Ch. 9 Slide 23, Section 9.3.2, BP

CONSUMERS

TWO PART TARIFFS WITH TWO FIRMS

UTILITY $u(q)$

indirect util. $v(p_i) = \max_q \; u(q) - p_i q$ $\boxed{\text{UMP}}$

walrasian demand $u'(q) = p_i$

Utility is then

$$w(p_i, m) = \underbrace{v(p_i)}_{\text{utility from consuming } q_i \text{ units (walrasian demand solving UMP)}} - \underbrace{m_i}_{\text{fixed fee (irrespective of units purchased)}}$$

$\Rightarrow r - \tau x + w(p_1, m_1)$ buying from firm 1

$\rightarrow r - \tau(1-x) + w(p_2, m_2)$ " " " 2

For compactness, $w_1 \equiv w(p_1, m_1)$

$w_2 \equiv w(p_2, m_2)$

2nd stage, consumers

Then, the indiff. cons is

$$r - \tau \hat{x} + w_1 = r - \tau(1-\hat{x}) + w_2$$

$$\Rightarrow \hat{x} = \frac{\tau + w_1 - w_2}{2\tau} \longleftarrow \text{Demand Firm 1}$$

$$1 - \hat{x} \longleftarrow \text{Demand Firm 2}$$

1st stage, Firms

Given demands, firm 1 solves

$$\max_{p_1, m_1} \; \pi_1 = \underbrace{\hat{x}}_{\text{volume of consumers}} \cdot \Big[\underbrace{(p_1 - c)}_{\text{margin}} \overbrace{q(p_1)}^{\text{output}} + \underbrace{m_1}_{\text{fixed fee}} \Big]$$

Since $w_1 = v(p_1) - m_1$, we can rewrite $m_1 = v(p_1) - w_1$. Inserting this,

$$\max_{p_1, w_1} \; \pi_1 = \underbrace{\frac{\tau + w_1 - w_2}{2\tau}}_{\hat{x}} \Big[(p_1 - c) q(p_1) + \underbrace{v(p_1) - w_1}_{m_1} \Big]$$

FOC p_1

$$q(p_1) + q'(p_1)(p_1-c) + v'(p_1) = 0$$

Since $v'(p_1) = -q_1$, we have that

$$\cancel{q(p_1)} + q'(p_1)(p_1 - c) \underbrace{\cancel{+\,q(p_1)}}_{v'(p_1)} = 0$$

$$\Rightarrow \underbrace{q'(p_1)}_{\substack{<0 \\ \text{by law} \\ \text{of demand}}}(p_1-c) = 0 \implies p_1^* = c$$

(same argument applies for firm 2)

Efficient consumption

FOC w_1

$$\left[(p_1-c)q(p_1) + v(p_1) - w_1\right] - (\tau + w_1 - w_2) = 0$$

Inserting $p_1 = c$, and invoking symmetry, $w_1 = w_2$, we obtain

$$\left[\underbrace{(c-c)q(p_1)}_{0} + v(p_1) - w_1\right] - (\tau + \cancel{w_1} - \cancel{w_1}) = 0$$

$$\Rightarrow v(p_1) - w_1 - \tau = 0$$

Recalling that $w_1 = v(p_1) - m_1$, yields

$$\cancel{v}(p_1) - \underbrace{\left[\cancel{v(p_1)} - m_1\right]}_{w_1} - \tau = 0 \implies m_1^* = \tau$$

(and same argument applies for firm 2)

Profits are:

$$\pi_1 = \frac{\tau + \overbrace{w_1 - w_2}^{0}}{2\tau}\left[\underbrace{(p_1-c)q_1}_{0} + \underbrace{m_1}_{\tau}\right] = \frac{\cancel{\tau}}{2\cancel{\tau}}\cdot\tau = \frac{\tau}{2}$$

so industry profits are $\Pi_M = \tau$

$$\implies \text{two-part tariff } T(q) = \underbrace{\tau}_{\text{fixed fee}} + \overbrace{c}^{\text{price per unit}}\, q$$

Comparing welfare ~~between~~ (totally with — two-part tariff (NL)
— linear tariff (L))

LINEAR PRICING

notation $\pi_i(p_i) \equiv (p_i - c)\, q(p_i)$

$$s(k) \equiv v\big(c + k(p-c)\big) \longrightarrow s(0) = v(c) \quad \text{"low price" or all surplus for cons.}$$
where $k \in [0,1]$ $\qquad s(1) = v(p) \quad$ "high price"

$$s_k(k) \equiv \frac{\partial s(k)}{\partial k} = (p-c)\, v'\big(c + k(p-c)\big)$$

Since $v'(p_i) = -q(p_i)$, we obtain

$$\pi\big(\underbrace{c+k(p-c)}_{p_i = ?}\big) = k\overbrace{(p-c)\,\underbrace{q\big(c+k(p-c)\big)}_{-v'(p_i)}}^{-s_k(k)} = -k \cdot s_k(k)$$

$$\underbrace{\big[\underbrace{c+k(p-c)}_{p_i} - c\big]}_{=k(p-c)}\, q\big(\underbrace{c+k(p-c)}_{p_i}\big)$$

Therefore, $\pi(p) = -\underbrace{s_k(1)}_{k=1}$

Suppose that firm j sets $p_j = p$. For p to be a symmetric equil, firm i must set $p_i = p$ to solve

$$\max_{p_i} \Pi_i = \frac{\tau + v(p_i) - v(p)}{2\tau}\, \pi_i(p)$$

alternatively,

$$\max_{k} \Pi_i = \frac{\tau + s(k) - s(1)}{2\tau}\big(-k \cdot s_k(k)\big)$$

We need that $p_i = p \Rightarrow k = 1$.

