1,2)$.
  - A car of quality q is valued as such by the buyer, and as q/Q by the seller.
  - Since $\frac{q}{Q} < q$, the buyer assigns a higher value to the car than the seller.
  - This allows both parties to exchange the car at a price p between q/Q and q and make a profit (for the seller) and a surplus (for the buyer).

Adverse Selection

- Akerloff's (1970) model:
  - If a car of quality q is exchanged at price p the buyer obtains a utility
    $$u(p,q) = q - p$$
    while the seller makes a profit of
    $$\pi(p,q,Q) = p - \frac{q}{Q}$$
  - Assume that there are a sufficient number of buyers so that all gains from trade are appropriated by the seller.

Symmetric Information

- When the buyer can **perfectly** observe the car quality q, he buys at a price p if and only if

$$q - p \geq 0 \text{ or } p = q$$

- That is, his utility from such a trade is positive.
- A seller with a car of quality q anticipates such an acceptance rule by the buyer and sets a price p that solves

$$\max_{p \geq 0} \; p - \frac{q}{Q}$$
$$\text{s.t. } p \leq q$$

where $p \leq q$ is the buyer's participation constraint (PC).

Symmetric Information

- Since condition (PC) must bind, $p = q$, the seller's objective function can be represented as unconstrained problem:

$$\max_{p\geq 0} \; p - \frac{p}{Q}$$

- Taking the FOC with respect to p yields

$$1 - \frac{1}{Q} > 0 \;\; \text{or} \;\; \frac{Q-1}{Q} > 0$$

- Since $Q > 1$ by definition, a corner solution exists whereby the seller raises the price p as much as possible

$$p^{SI} = q$$

Asymmetric Information

- When the buyer is **unable** to observe the car's true quality q, he forms an expectation $E(q)$.
- The buyer accepts a trade if the car's asking price p satisfies

$$p = E(q)$$

- The seller anticipates such an acceptance rule by the buyer and sets a price p that solves

$$\max_{p \geq 0} \; p - \frac{q}{Q}$$

$$\text{s.t. } \; p \leq E(q)$$

where $p \leq E(q)$ is the buyer's PC constraint.

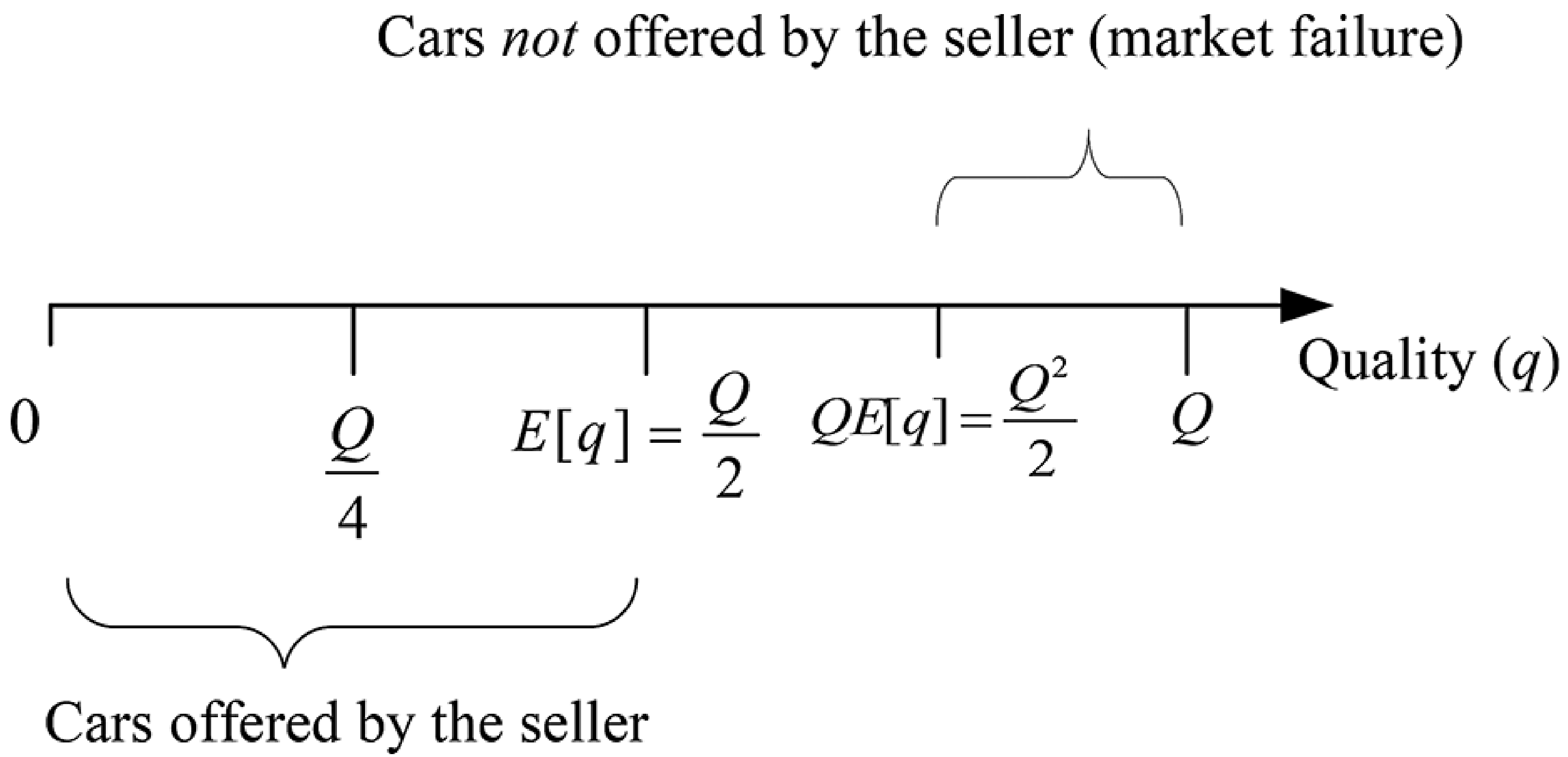
Asymmetric Information

- Since condition (PC) must bind, $p = E(q)$, the price that the seller sets

$$p - \frac{q}{Q} = E(q) - \frac{q}{Q} \geq 0$$

$$q \leq Q \cdot E(q)$$

Asymmetric Information



Asymmetric Information

- When q is uniformly distributed, that is, $q \sim U[0, Q]$, its expected value becomes
$$E(q) = \frac{Q - 0}{2} = \frac{Q}{2}$$
- Then, $Q \cdot E(q) = Q^2/2$.
- Hence all cars with relatively **low quality**, $q \leq Q^2/2$, are offered by the seller at a price
$$p = E(q) = \frac{Q}{2}$$
yielding profit of $\frac{Q}{2} - \frac{q}{2}$ for the seller and a zero (expected) utility for the buyer since $p = E(q)$.

