HOMEWORK #3, ECONS 594

#3, CH. 8

Exercise 5 *Spatial market segmentation [part 2 and 3 of the exercise are included in the 2nd edition of the book]*

Consider a horizontally differentiated product market in which two firms are located at points $l_1 = 0$ and $l_2 = 1$ on the line. Firms produce at marginal costs c. There is a continuum of consumers of mass 1 who are uniformly distributed on the unit interval. They have unit demand and have an outside utility of $-\infty$. A consumer located at $x \in [0,1]$ obtains indirect utility $v_1 = r - \tau(x)^2 - p_1$ if she buys one unit from firm 1 and $v_2 = r - \tau(1-x)^2 - p_2$ if she buys from firm 2. Firms have marginal costs equal to c.

1. Suppose that firms simultaneously set a uniform price for all consumers. Characterize the equilibrium of the game. Determine equilibrium profits.
2. Suppose now that firms can price-discriminate between consumers located on $[0, 1/2]$ (segment A) and $[1/2, 1]$ (segment B). Determine the profit function of each firm. Characterize the pure-strategy Nash equilibrium of the game in which firms simultaneously set prices. **Note:** You are allowed to restrict attention to the part of the demand function, which is relevant for the equilibrium analysis.
3. Compare your result in (2) to the game in which firms cannot discriminate. In which environment obtain firms larger profits? Explain your findings.
4. Suppose now that firm 1 can discriminate between the two consumer segments and that firm 2 cannot. Characterize the Nash equilibrium of the price game in which firm 1 sets a possibly different price for each consumer segment, while firm 2 sets the same price to all consumers.
5. Compare the firms' profits in situation (4) to those in situation (2).
6. Consider the possibility for firms to "invest" (non-negative number) into the possibility to price discriminate between consumers in segments A and B at investment cost I. Characterize the equilibrium of the two-stage game in which firms simultaneously decide whether to invest in stage 1 and simultaneously set prices in stage 2 depending on the level of the investment cost I. Comment on your result.

Solutions to Exercise 5 #3, CH. 8

1. The indifferent consumer is located at

$$\hat{x} = \frac{p_2 - p_1}{2\tau} + \frac{1}{2}.$$

Hence, demand is

$$\begin{aligned} q_1(p_1, p_2) &= \frac{p_2 - p_1}{2\tau} + \frac{1}{2}, \\ q_1(p_1, p_2) &= 1 - \frac{p_2 - p_1}{2\tau} - \frac{1}{2}. \end{aligned}$$

Firms maximize profits $(p_i - c)q_i(p_i, p_j)$ with respect to p_i. Rewriting the first-order conditions we obtain

$$\begin{aligned} p_1 &= \frac{p_2 + c}{2} + \frac{\tau}{2}, \\ p_2 &= \frac{p_1 + c}{2} + \frac{\tau}{2}. \end{aligned}$$

Solving this system of two equations we obtain

$$\begin{aligned} p_1^* &= c + \tau, \\ p_2^* &= c + \tau. \end{aligned}$$

Equilibrium profits are $\tau/2$.

2. In each segment, there is an indifferent consumer,

$$\widehat{x}^A = \frac{p_2^A - p_1^A}{2\tau} + \frac{1}{2} \text{ and } \widehat{x}^B = \frac{p_2^B - p_1^B}{2\tau} + \frac{1}{2}.$$

Profit functions are $\pi_1 = (p_1^A - c)\widehat{x}^A + (p_1^B - c)(\widehat{x}^B - 1/2)$ and $\pi_2 = (p_2^A - c)(1/2 - \widehat{x}^A) + (p_2^B - c)(1 - \widehat{x}^B)$. Maximizing π_i with respect to p_i^A and p_i^B for each firm i gives rise to four first-order conditions. We note that segments A and B are independent. First-order conditions in segment A can be rewritten as

$$p_1^A = \frac{1}{2}(c + p_2^A + \tau) \text{ and } p_2^A = \frac{1}{2}(c + p_1^A).$$

Hence, equilibrium prices are $p_1^{A*} = c + \frac{2}{3}\tau$ and $p_2^{A*} = c + \frac{1}{3}\tau$. Correspondingly, in segment B we obtain $p_1^{B*} = c + \frac{1}{3}\tau$ and $p_2^{B*} = c + \frac{2}{3}\tau$.