FOC$_k$

$$-k\left(s_k(k)\right)^2 - \left[\tau + s(k) - s(1)\right]\left[s_k(k) + k \cdot s_{kk}(k)\right] = 0$$

If $k=1$, we obtain

$$-\left[s_k(1)\right]^2 - \tau\left[s_k(1) + s_{kk}(1)\right] = 0$$

$$\Rightarrow \quad -s_k(1) = s_{kk}(1) + \frac{1}{\tau}\left[s_k(1)\right]^2$$

Replacing $\pi(p) = -s_k(1)$, we can express the industry profit as

Therefore, industry profit is

$$\pi_L = -s_K(1) \underset{\substack{\uparrow \\ \text{from FOC}_K}}{=} s_{KK}(1) + \frac{1}{\tau}\left[s_K(1)\right]^2$$

Since $s_K(K) = (p-c)\,v'(c+K(p-c))$ and $v'(p_i) = -q(p_i)$, we have that

$$s_K(K) = -(p-c)\,q(c+K(p-c))$$

$$\xrightarrow[\text{differentiating } s_K(K) \text{ w.r.t. } K]{} s_{KK}(K) = -(p-c)\,q'(c+K(p-c))\cdot(p-c)$$

$$= -\underbrace{(p-c)^2}_{+}\,\underbrace{q'(c+K(p-c))}_{-\text{ by Law of demand}}$$

$$= +$$

Therefore,

$$\pi_L = \underbrace{s_{KK}(1)}_{+} + \frac{1}{\tau}\left[s_K(1)\right]^2$$

$$\Rightarrow \pi_L > \frac{1}{\tau}\left[s_K(1)\right]^2 = \frac{1}{\tau}\pi_L^2$$

Because $\pi_L = -s_K(1)$, so $\left[s_K(1)\right]^2 = \pi_L^2$

$$\Rightarrow \cancel{\pi_L} > \frac{1}{\tau}\pi_L^{\cancel{2}}$$

$$\Rightarrow \tau_L > \pi_L$$

$$\| $$

π_{NL} (top of page 238)

industry profits are higher without NL than L pricing.

Consumers

At $\hat{p} = c + K(p-c)$, welfare is

$$W(K) = \underbrace{\pi(c + K(p-c))}_{= -K \cdot s_K(K)} + \underbrace{v(c+K(p-c))}_{= s(K)}$$

PS+CS

$\boxed{\text{Under NL}} \rightarrow \hat{p} = c$, so $K=0$, yielding

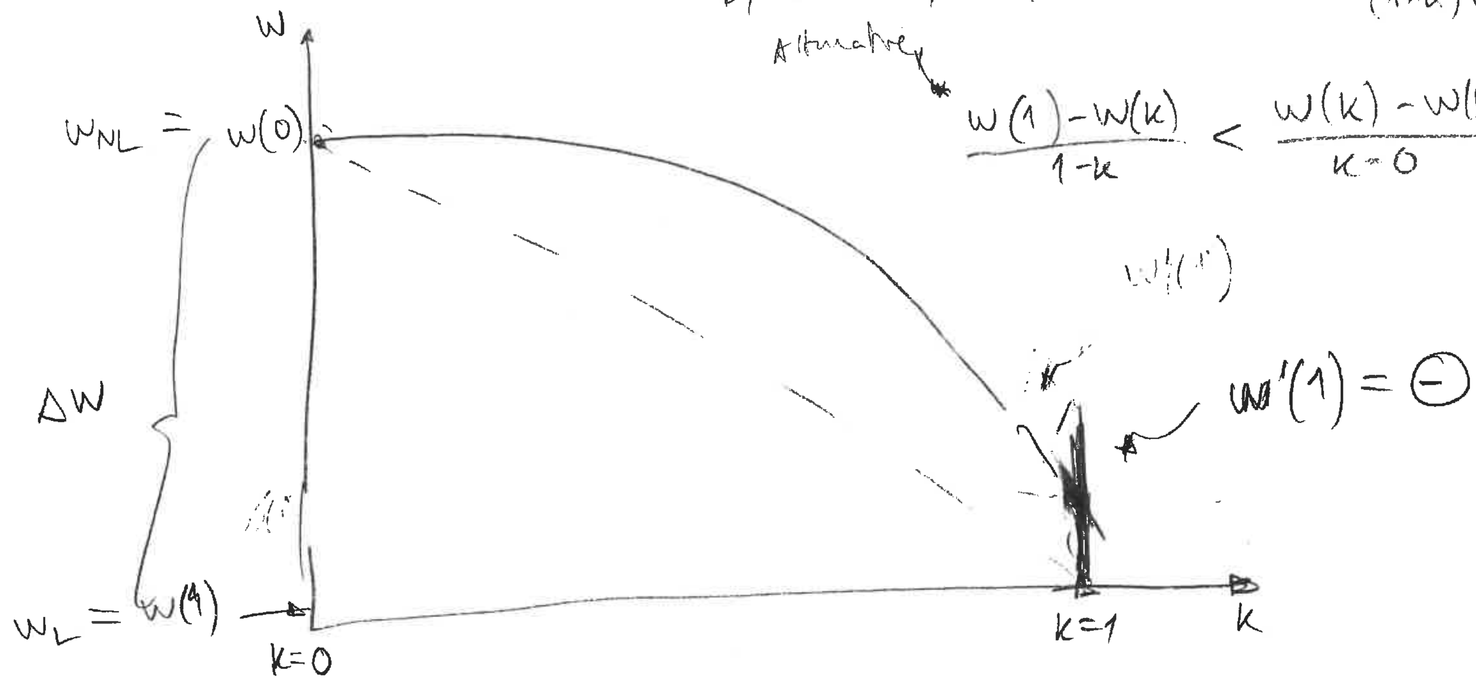
$$W(0) = \underbrace{-0 \cdot s_K(0)}_{0} + s(0) = s(0) \rightarrow v(c)$$