Asymmetric Information

- Cars with relatively **high quality**, $q \geq Q^2/2$, are not offered by the seller since the highest price he can charge to the uninformed buyer, $p = E(q)$, does not compensate the seller's costs.
- This is problematic.
- The buyer's inability to observe q leads to the non-existence of the market for good cars ("peaches"), whereas only bad cars ("lemons") exist in the market.

Asymmetric Information

- A fully rational buyer would anticipate such a pricing decision by the seller
  - That the seller finds it worthy to only offer low quality cars, $p \leq Q^2/2$.
- In that case, the buyer anticipates that only cars of quality $q \in (0, Q^2/2)$ are offered.
- Then, if $q \sim U[0, Q]$, buyers can compute the expected quality of those offered cars

$$E\left[q \middle| q \leq \frac{Q^2}{2}\right] = \frac{\frac{Q^2}{2} - 0}{2} = \frac{Q^2}{4}$$

Asymmetric Information

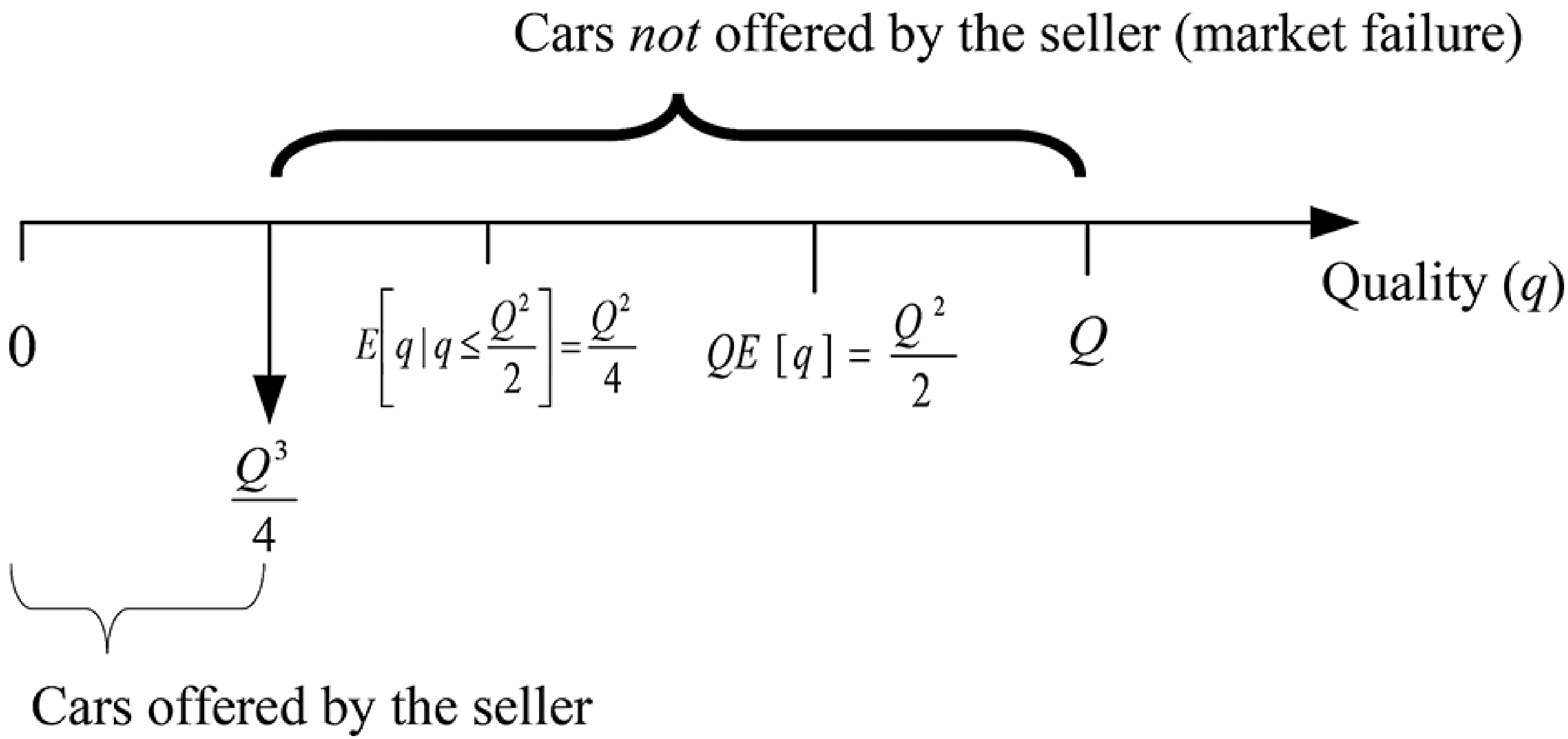
- Hence the buyer would only buy cars whose price satisfies $p = Q^2/4$.
- The seller would then set the price at $p = Q^2/4$, yielding a profit of

$$p - \frac{q}{Q} = \frac{Q^2}{4} - \frac{q}{Q}$$

which is positive only if quality q satisfies

$$q \leq \frac{Q^3}{4}$$

Asymmetric Information



Asymmetric Information

- A rational buyer would now update its expected car quality to those satisfying $Q^3/4$
- This yields an expected quality of only

$$E\left[q \middle| q \leq \frac{Q^3}{4}\right] = \frac{\frac{Q^3}{4} - 0}{2} = \frac{Q^3}{8}$$

- The seller offers cars that yield a positive profit, that is, those with quality q satisfying

$$p - \frac{q}{Q} = \frac{Q^3}{8} - \frac{q}{Q} \geq 0 \quad \text{or} \quad q \leq \frac{Q^4}{8}$$

which lies closer to zero than cutoff $\frac{Q^3}{4}$.