3. When firms can discriminate between the two segments they set lower prices in equilibrium, as competition has become more intense (as a price cut leads to a relatively lower reduction in revenues since there are relatively fewer inframarginal consumers). Since the overall market shares are the same, under price discrimination profits are unambiguously lower than without price discrimination.

4. There continues to be an indifferent consumer in each segment,

$$\widehat{x}^A(p_1^A, p_2) = \frac{p_2 - p_1^A}{2\tau} + \frac{1}{2} \text{ and } \widehat{x}^B(p_1^B, p_2) = \frac{p_2 - p_1^B}{2\tau} + \frac{1}{2}.$$

Profit functions are $\pi_1 = (p_1^A - c)\widehat{x}^A(p_1^A, p_2) + (p_1^B - c)(\widehat{x}^B(p_1^B, p_2) - 1/2)$ and $\pi_2 = (p_2 - c)(1/2 - \widehat{x}^A(p_1^B, p_2)) + (p_2 - c)(1 - \widehat{x}^B(p_1^B, p_2))$. Firm 1 maximizes π_1 with respect to p_1^A and p_1^B, while firm 2 maximizes π_2 with respect to p_2. We can write first-order conditions as

$$\begin{aligned} \frac{1}{2} - \frac{p_1^A - c}{2\tau} - \frac{p_1^A - p_2}{2\tau} &= 0, \\ -\frac{p_1^B - c}{2\tau} + \frac{p_2 - p_1^B}{2\tau} &= 0, \\ \frac{1}{2} - \frac{p_2 - c}{2\tau} + \frac{p_1^A - p_2}{2\tau} - \frac{p_2 - p_1^B}{2\tau} &= 0. \end{aligned}$$

Solving these three equations we obtain $p_1^{A*} = c + 3\tau/4$, $p_1^{B*} = c + \tau/4$, and $p_2^* = c + \tau/2$.

5. If both firms discriminate, each firm obtains equilibrium profit $5\tau/18$. If firm 2 does not discriminate, firm 1 obtains equilibrium profit $5\tau/16$, while firm 2 obtains a profit of $\tau/4$. Hence, firm1 is better off, if firm 2 cannot discriminate, while firm 2 is worse off.

6. Equilibrium profits without discrimination are $\tau/2$. We thus have

	I	no I
I	$5\tau/18 - I$, $5\tau/18 - I$	$5\tau/16 - I$, $\tau/4$
no I	$\tau/4$, $5\tau/16 - I$	$\tau/2$, $\tau/2$

Hence, for any $I \geq 0$, there exists a subgame-perfect equilibrium in which none of the firms invests in the ability to price discriminate. If $5\tau/18 - I > \tau/4$ there is also an equilibrium in which both firms invest in the ability to price disriminate. Thus, the condition for this alternative equilibrium to exist is $I < \tau/36$. Firms may fail to coordinate and both invest in the ability to spatially discriminate which leads to lower prices and lower profits.

Exercise 6 *Price discrimination in duopoly*

Consider a duopoly market with two firms and a continuum of consumers. Each firm $i \in \{1, 2\}$ sells its product at price p_i and incurs marginal costs equal to zero. Consumers are of measure 1 and have unit demand. When buying one unit of product i a consumer of type (t, x) obtains utility $r - t|x - l_i| - p_i$ where l_i is the location of firm i and p_i is its price; if she does not buy her utility is set equal to $-\infty$. Half of consumers belong to the group with type t_A and half of consumers to the other group with type t_B; $t_A \geq t_B$. Within each group, consumers are uniformly distributed on the unit interval, $x \in [0, 1]$. Firms are located at 0 and 1, respectively.

1. Suppose that $t_A = t_B$. Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Report equilibrium prices, outputs, and profits.

2. Suppose that $t_A > t_B$. Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Report equilibrium prices, outputs, and profits.

3. Suppose that $t_A > t_B$. Suppose furthermore that both firms observe consumer type t and that they can condition their price on this type; i.e., firm i set $p_i(t)$. (Consumers are assumed not to be able to trade among each other). Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Compare your result to the previous setting. Discuss whether firms benefit from regulation that requires them to set uniform prices.

