EconS 594 - Industrial Organization
Midterm Exam - Due date: Saturday, October 17th.

Instructions: This exam has 5 exercises. Write your answers to each exercise in a different page. Show all your work and be as clear as possible in your answer. You can work in groups, but each student must submit his/her exam. The due date of this take-home exam is Saturday, October 17th, submitted via email before 11:00am. Since this is a take-home exam, late submission will be subject to significant grade reduction.

1. **Leniency programs, based on Motta and Polo (2003).**[1] Consider with n identical firms. In every period t, every firm simultaneously and independently chooses whether to collude, earning monopoly profit π_M, or unilaterally deviate, earning profit π_D, where $\pi_D > \pi_M$. Deviations trigger an infinite punishment (standard grim-trigger strategy) where all firms choose the Nash equilibrium of the unrepeated game, earning profit π_N, where $\pi_D > \pi_M > \pi_N$. For simplicity, assume that firms' common discount factor, δ, satisfies $\delta \geq \frac{\pi_D - \pi_M}{\pi_D - \pi_N}$, which guarantees that collusion emerges in equilibrium when firms are not subject to fines from the antitrust authority (AA).

   In this exercise, we seek to understand the effects of leniency programs, where the AA lowers the fine that collusive firms have to pay when found guilty of collusion, from F to R, where $R < F$. Assume that the AA can perfectly observe collusion, but it needs hard evidence when bringing the case to court. Consider the following time structure: In period $t = 1$, every firm i independently and simultaneously chooses whether to collude, facing the above grim-trigger strategies. In all subsequent periods, $t \geq 2$, the AA then opens an investigation with probability $\alpha \in [0, 1]$ against the collusive firms.

   - If the inquiry is not open, every collusive firm earns profit π_M.
   - If the inquiry is opeed, every firm simultaneously and independently chooses whether or not to reveal information to the AA.
   - If at least one firm reveals, the AA is able to prove them all guilty of collusion.
   - A firm which cooperates with the AA pays a reduced fine of R, whereas every firm that does not cooperate with the AA pays the full fine F.
   - If no firm reveals information, the AA is able to prove the firms guilty of collusion with probability $p \in [0, 1]$.

   If the AA has not been able to prove the firms guilty of collusion at the end of this inquiry period t, the firms will be never investigated again in the future. However, if the firms are found guilty in period t, they will never collude again in the future, earning π_N in all subsequent periods.

   (a) *Reveal game.* Show that, in the subgame that ensues after the AA opens an investigation, we can sustain two Nash equilibria: one where all firms reveal, and one where all do not reveal information to the AA. Discuss under which parameter conditions each Nash equilibrium can be sustained. Interpret your results.

[1] Motta, M. and M. Polo (2003) "Leniency programs and cartel prosecution", International Journal of Industrial Organization, 21, pp. 347-79.

(b) *Collude and reveal.* In the first stage, identify under which conditions on probability α every firm chooses to collude in $t = 1$ and reveal in $t = 2$ (rather than deviating in $t = 1$ which triggers π_N profits thereafter).

(c) *Collude and not reveal.* In the first stage, identify under which conditions on probability α every firm chooses to collude in $t = 1$ and not reveal in $t = 2$ (rather than deviating in $t = 1$ which triggers π_N profits thereafter).

(d) Draw a figure with probabily α in the vertical axis and probability p in the horizontal axis, and identify the regions of parameter values that sustain the subgame perfect equilibria found in parts (b) and (c). Interpret your results.

(e) Using the figure in part (c), what happens when no leniency programs are in place (i.e., $R = F$). How does a reduction in R, for a given F, affect the regions supporting each equilibrium outcome? Interpret. Is a leniency program always optimal?

2. **Switching costs, based on Klemperer (1995).**[2] Consider an industry with N consumers distributed along the $[0, 1]$ line, which measures consumers' linear cost of learning how to use a product. Firms A and B sells the same homogeneous product but are located at 0 and 1, respectively, and have the same unit cost $c = 0$ in each period. Therefore, a consumer located at point x has a learning cost tx of using firm A's product and $t(1-x)$ of using B's product, where $t > 0$. Consumers do not have any physical transportation cost.