Asymmetric Information

- Intuition:
  - The seller would shift the set of offered cars even more to the left of the quality line toward worse cars (closer to zero).
  - Repeating the same argument enough times, we find that the market "unravels."
  - It only offers cars of the worse possible quality, $q = 0$.
  - The buyer is only willing to pay a price of $p = 0$, leaving all other types of cars unsold.

Asymmetric Information

- Example 5:
  - Consider a market of used cars with maximum available quality $Q = 1.9$, and that $q \sim U[0, Q]$.
  - Recall that $Q \in (1,2)$, i.e., the availability of several cars of relatively good quality.
  - The buyer's expected value is $\frac{1.9}{2} = 0.95$.
  - The cutoff $Q \cdot E(q)$ of cars offered by the seller is $1.9 \cdot 0.95 = 1.805$.
  - Unoffered cars $(1.805, 1.9)$.
  - Under complete information, these cars would have been bought by the buyer who values them at q, and sold by the seller who values them at only $\frac{q}{1.9} = 0.52q$.

Asymmetric Information

- Example 5: (con't)
  - A rational buyer will anticipate that cars in the interval $(1.805, 1.9)$ are unoffered by the seller.
  - Thus buyer updates expected value of offered cars to
$$E[q|q \leq 1.805] = \frac{1.805 - 0}{2} = 0.9$$
  - This leads the seller to only offer those cars with quality
$$q \leq \frac{Q^3}{4} = 1.71$$
  - The set of offered cars is thus restricted from $(0, 1.805)$ to $(0, 1.71)$.
  - A similar argument applies to further iterations in the buyer's expected car quality.
  - The presence of asymmetric information between buyer and seller prevents mutually beneficial trades from occurring.

Asymmetric Information

- Application to Labor Markets
  - Consider a competitive labor market with many firms seeking to hire a worker for a specific position.
  - The worker (seller of labor services) privately observes his own productivity θ, but firms (the buyer of labor) cannot observe it.
  - Firms offer a wage according to the worker's expected productivity

$$E(\theta) = 1/2,\ \ \theta \sim U[0,1]$$

  - For this salary, only workers with a productivity $\theta < 1/2$ would be interested in accepting the position, while those with $\theta > 1/2$ will be left unemployed.

Asymmetric Information

- Application to Labor Markets
  - A fully rational manager will only offer a salary of
$$w = E\left(\theta | \theta \leq \frac{1}{2}\right) = \frac{1}{4}$$
  - Then only those workers with productivity $\theta \leq \frac{1}{4}$ accept the job.
  - Extending the argument infinite times, workers with lowest productivity level $\theta = 0$ are employed, while the labor market for all other worker types $\theta > 0$ unravels.

Solutions to Adverse Selection

- The market failure described above can be overcome by a number of tools.
  - Sellers can offer warranties for their cars in order to signal their quality.
  - **Screening**: The principal (buyer) offers a menu of contracts to the agent (seller) that induce each type of agent to voluntarily select only one contract, whereby the contracts induce self-selection.

Adverse Selection
The Principal–Agent Problem

The Principal–Agent Problem

- Consider a setting where a firm (the principal) seeks to hire a worker (an agent).
- The firm cannot observe the worker's cost of effort
  - This affects the amount of effort that the worker exerts and thus the firm's profits.
- The firm's manager would like to know the worker's cost of effort in order to design his salary.
- The firm's profit function is

$$\pi(e, w) = x(e) - w$$

  where $x(e)$ is the benefit that the firm obtains when the worker supplies e units of effort, $x'(e) \geq 0$, $x''(e) \leq 0$.

The Principal–Agent Problem

- The worker's utility function is
$$v(w, e|\theta) = u(w) - c(e, \theta)$$
where $u(w)$ is the value from the salary w, $u'(w) > 0$, $u''(w) \leq 0$; $c(e, \theta)$ is the worker's cost of exerting e units of effort when his type is θ.
- Assume the worker can only be of two types, θ_L and θ_H, where $\theta_L < \theta_H$, with probabilities p and $1 - p$.
- A high-type worker faces a higher total and marginal cost of effort
$$c(e, \theta_L) < c(e, \theta_H)$$
$$c'(e, \theta_L) < c'(e, \theta_H)$$
for every e.

Symmetric Information

- When the principal (firm) **knows** that the agent is type $i = \{L, H\}$, it solves

$$\max_{w_i, e_i} \; x(e_i) - w_i$$

$$\text{s.t.} \quad u(w_i) - c(e_i, \theta_i) \geq 0 \qquad \text{(PC)}$$

- (PC) constraint guarantees that the worker willingly accepts the contract.
- Since the firm can reduce w_i until (PC) holds with equality, (PC) must bind