4. Suppose that $t_A > t_B$. Suppose now that only firm 1 observes consumer type t and that it can condition its price on this type – i.e., firm 1 set $p_1(t)$ – whereas firm 2 has to charge the same price to all consumers. (Consumers are assumed not to be able to trade among each other). Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game.

5. Suppose that $t_A = 2 > t_B = 1$. Consider the two-stage game in which, in the first stage, firms acquire the ability to identify a consumer's type t at cost C and, in the second stage, they compete in prices. Using your insights from parts 2 to 4, characterize the subgame-perfect equilibria as a function of C.

6. Discuss your findings.

Solutions to Exercise 6 2

1. Standard linear Hotelling model. Set $t \equiv t_A = t_B$. Demand for firm i:

$$\frac{1}{2} - \frac{p_i - p_j}{2t}, j \neq i$$

Firm i's maximization problem is

$$\max_{p_i} p_i \left(\frac{1}{2} - \frac{p_i - p_j}{2t} \right).$$

The first-order condition is

$$\frac{1}{2} - \frac{2p_i - p_j}{2t} = 0,$$

which gives rise to the best response

$$p_i = \frac{t}{2} + \frac{p_j}{2}.$$

In symmetric equilibrium, $p_1^* = p_2^* = t$. Equilibrium demand is $1/2$, and equilibrium profits are $\pi_1^* = \pi_2^* = t/2$.

2. The maximization problem now becomes

$$\max_{p_i} \frac{1}{2} p_i \left(\frac{1}{2} - \frac{p_i - p_j}{2t_A} \right) + \frac{1}{2} p_i \left(\frac{1}{2} - \frac{p_i - p_j}{2t_B} \right).$$

The first-order condition is

$$\frac{1}{2} - \frac{2p_i - p_j}{4t_A} - \frac{2p_i - p_j}{4t_B} = 0,$$

which can be rewritten as

$$2 = \frac{2p_i - p_j}{t_A} + \frac{2p_i - p_j}{t_B}.$$

In symmetric equilibrium, $p_i^* = p_j^* = p^*$. Thus, we have

$$\begin{aligned} 2 &= p^* \left(\frac{1}{t_A} + \frac{1}{t_B} \right) \\ 2 &= p^* \frac{t_A + t_B}{t_A t_B} \\ p^* &= 2 \frac{t_A t_B}{t_A + t_B}. \end{aligned}$$

Equilibrium demand is $1/2$, and equilibrium profits are $\pi_1^* = \pi_2^* = t_A t_B / (t_A + t_B)$.

3. Using the results in 1, we obtain $p_1^{A*} = p_2^{A*} = t_A$ and $p_1^{B*} = p_2^{B*} = t_B$. Equilibrium demand is $1/4$ for each consumer type t, and equilibrium profits are $\pi_1^* = \pi_2^* = (t_A + t_B)/4$.

   Profits under uniform pricing are smaller than under discriminatory pricing if

$$\begin{aligned} \frac{t_A t_B}{(t_A + t_B)} &< \frac{t_A + t_B}{4} \\ 4 t_A t_B &< t_A^2 + 2 t_A t_B + t_B^2 \\ 0 &< t_A^2 - 2 t_A t_B + t_B^2 \\ 0 &< (t_A - t_B)^2 \end{aligned}$$

   which is always satisfied. Hence firms suffer from the regulatory intervention.

4. For firm 1, the maximization problem is

$$\max_{p_1^A, p_1^B} \frac{1}{2} p_1^A \left(\frac{1}{2} - \frac{p_1^A - p_2}{2 t_A} \right) + \frac{1}{2} p_1^B \left(\frac{1}{2} - \frac{p_1^B - p_2}{2 t_B} \right).$$

   For firm 2, the problem is

$$\max_{p_2} \frac{1}{2} p_2 \left(\frac{1}{2} - \frac{p_2 - p_1^A}{2 t_A} \right) + \frac{1}{2} p_2 \left(\frac{1}{2} - \frac{p_2 - p_1^B}{2 t_B} \right).$$

   The system of first-order conditions is then

$$\begin{aligned} \frac{1}{2} - \frac{2 p_1^A - p_2}{2 t_A} &= 0 \\ \frac{1}{2} - \frac{2 p_1^B - p_2}{2 t_B} &= 0 \\ \frac{1}{2} - \frac{2 p_2 - p_1^A}{4 t_A} - \frac{2 p_2 - p_1^B}{4 t_B} &= 0 \end{aligned}$$