$\boxed{\text{Under L}} \rightarrow \hat{p} = p$, so $K=1$, entailing

$$W(1) = -1 \cdot s_K(1) + s(1) = s(1) - s_K(1)$$

Suppose that welfare is concave in prices, so $W(K)$ is concave in K. Therefore, By concavity, $w(\alpha 0 + (1-\alpha)k) > \alpha w(0) + (1-\alpha) w(1)$

Alternatively, $\dfrac{w(1)-w(k)}{1-k} < \dfrac{w(k)-w(0)}{k-0}$



$$s_{kk}(1) = \underbrace{-w'(1)}_{+} \geq \Delta W \quad \text{by concavity}$$

From last page $s_{kk}(1) = \pi_L - \frac{1}{\tau}\pi_L^2 = \frac{\pi_L}{\tau}(\tau - \pi_L)$

In addition, $\pi_L < \tau$, or $\frac{\pi_L}{\tau} < 1$, so that

$$\underbrace{\frac{\pi_L}{\tau}}_{<1}\underbrace{(\tau - \pi_L)}_{+} < \underbrace{\tau - \pi_L}_{+}$$

Overall,

$$\Delta W \equiv w_{NL} - w_L < \underbrace{s_{kk}(1)}_{\underbrace{\frac{\pi_L}{\tau}}_{<1}\underbrace{(\tau-\pi_L)}_{+}} < \pi_{NL} - \pi_L = \tau - \pi_L$$

Finally, since

$W = CS + \pi \Rightarrow CS = W - \pi$, so we can rewrite the above result as

$$W_{NL} - W_L < \pi_{NL} - \pi_L$$

$$\Rightarrow \underbrace{W_{NL} - \pi_{NL}}_{CS_{NL}} < \underbrace{W_L - \pi_L}_{CS_L} \Rightarrow CS_{NL} < CS_L$$

consumers are worse off with NL than L

(Intuition on page 240)

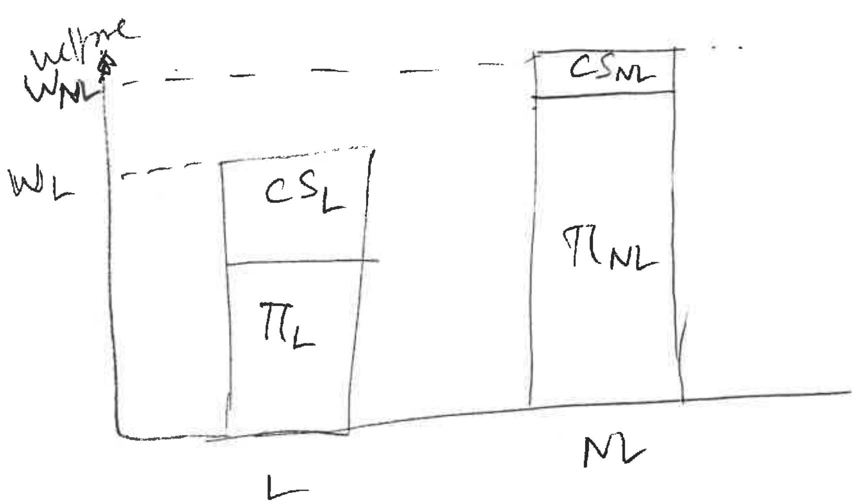
Intuition

In NL pricing, lowering fee has two effects → (+) stealing ~~more~~ customers from my rival
→ (−) lower revenue

In L pricing, lowering price has the above two effects and a third effect:

$\downarrow p$ expands demand from each type of customer.

Overall, there is more incentives to lower prices with L than NL



$W_{NL} > W_L$ and

$\pi_{NL} > \pi_L$, but

$CS_{NL} < CS_L$