$$u(w_i) = c(e_i, \theta_i)$$

$$w_i = u^{-1}[c(e_i, \theta_i)]$$

Symmetric Information

- The principal's unconstrained maximization problem can then be written as

$$\max_{e_i} \; x(e_i) - u^{-1}[c(e_i, \theta_i)]$$

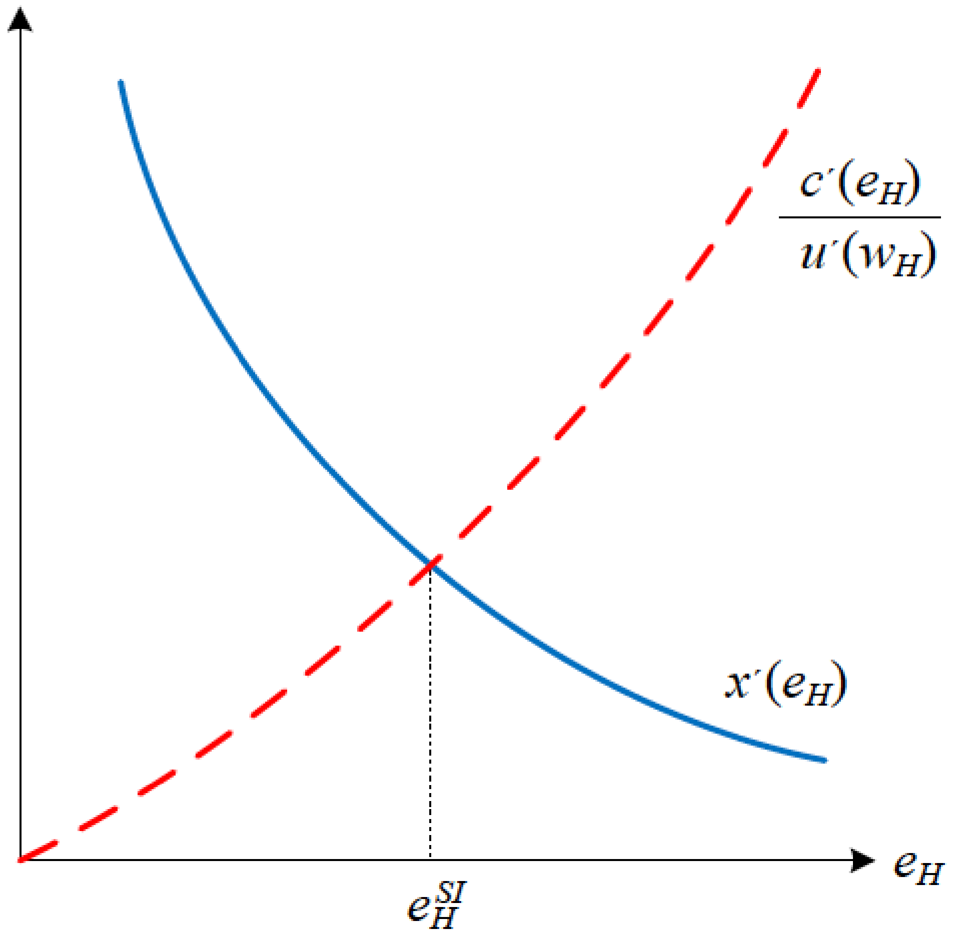
- Taking FOC with respect to e_i yields

$$x'(e_i) = \frac{1}{u'\{u^{-1}[c(e_i, \theta_i)]\}} c'(e_i, \theta_i)$$

$$x'(e_i) = \frac{c'(e_i, \theta_i)}{u'\{w_i\}}$$

- Hence effort is increased until the point at which the marginal rate of substitution of effort and wage for the firm (left-hand side) coincides with that of the worker (right-hand side).

Symmetric Information



Symmetric Information

- Example 6:
  - Consider a principal and an agent of type $\theta_L = 1$, $\theta_H = 2$.
  - The probability of facing a low type is $p = 1/2$.
  - Productivity of effort is $x(e) = \log(e)$, and $u(w) = w$.
  - The cost of effort is $c(e, \theta) = \theta_i e^2$, with the marginal cost of effort of $2\theta_i e$, which is positive and increasing in e.
  - The principal's profit function is

$$\pi(e, w) = \log(e) - w$$

  - The agent's utility is

$$v(w, e|\theta_i) = w - \theta_i e^2$$

Symmetric Information

- Taking FOC

$$x'(e_i) = \frac{c'(e_i, \theta_i)}{u'\{w_i\}} \Rightarrow \frac{1}{e_i} = \frac{2\theta_i e_i}{1}$$

- Solving for e_i

$$e_i^2 = \frac{1}{2\theta_i} \rightarrow e_i^{SI} = \left(\frac{1}{2\theta_i}\right)^{1/2}$$

- Use the (PC) constraint, $u(w_i) = c(e_i, \theta_i)$, to find optimal salary

$$w_i = \theta_i \left(e_i^{SI}\right)^2 = \theta_i \left(\left(\frac{1}{2\theta_i}\right)^{1/2}\right)^2 = \frac{1}{2}$$

Symmetric Information

- Plugging in $\theta_L = 1$ and $\theta_H = 2$, we find optimal contracts

$$(w_H^{SI}, e_H^{SI}) = \left(\frac{1}{2}, \frac{1}{2}\right) = (0.5, 0.5)$$

$$(w_L^{SI}, e_L^{SI}) = \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right) = (0.5, 0.707)$$

- The firm will pay both types of workers the same wage under symmetric information, but expect a higher effort level from the low-cost worker, $e_L^{SI} > e_H^{SI}$.

Asymmetric Information

- When the firm **cannot** observe the worker's type, it seeks to maximize the expected profits by designing a pair of contracts, (w_H, e_H) and (w_L, e_L), that satisfy four constraints:
  1. voluntary participation of the high-type worker;
  2. voluntary participation of the low-type worker;
  3. the high-type worker prefers the contract (w_H, e_H) rather than that for the low-type, (w_L, e_L);
  4. the low-type worker prefers the contract (w_L, e_L) rather than that for the high-type worker (w_H, e_H).
- Since every type of worker has an incentive to select the contract meant for him, these contracts induce "**self-selection**."

Asymmetric Information

- The firm solves the following profit maximization problem

$$\max_{w_L, e_L, w_H, e_H} \quad p[x(e_L) - w_L] + (1-p)[x(e_H) - w_H]$$

$$\text{s.t.} \quad u(w_H) - c(e_H, \theta_H) \geq 0 \qquad (\text{PC}_\text{H})$$

$$u(w_L) - c(e_L, \theta_L) \geq 0 \qquad (\text{PC}_\text{L})$$

$$u(w_H) - c(e_H, \theta_H) \geq u(w_L) - c(e_L, \theta_H) \qquad (\text{IC}_\text{H})$$

$$u(w_L) - c(e_L, \theta_L) \geq u(w_H) - c(e_H, \theta_L) \qquad (\text{IC}_\text{L})$$

Asymmetric Information

- Note that (PC_L) is implied by (IC_L) and (PC_H)

$$u(w_L) - c(e_L, \theta_L) \geq u(w_H) - c(e_H, \theta_L)$$
$$> u(w_H) - c(e_H, \theta_H) \geq 0$$

  - The first (weak) inequality stems from (IC_L).
  - The second (strict) inequality stems from the assumption $c(e_H, \theta_L) < c(e_H, \theta_H)$.
  - The third (weak) inequality stems from (PC_H).
- Hence we obtain (PC_L)

$$u(w_L) - c(e_L, \theta_L) > 0$$

Asymmetric Information

- The Lagrangian is

$$\begin{aligned}\mathcal{L} &= p[x(e_L) - w_L] + (1-p)[x(e_H) - w_H] \\ &+ \lambda_1[u(w_H) - c(e_H, \theta_H)] \\ &+ \lambda_2[u(w_H) - c(e_H, \theta_H) - u(w_L) + c(e_L, \theta_H)] \\ &+ \lambda_3[u(w_L) - c(e_L, \theta_L) - u(w_H) + c(e_H, \theta_L)]\end{aligned}$$

