

# Vertical differentiation and natural monopoly<sup>1</sup>

1. **Vertical differentiation and natural monopoly.** Consider the following model of vertically differentiated products with two firms. In the first stage, every firm  $i$  chooses its quality,  $s_i$ , in the interval  $[0, 1]$ , where  $i = \{1, 2\}$ . In the second stage, every firm  $i$ , observing the quality pair  $(s_1, s_2)$  from the first stage, responds setting a price  $p_i$ . In the third stage, given firms' location and prices, consumers buy one unit of the good from either firm 1 or 2. Every consumer when buying from firm  $i$  enjoys utility

$$r - p_i + \theta s_i$$

where parameter  $\theta \sim U[\underline{\theta}, \bar{\theta}]$  denotes how much this consumer cares about quality, where  $\bar{\theta} > \underline{\theta}$ . Intuitively, a consumer with  $\theta = \underline{\theta}$  does not assign any concern to quality, while  $\theta = \bar{\theta}$  assigns the maximal importance to quality. Assume that, for simplicity, both firms' marginal production cost is  $c = 0$ . Consider that qualities are given, where  $s_2 > s_1$ , but firms compete in prices.

- (a) *Third stage - Finding demand.* For given locations and prices, find the demand for each firm faces in the third stage of the game.

- If a consumer purchases from firm 1, his utility is  $r - p_1 + \theta s_1$ , while purchasing from firm 2 yields  $r - p_2 + \theta s_2$ . Therefore, the indifferent consumer  $\hat{\theta}$  solves

$$r - p_1 + \hat{\theta} s_1 = r - p_2 + \hat{\theta} s_2$$

which yields

$$\hat{\theta} = \frac{p_2 - p_1}{s_2 - s_1}.$$

Therefore, firm 1's demand is  $\hat{\theta} - \underline{\theta}$ , while firm 2's demand is  $\bar{\theta} - \hat{\theta}$ . Graphically, cutoff  $\hat{\theta}$  shifts rightward along the interval  $[\underline{\theta}, \bar{\theta}]$  when the price differential  $p_2 - p_1$  increases (meaning that firm 2 sets higher prices than firm 1), expanding the demand for firm 1 while shrinking that of firm 2. In contrast, cutoff  $\hat{\theta}$  shifts leftward when the quality differential  $s_2 - s_1$  increases (that is, firm 2 offers a higher quality than firm 1), shrinking the demand of firm 1 and expanding that of firm 2.

- (b) *Second stage - Prices.* For given locations, find the prices that each firm charge in the second stage.

- *Firm 1's best response function.* Firm 1 chooses the price  $p_1$  that solves

$$\max_{p_1} (p_1 - 0) \underbrace{\left( \frac{p_2 - p_1}{s_2 - s_1} - \underline{\theta} \right)}_{\text{Demand, } \hat{\theta} - \underline{\theta}}$$

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Differentiating with respect to  $p_1$ , we obtain

$$\frac{\partial \pi_1}{\partial p_1} = \frac{p_2 - p_1}{s_2 - s_1} - \underline{\theta} - \frac{p_1}{s_2 - s_1} = 0$$

Solving for  $p_1$ , we find firm 1's best response function

$$p_1(p_2) = \frac{\underline{\theta}(s_1 - s_2)}{2} + \frac{1}{2}p_2$$

with vertical intercept at  $\frac{\underline{\theta}(s_1 - s_2)}{2}$  and slope  $\frac{1}{2}$ . Intuitively, when firm 2 increases its price by \$1, firm 1 responds increasing its own by \$0.5.