   The time structure of the game is the following:

   i) At period 1, consumer utility is given by

   $$u = r - p_i - t\,|l_i - x|$$

   for every firm $i = \{A, B\}$, where l_i denotes the location of firm i.

   ii) At period 2, each consumer has a reservation price r to buy the good.

   Goods cannot be stored and consumers do not discount future payoffs. Goods are perceived as perfectly homogeneous, but there are switching costs: if a consumer changes provider, he has to pay a switching cost $s > 0$, which for simplicity is assumed to be independent of the distance that separates the consumer from the firm. Firms set simultaneously prices in each period. Assume that $r > c$, that $s \geq r - c$, and that $r - 2t > c$.

   (a) *Second period.* Find equilibrium prices in the second stage for every firm i, $p^2_{i,S}$, where subscript S denotes that our setting considers switching costs. Find equilibrium profits in the second period, for given first-period sales q^1_A by firm A and q^1_B by firm B.

   (b) *First period.* Find the equilibrium prices in the first period of the game, $p^1_{i,S}$ for every firm i.

   (c) *No switching costs.* Find the equilibrium price for the game where the first period is like the above, but in the second period there are no switching costs at all. Label prices $p^1_{i,NS}$ and $p^2_{i,NS}$ for every firm i, where subscript NS denotes that our setting assumes no switching costs.

   (d) *Price comparison.* Show that first-period prices are lower with switching costs, $p^1_{i,S} < p^1_{i,NS}$, but second-period prices are higher, $p^2_{iS} > p^2_{iNS}$, for every firm i.

[2] Klemperer, P. (1995) "Competition when consumers have switching costs: An overview" Review of Economic Studies, 62, pp. 515-39.

3. **Pay to switch or pay to stay?, based on Schaffer and Zhang (2000).**[3] Consider an industry with two firms, A and B, selling a homogeneous good, and both facing the same marginal cost of production, $c > 0$. Consumers buy at most one unit of either firm. Consumers are partitioned in the following way: Group a of consumers represents a fraction $\theta \in \left[\frac{1}{2}, 1\right]$ of all consumers, while group b represents a fraction $1-\theta$ of all consumers. If both firms charge the same price, all consumers in group a (group b) purchase from firm A (firm B, respectively). This entails that, if both firms charge the same price to both groups, firm A captures a larger market share.

   In addition, let $F_k(x)$ denote the fraction of consumers in group $k = \{a, b\}$, with loyalty l_k less or equal to x, and

$$F_k(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{l_k} & \text{if } 0 \leq x \leq l_k \\ 1 & \text{if } x > l_k. \end{cases}$$

   To understand the intuition behind this probability, consider, as an illustration, consumers in group a. If firm A charges a price premium $x = p_A - p_B$, then consumers in group a purchase from firm A if and only if $x < l_a$ (that is, if $p_A - p_B < l_a$ or $p_A < l_a + p_B$), which occurs with probability $1 - F_a(x)$. In other words, the fraction of consumers in group a who purchase from firm A is $1 - F_a(x)$ while the remaining fraction of consumers purchase from firm B, that is, $F_a(x)$. In contrast, the fraction of consumers in group b who purchase from firm A is $F_b(x)$ while those buying from firm B is $1 - F_b(x)$.

   Let p_i denote the price that firm $i = \{A, B\}$ sets on group a, and let $\widetilde{p}_i$ represent the price that firm i sets on group b. Firm A's demand is then

$$D_A = \theta \left[1 - F_a(p_A - p_B)\right] + (1-\theta) F_b(\widetilde{p}_B - \widetilde{p}_A)$$

   where the first (second) term indicates the sales from group a (group b). Similarly, firm B's demand is

$$D_B = \theta F_a(p_A - p_B) + (1-\theta)\left[1 - F_b(\widetilde{p}_B - \widetilde{p}_A)\right]$$

   where, similarly, the first (second) term indicates the sales from group a (group b).

   (a) *No price discrimination.* If firms cannot price discriminate, $p_i = \widetilde{p}_i$ for every firm i, show that firm A sets a strictly higher price than firm B if and only if $\theta > 1/2$.