Asymmetric Information

- Taking FOCs

$$\frac{\partial \mathcal{L}}{\partial w_L} = -p - \lambda_2 u'(w_L) + \lambda_3 u'(w_L) = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_H} = -(1-p) + \lambda_1 u'(w_H) + \lambda_2 u'(w_H) - \lambda_3 u'(w_H) = 0$$

$$\frac{\partial \mathcal{L}}{\partial e_L} = px'(e_L) + \lambda_2 c'(e_L, \theta_H) - \lambda_3 c'(e_L, \theta_L) = 0$$

$$\frac{\partial \mathcal{L}}{\partial e_H} = (1-p)x'(e_H) - \lambda_1 c'(e_H, \theta_H) - \lambda_2 c'(e_H, \theta_H) + \lambda_3 c'(e_H, \theta_L) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = u(w_H) - c(e_H, \theta_H) \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = u(w_H) - c(e_H, \theta_H) - u(w_L) + c(e_L, \theta_H) \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = u(w_L) - c(e_L, \theta_L) - u(w_H) + c(e_H, \theta_L) \geq 0$$

Asymmetric Information

- For simplicity, consider that the cost of effort takes the following form
$$c(e,\theta_i) = \theta_i c(e) \quad \text{for all } \; i = \{H, L\}$$
where $c(e)$ is increasing and convex in effort, $c'(e) \geq 0$ and $c''(e) \geq 0$.
- Rearranging the first two FOCs yields
$$-\lambda_2 + \lambda_3 = \frac{p}{u'(w_L)}$$
$$\lambda_1 + \lambda_2 - \lambda_3 = \frac{1-p}{u'(w_H)}$$
- Then adding them together
$$\lambda_1 = \frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}$$

Asymmetric Information

- Hence $\lambda_1 > 0$, implying that the constraint associated with Lagrange multiplier λ_1, (PC_H), binds:

$$u(w_H) - c(e_H, \theta_H) = 0$$

Asymmetric Information

- The third FOC can be written as
$$px'(e_L) = \lambda_3\theta_L c'(e_L) - \lambda_2\theta_H c'(e_L)$$
- Rearranging
$$\frac{px'(e_L)}{c'(e_L)} = \lambda_3\theta_L - \lambda_2\theta_H$$
- The fourth FOC can be written as
$$(1-p)x'(e_H) = \lambda_1\theta_H c'(e_H) - \lambda_3\theta_L c'(e_H) + \lambda_2\theta_H c'(e_H)$$
- Rearranging
$$\frac{(1-p)x'(e_H)}{c'(e_H)} = \lambda_1\theta_H - (\lambda_3\theta_L - \lambda_2\theta_H)$$

Asymmetric Information

- Combining the two (rearranged) FOCs yields

$$\frac{(1-p)x'(e_H)}{c'(e_H)} = \lambda_1 \theta_H - \frac{px'(e_L)}{c'(e_L)}$$

- Solving for $\lambda_1 \theta_H$ and using $\lambda_1 = \frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}$ from our results above, we obtain

$$\left[\frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}\right]\theta_H = \frac{px'(e_L)}{c'(e_L)} + \frac{(1-p)x'(e_H)}{c'(e_H)}$$

Asymmetric Information

- Moreover, $\lambda_3 > \lambda_2$, since otherwise the first FOC, $(\lambda_3 - \lambda_2)u'(w_L) = p$, could not hold.
- Therefore, $\lambda_3 > 0$, which means (IC_L) binds:
$$u(w_L) - \theta_L c(e_L) = u(w_H) - \theta_L c(e_H)$$
- Rearranging the right-hand side
$$u(w_L) - \theta_L c(e_L)$$
$$= u(w_H) - \theta_H c(e_H) + (\theta_H - \theta_L)c(e_H)$$
- Since (PC_H), binds, $u(w_H) - c(e_H, \theta_H) = 0$, hence
$$u(w_L) - \theta_L c(e_L) = (\theta_H - \theta_L)c(e_H)$$

Asymmetric Information

- Intuition:
  - The most efficient agent, θ_L, obtains in equilibrium a positive utility level, $(\theta_H - \theta_L)c(e_H)$, that increases in his difference with respect to the least efficient worker, $\theta_H - \theta_L$.

Asymmetric Information

- The incentive compatibility condition of the least efficient worker, (IC_H) , does not bind, implying that its associated Lagrange multiplier $\lambda_2 = 0$.
- Using this result in the first and third FOCs yields

$$\lambda_3 = \frac{p}{u'(w_L)} \quad \text{and} \quad \frac{px'(e_L)}{c'(e_L)} = \lambda_3 \theta_L$$

- Solving for λ_3 and combining the two FOCs

$$\frac{p}{u'(w_L)} = \frac{px'(e_L)}{\theta_L c'(e_L)}$$

- Solving for $x'(e_L)$

$$x'(e_L) = \frac{\theta_L c'(e_L)}{u'(w_L)}$$

Asymmetric Information

- Intuition:
  - For the most efficient worker, the equilibrium outcome under asymmetric information **coincides** with the socially optimal result under symmetric information.

Asymmetric Information

- Using $\lambda_1 = \frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}$, $\lambda_2 = 0$, $\lambda_3 = \frac{p}{u'(w_L)}$ in the fourth FOC, we obtain

$$(1-p)x'(e_H) - \left[\frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}\right]\theta_H c'(e_H) + \left[\frac{p}{u'(w_L)}\right]\theta_L c'(e_H) = 0$$

- Rearranging

$$\frac{(\theta_H - \theta_L)p}{1-p}\frac{c'(e_H)}{u'(w_L)} + \frac{\theta_H c'(e_H)}{u'(w_H)} = x'(e_H)$$

- The effort level that solves this equation is the **optimal** effort under asymmetric information, e_H^{AI}.