- *Firm 2's best response function.* Operating similarly for firm 2, we have that this firm chooses price  $p_2$  to solve

$$\max_{p_2} (p_2 - 0) \underbrace{\left( \bar{\theta} - \frac{p_2 - p_1}{s_2 - s_1} \right)}_{\text{Demand, } \bar{\theta} - \bar{\theta}}$$

Differentiating with respect to  $p_2$ , we obtain

$$\frac{\partial \pi_2}{\partial p_2} = \frac{p_1 - 2p_2}{s_2 - s_1} + \bar{\theta} = 0$$

Solving for  $p_2$ , we find firm 2's best response function

$$p_2(p_1) = \frac{\bar{\theta}(s_2 - s_1)}{2} + \frac{1}{2}p_1$$

with vertical intercept at  $\frac{\bar{\theta}(s_2 - s_1)}{2}$  and slope  $\frac{1}{2}$ . Intuitively, when firm 1 increases its price by \$1, firm 2 responds increasing its own by \$0.5.

- *Finding equilibrium prices.* Simultaneously solving for  $p_1$  and  $p_2$  in the above best response functions, we find equilibrium prices

$$\begin{aligned} p_1^*(s_1, s_2) &= \frac{(\bar{\theta} - 2\underline{\theta})(s_2 - s_1)}{3}, \text{ and} \\ p_2^*(s_1, s_2) &= \frac{(2\bar{\theta} - \underline{\theta})(s_2 - s_1)}{3}. \end{aligned}$$

Since quality is given and satisfies  $s_2 > s_1$  by definition, we can claim that  $p_1^*(s_1, s_2) > 0$  if and only if  $\bar{\theta} > 2\underline{\theta}$ . Similarly,  $p_2^*(s_1, s_2) > 0$  if and only if  $2\bar{\theta} > \underline{\theta}$ , which can be rewritten as  $\bar{\theta} > \frac{\underline{\theta}}{2}$ . However, this inequality holds by the initial condition  $\bar{\theta} > \underline{\theta}$  since  $\underline{\theta} > \frac{\underline{\theta}}{2}$ , implying that  $p_2^*(s_1, s_2) > 0$  holds under all admissible parameter values. We can, therefore, identify the following two regions of parameter  $\bar{\theta}$ , as depicted in the figure below:

- When  $\bar{\theta} > 2\underline{\theta}$ , condition  $\bar{\theta} > \frac{\underline{\theta}}{2}$  also holds, implying that both firms set positive prices. Intuitively, individuals with the highest concern for quality (those with  $\theta = \bar{\theta}$ ) have such a high concern, relative to the individuals with the lowest concern (those with  $\theta = \underline{\theta}$ ), that both firms can sell positive units and make a profit.

- When  $\bar{\theta} \leq 2\theta$ , the price of firm 1 is zero while that of firm 2 is positive (i.e., only firm 2 is active). In this case, the “quality concern differential” between the individuals with the highest and lowest quality concern,  $\bar{\theta} - \underline{\theta}$ , is lower, implying that only firm 2 (the firm with the highest quality) can make a profit.

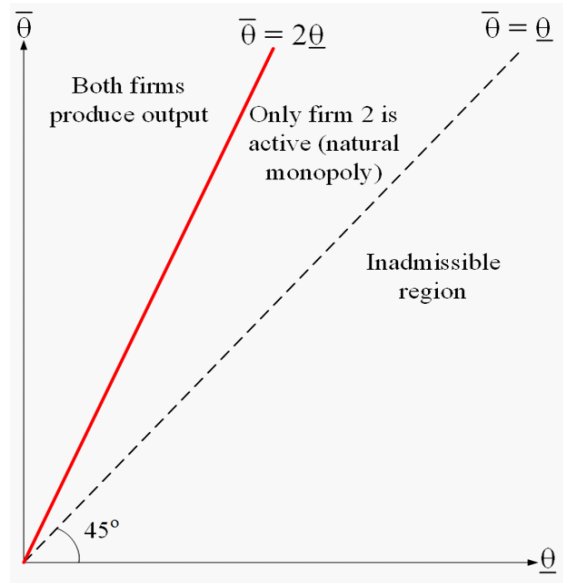


Figure 1. Regions of  $\underline{\theta}$  and  $\bar{\theta}$  pairs.

Therefore, when parameter  $\bar{\theta}$  takes relatively low values,  $\bar{\theta} \leq 2\theta$ , we can then claim that firm 2 is a “natural monopoly” since firm 1 voluntarily exits the market.