   (b) Still in the setting of part (a), write each firm's profit maximization problem and find equilibrium prices and profits.

   (c) Evaluate equilibrium prices you found in part (b) at $\theta = 1/2$. Interpret your results.

   (d) Evaluate equilibrium prices you found in part (b) at $\theta = 1$. Interpret your results.

   (e) *Price discrimination.* When firms can price discriminate, $p_i \neq \widetilde{p}_i$ for every firm i, show that $p_A > \widetilde{p}_A$ for firm A but $p_B < \widetilde{p}_B$ for firm B.

[3] Shaffer, G. and Zhang, Z. (2000), "Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs," Journal of Economics & Management Strategy 9, pp. 397-424.

(f) Still in the setting of part (e), write each firm's profit maximization problem and find equilibrium prices and profits.

(g) *Comparison.* Compare equilibrium prices from parts (b) and (f), to compute the price discount that firm A offers, $d_A = p_A - \widetilde{p}_A$. When positive, firm A applies a discount to consumers in group b, which Schaffer and Zhang (2000) refer to as firm A "pays to switch". In contrast, when $d_A < 0$, firm A applies a discount to consumers in its own group a, informally known as that the firm "pays to stay." Show that firm A practices "pay to stay" only when the loyalty ratio $\frac{l_b}{l_a}$ satisfies $\frac{l_b}{l_a} \geq 2$.

(h) Compute the price discount that firm B offers, $d_B = \widetilde{p}_B - p_B$, showing that it "pays to stay" only when the loyalty ratio $\frac{l_b}{l_a}$ satisfies $\frac{l_b}{l_a} < \frac{1}{2}$. Combine your results with those in part (e) to argue that it is never optimal for both firms to practice "pay to stay." Interpret.

4. **Stable cartel and product differentiation.** Consider the example that we discussed in class on October 1st, about an industry with $n = 3$ firms competing a la Cournot and forming a cartel between two of the firms (see pages 352-353 in BP for more details). In that model, we considered that every firm i faces an inverse demand function $p_i = a - q_i - \gamma(q_j + q_k)$, but then assumed that $\gamma = \frac{1}{2}$.

   (a) Reproduce all the steps of our analysis in class, in detail, but allowing for any value of parameter $\gamma \in [0, 1]$. How is the internal and external stability of the cartel affected by a marginal increase in γ? Interpret.

   (b) Reproduce all the steps of our analysis in class, in detail, but allowing for any number of firms, $n \geq 3$. How is the internal and external stability of the cartel affected by a marginal increase in n? Interpret.

5. **The "tragedy of the anticommons," based on Heller and Eisenberg (1998).**[4] Consider a setting with two firms, A and B, selling complementary goods. Every firm i faces a linear inverse demand function

$$p_i(q_i, q_j) = a - bq_i + dq_j$$

where $b > d \geq 0$. Intuitively, parameter $b > 0$ indicates that firm i faces a downward sloping demand curve for its product. When $d = 0$, the products of firm i and j are regarded as independent by consumers, but when $d > 0$ customers see the goods as complementary, so more sales by firm j increase the demand for firm i's product. In addition, we assume that own-price effects dominate cross-price effects, that is, $b > d$. For simplicity, assume that firms face a symmetric marginal cost of production c, where $a > c \geq 0$.

   (a) *Nash equilibrium.* Assuming that every firm i simultaneously and independently chooses its output q_i, find its best response function, the equilibrium output, price, and profits.

   (b) *Coordinating output decisions in a cartel.* Assume now that both firms coordinate their production decisions to maximize their joint profits. Find their equilibrium output, price, and profits.

   (c) Compare equilibrium profits in the cartel against those in the Nash equilibrium of part (a).

   (d) *Numerical example.* Evaluate equilibrium profits in the cartel and in the Nash equilibrium of the game at $a = b = 1$ and $c = 1/2$. Then, evaluate them at $d = 0$ and at $d = 1/2$. Interpret your results.

[4] Heller, M. and R. Eisenberg (1998) "Can Patents Deter Innovation? The Anticommons in Biomedical Research," Science, 280, pp. 698–701.