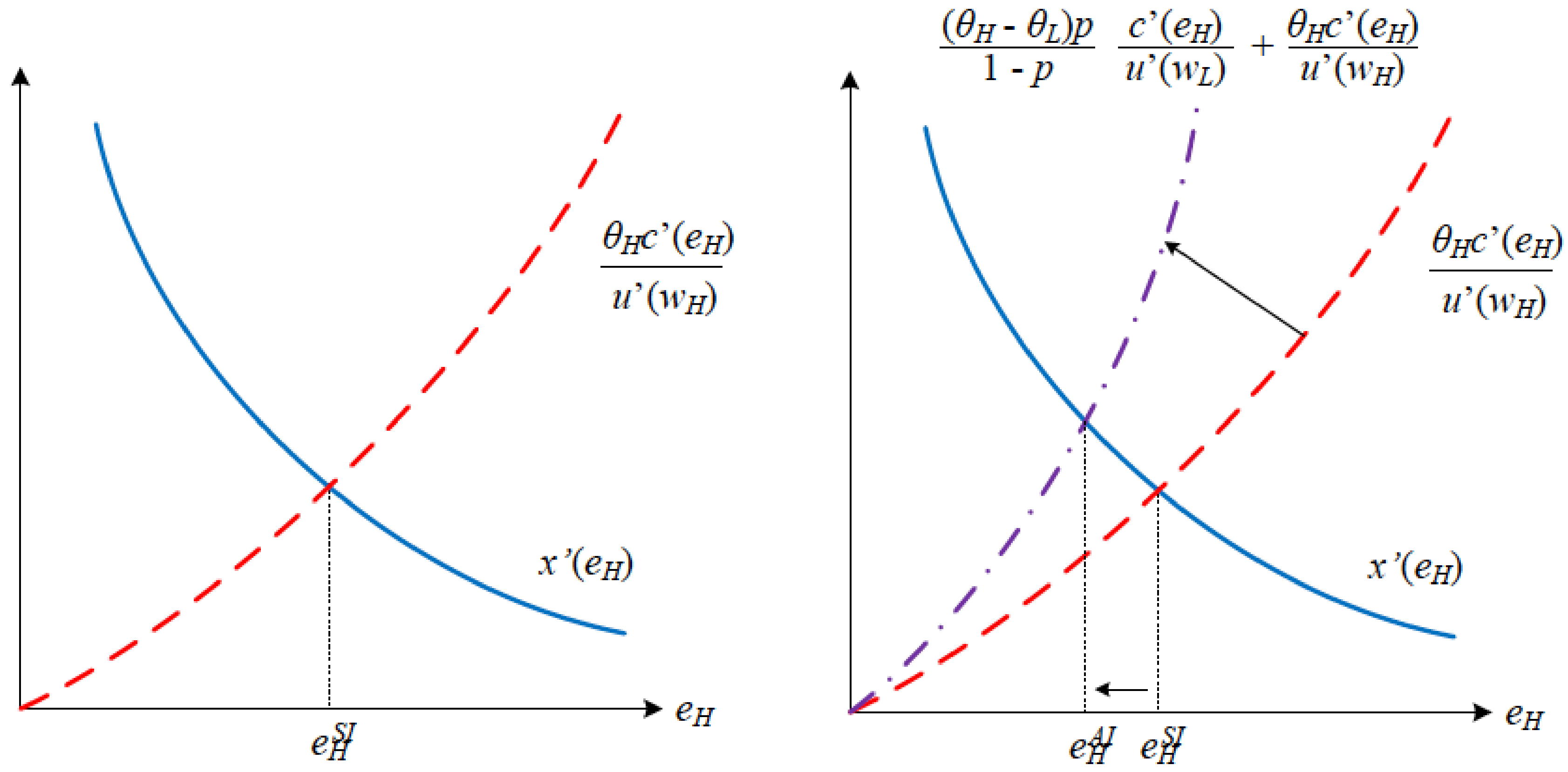
Asymmetric Information

- Compare e_H^{AI} against the effort arising under symmetric information e_H^{SI}, $\frac{\theta_H c'(e_H)}{u'(w_H)} = x'(e_H)$.
- Given $\theta_H - \theta_L > 0$, $p > 0$, $c'(e_H) > 0$ and $u'(w_L) > 0$,

$$\frac{(\theta_H - \theta_L)p}{1-p}\frac{c'(e_H)}{u'(w_L)} + \frac{\theta_H c'(e_H)}{u'(w_H)} > \frac{\theta_H c'(e_H)}{u'(w_H)}$$

- Hence the effort level under asymmetric information is **lower** than that under symmetric information, $e_H^{AI} < e_H^{SI}$.

Asymmetric Information



Asymmetric Information

- In summary, the pair of contracts (w_H, e_H) and (w_L, e_L) must satisfy the following equations

$$u(w_L) - \theta_L c(e_L) = (\theta_H - \theta_L)c(e_H)$$

$$u(w_H) - c(e_H, \theta_H) = 0$$

$$\frac{\theta_L c'(e_L)}{u'(w_L)} = x'(e_L)$$

$$\frac{(\theta_H - \theta_L)p}{1 - p} \frac{c'(e_H)}{u'(w_L)} + \frac{\theta_H c'(e_H)}{u'(w_H)} = x'(e_H)$$

Monotonicity in Effort

- Consider that effort levels satisfy $e_L \geq e_H$.
  - That is, the worker with the lowest cost of effort exerts a larger effort level than the worker with a high cost of effort.
- Combining (IC_L) and (IC_H) to obtain

$$u(w_L) - c(e_L, \theta_L) \geq u(w_H) - c(e_H, \theta_L) >$$
$$u(w_H) - c(e_H, \theta_H) \geq u(w_L) - c(e_L, \theta_H)$$

  - The first inequality stems from (IC_L).
  - The second inequality is due to $c(e_L, \theta_L) < c(e_H, \theta_H)$.
  - The third inequality is due to (IC_H).
- Hence, the above inequality can be rearranged as

$$c(e_H, \theta_L) - c(e_L, \theta_L) \geq u(w_H) - u(w_L)$$
$$> c(e_H, \theta_H) - c(e_L, \theta_H)$$

Monotonicity in Effort

- Multiplying this expression by -1, and using the first and last terms
$$c(e_L, \theta_L) - c(e_H, \theta_L) < c(e_L, \theta_H) - c(e_H, \theta_H)$$
- This condition indicates that the marginal cost of increasing effort from e_H to e_L is higher for the high-type than for the low-type worker.
- Evaluating this condition in the cost of effort function $c(e, \theta) = \theta c(e)$
$$\theta_L[c(e_L) - c(e_H)] < \theta_H[c(e_L) - c(e_H)]$$
- Since $\theta_L < \theta_H$, we must have $c(e_L) > c(e_H)$.
- Hence effort is larger for the worker with the low cost of effort, $e_L > e_H$

Monotonicity in Effort

- Example 7:
  - Let us use Example 6 to calculate the optimal contracts under asymmetric information.
  - Taking FOCs from above

$$u(w_L) - \theta_L c(e_L) = (\theta_H - \theta_L)c(e_H) \Rightarrow w_L - e_L^2 = e_H^2$$

$$u(w_H) - \theta_H c(e_H) = 0 \Rightarrow w_H = 2e_H^2$$

$$x'(e_L) = \frac{\theta_L c'(e_L)}{u'(w_L)} \Rightarrow \frac{1}{e_L} = \frac{2e_L}{1} \rightarrow 2e_L^2 = 1 \rightarrow e_L = \frac{1}{\sqrt{2}}$$

$$\frac{(\theta_H - \theta_L)p}{1-p}\frac{c'(e_H)}{u'(w_L)} + \frac{\theta_H c'(e_H)}{u'(w_H)} = x'(e_H) \Rightarrow \frac{2e_H}{1} + \frac{4e_H}{1}$$

$$= \frac{1}{e_H} \rightarrow e_H = \frac{1}{\sqrt{6}}$$

Monotonicity in Effort

- Example 7: (con't)
  - From the last two FOCs, we obtain the equilibrium effort levels $e_L = \frac{1}{\sqrt{2}}$ and $e_H = \frac{1}{\sqrt{6}}$.
  - From the first equation
$$w_L - \frac{1}{2} = \frac{1}{6} \rightarrow w_L = \frac{2}{3}$$
  - From the second equation
$$w_H = 2 \cdot \frac{1}{6} \rightarrow w_H = \frac{1}{3}$$
  - Therefore, the optimal pair of contracts is
$$\left(w_H^{AI}, e_H^{AI}\right) = \left(\frac{1}{3}, \frac{1}{\sqrt{6}}\right) = (0.333, 0.408)$$
$$\left(w_L^{AI}, e_L^{AI}\right) = \left(\frac{2}{3}, \frac{1}{\sqrt{2}}\right) = (0.667, 0.707)$$

Monotonicity in Effort

- Example 7: (con't)
  - The introduction asymmetric information entails:
    - *No changes* in effort for the low-cost worker relative to symmetric information
    $$e_L^{AI} = e_L^{SI} = 0.707$$
    - *Lower* effort for the high-cost worker than under symmetric information
    $$e_H^{AI} = 0.408 < 0.5 = e_H^{SI}$$
    - *Higher* salaries for the low-cost worker than under symmetric information
    $$w_L^{AI} = 0.667 > 0.5 = w_L^{SI}$$
    - *Lower* salaries for the high-cost worker
    $$w_H^{AI} = 0.333 < 0.5 = w_H^{SI}$$

Monotonicity in Effort

- Example 7: (con't)
  - The net utility that each type of worker obtains under asymmetric information is

$$u_H^{AI} = w_H - 2e_H^2 = 0$$
$$u_L^{AI} = w_L - e_L^2 = 0.167$$

  - Hence the worker with a low cost of effort captures an information rent

$$u_L^{AI} - u_L^{SI} = 0.167 - 0 = 0.167$$

  - The worker with a high cost of effort does not

$$u_H^{AI} = u_H^{SI} = 0$$

  - Intuitively, the firm must compensate the low-cost worker above symmetric information terms in order for him to reveal his type.

Application of Adverse Selection—Regulation

Regulation

- Regulatory agencies often cannot observe some characteristics of the regulated firm or of individual.
- Examples:
  – A firm's production costs
  – A firm's costs from pollution abatement
  – A consumer's willingness to pay for certain products
- In these scenarios the privately informed party (e.g., firm) has incentives to overstate its costs.
- Hence the regulator cannot directly ask firms about their production costs since responses would be unreliable.
- Adverse selection models offer an alternative contracting tool to extract information from privately informed firms (or consumers).

Regulation

- Consider that a government regulating a monopoly with cost function
$$c(q) = C + cq$$
where C is fixed costs and $c > 0$ is marginal costs.
- The consumer pays F for the bulk of q units consumed, and the monopolist may receive a lump-sum subsidy from the government of S.
- Assume that the shadow cost of raising public funds is $g \in (0,1)$, thus implying that the total cost of providing a subsidy S to the monopolist is $(1 + g)S$.
- Analyze settings where government has symmetric and asymmetric information about the monopolist's costs.

Regulation- Symmetric Information

- Consider that the government can **perfectly** observe the monopolist's marginal cost of production c.
- The government solves the following problem subject to PCs of both the monopolist and the consumer:

$$\max_{F,S,q} \; [u(q) - F] + [F + S - C - cq] - (1 + g)S$$

$$\text{s.t.} \quad F + S - C - cq \geq 0 \qquad (\text{PC}_{\text{Monop}})$$

$$u(q) - F \geq 0 \qquad (\text{PC}_{\text{Consum}})$$

where $u(q) - F$ is the consumer's utility after paying F for q units; and $F + S - C - cq$ is the monopolist's profits.

- The Lagrangian is

$$\mathcal{L} = [u(q) - F] + [F + S - C - cq] - (1 + g)S + \lambda_1[F + S - C - cq] + \lambda_2[u(q) - F]$$

Regulation- Symmetric Information

- Taking FOCs yields

$$\frac{\partial \mathcal{L}}{\partial F} = \lambda_1 - \lambda_2 = 0 \rightarrow \lambda_1 = \lambda_2$$

$$\frac{\partial \mathcal{L}}{\partial S} = 1 - (1 + g) + \lambda_1 = 0 \rightarrow \lambda_1 = g$$

$$\frac{\partial \mathcal{L}}{\partial q} = u'(q) - c - \lambda_1 c + \lambda_2 u'(q) = 0$$

$$\lambda_1[F + S - C - cq] = 0$$

$$\lambda_2[u(q) - F] = 0$$

- Combining the first and second FOC

$$\lambda_1 = \lambda_2 = g$$

Regulation- Symmetric Information

- Plugging this result into the third FOC yields

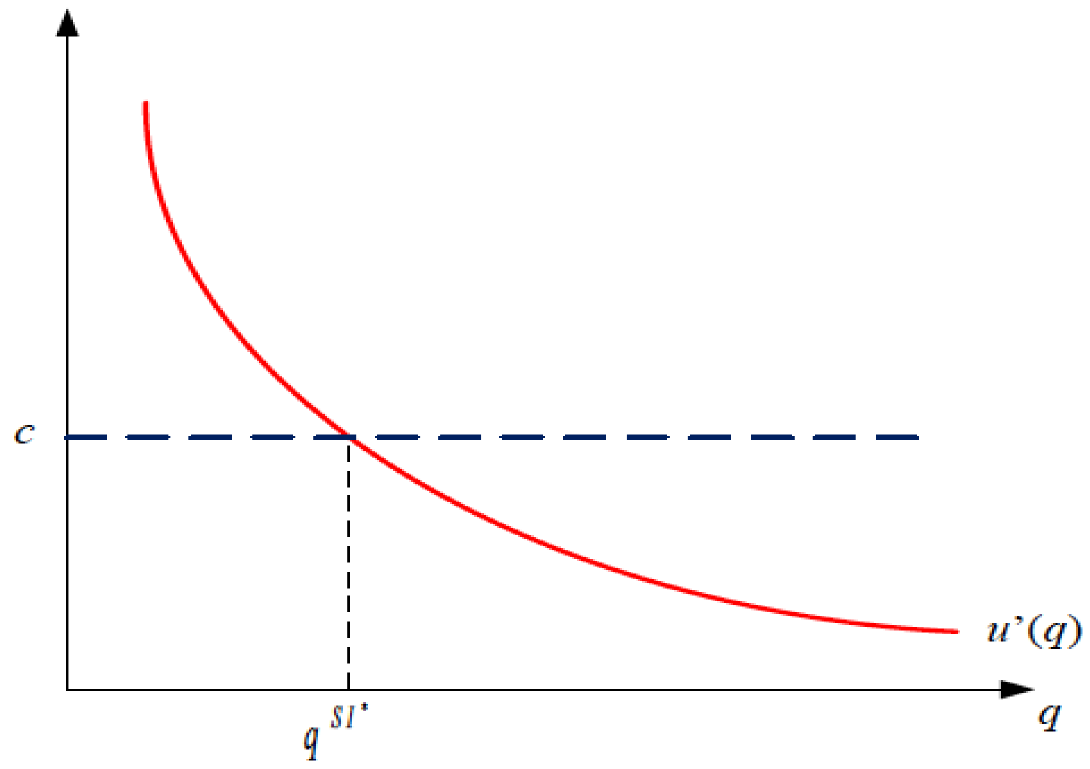
$$u'(q) - c - gc + gu'(q) = 0$$

- Rearranging

$$(1+g)u'(q) = (1+g)c \Leftrightarrow u'(q) = c$$

- That is, q is increased until the point where marginal utility from further units coincides with its marginal cost.
- Hence, under symmetric information, the monopolist's production is **efficient**.

Regulation- Symmetric Information



Regulation- Asymmetric Information

- Consider now that the government **cannot** observe the monopolist's marginal cost of production c.
- Marginal cost can be low or high $c = \{c_L, c_H\}$, where $c_L < c_H$, with associated probabilities p and $1 - p$, respectively.
- The government offers two menus (F_L, S_L, q_L) and (F_H, S_H, q_H) to maximize the expected social welfare subject to PCs of both the monopolist and the consumer.

Regulation- Asymmetric Information

- The government's maximization problem is

$$\max_{(F_L, S_L, q_L),(F_H, S_H, q_H)} p[u(q_L) - F_L + F_L + S_L - C - c_L q_L - (1+g)S_L]$$
$$+ (1-p)[u(q_H) - F_H + F_H + S_H - C - c_H q_H - (1+g)S_H]$$

s.t.

$$F_L + S_L - C - c_L q_L \geq 0 \qquad (\text{PC}_{\text{Monop},L})$$

$$F_H + S_H - C - c_H q_H \geq 0 \qquad (\text{PC}_{\text{Monop},H})$$

$$F_L + S_L - C - c_L q_L \geq F_H + S_H - C - c_L q_H \qquad (\text{IC}_{\text{Monop},L})$$

$$F_H + S_H - C - c_H q_H \geq F_L + S_L - C - c_H q_L \qquad (\text{IC}_{\text{Monop},H})$$

$$u(q_L) - F_L \geq 0 \qquad (\text{PC}_{\text{Consum},L})$$

$$u(q_H) - F_H \geq 0 \qquad (\text{PC}_{\text{Consum},H})$$

Regulation- Asymmetric Information

- Timeframe:
  - The government offers contracts
  - The monopolist chooses one of contracts, and then the K-type monopolist offers q_K units to the consumer at a lump-sum price of F_K where $K = \{L, H\}$.
  - The consumer can accept or reject the offer.
- *Practice:* Solve the problem on your own.
  - Output of the low type coincides with that under symmetric information, whereas, that of the high type is smaller.
  - However, the subsidy that the high-cost firm receives is lower than under symmetric information, while that of the low-cost firm is the same.

Regulation- Asymmetric Information

- Example 8:
  - Consider consumers with utility function $u(q) = \sqrt{q}$, a monopoly with cost function $c(q) = \frac{1}{4} + cq$, where marginal costs can be high $c_H = \frac{1}{8}$ or low $c_L = \frac{1}{16}$, with probability $p = \frac{1}{2}$.
  - The shadow cost of raising public funds is $g = \frac{1}{24}$.
  - Symmetric information entails an output level that solves

$$\frac{1}{2\sqrt{q}} = c_K$$

    which yields $q_H^{SI} = 16$ and $q_L^{SI} = 64$.
  - Asymmetric information entails output levels

$$q_L^{AI} = q_L^{SI} = 64$$

$$q_H^{AI} \cong 15.38 < q_H^{SI} = 16$$