   The first two equations can be written as $p_1^A = \frac{t_A}{2} + \frac{p_2}{2}$ and $p_1^B = \frac{t_B}{2} + \frac{p_2}{2}$. Substituting these expressions in the first-order condition of firm 2, we obtain that, in equilibrium,

$$\begin{aligned}
\frac{1}{2}-\frac{2p_2^*-\left(\frac{t_A}{2}+\frac{p_2^*}{2}\right)}{4t_A}-\frac{2p_2^*-\left(\frac{t_B}{2}+\frac{p_2^*}{2}\right)}{4t_B} &= 0\\
\frac{1}{2}-\frac{3p_2^*-t_A}{8t_A}-\frac{3p_2^*-t_B}{8t_B} &= 0\\
\frac{3}{4}-\frac{3p_2^*}{8t_A}-\frac{3p_2^*}{8t_B} &= 0\\
1-\frac{p_2^*}{2t_A}-\frac{p_2^*}{2t_B} &= 0\\
\frac{1}{2}p_2^*\frac{t_A+t_B}{t_At_B} &= 1\\
p_2^* &= \frac{2t_At_B}{t_A+t_B}
\end{aligned}$$

Substituting this equilibrium price in the best-response function of firm 1, we obtain

$$\begin{aligned}
p_1^{A*} &= \frac{t_A}{2}+\frac{t_At_B}{t_A+t_B}\\
&= \frac{t_A^2+3t_At_B}{2(t_A+t_B)}
\end{aligned}$$

and

$$p_1^{B*}=\frac{t_B^2+3t_At_B}{2(t_A+t_B)}.$$

Equilibrium demand for firm 1 among consumers of type t_A is

$$\begin{aligned}
&\frac{1}{2}-\frac{p_1^{A*}-p_2^*}{2t_A}\\
=\ &\frac{1}{2}-\frac{1}{2t_A}\left(\frac{t_A^2+3t_At_B}{2(t_A+t_B)}-\frac{2t_At_B}{t_A+t_B}\right)\\
=\ &\frac{1}{2}-\frac{1}{2t_A}\frac{t_A^2-t_At_B}{2(t_A+t_B)}\\
=\ &\frac{1}{2}-\frac{1}{4}\frac{t_A-t_B}{t_A+t_B}
\end{aligned}$$

Similarly, its demand among consumers of type t_B is

$$\frac{1}{2}-\frac{1}{4}\frac{t_B-t_A}{t_A+t_B}.$$

Hence, firm 1's equilibrium profit is

$$\begin{aligned}
& \frac{1}{2}\frac{t_A^2+3t_At_B}{2(t_A+t_B)}\left(\frac{1}{2}-\frac{1}{4}\frac{t_A-t_B}{t_A+t_B}\right)+\frac{1}{2}\frac{t_B^2+3t_At_B}{2(t_A+t_B)}\left(\frac{1}{2}-\frac{1}{4}\frac{t_B-t_A}{t_A+t_B}\right)\\
= & \frac{1}{16}\frac{t_A^2+3t_At_B}{t_A+t_B}\left(2-\frac{t_A-t_B}{t_A+t_B}\right)+\frac{1}{16}\frac{t_B^2+3t_At_B}{t_A+t_B}\left(2-\frac{t_B-t_A}{t_A+t_B}\right)\\
= & \frac{1}{16}\frac{t_A^2+3t_At_B}{(t_A+t_B)^2}(2(t_A+t_B)-(t_A-t_B))+\frac{1}{16}\frac{t_B^2+3t_At_B}{(t_A+t_B)^2}(2(t_A+t_B)-(t_B-t_A))\\
= & \frac{1}{16}t_A\frac{(t_A+3t_B)^2}{(t_A+t_B)^2}+\frac{1}{16}t_B\frac{(t_B+3t_A)^2}{(t_A+t_B)^2}\\
= & \frac{1}{16(t_A+t_B)^2}(t_A(t_A+3t_B)^2+t_B(t_B+3t_A)^2)\\
= & \frac{1}{16(t_A+t_B)^2}(t_A[(t_A+t_B)^2+4t_B^2+4t_B(t_A+t_B)]+t_B(t_B+3t_A)^2)
\end{aligned}$$

Firm 2's equilibrium profit is

$$\begin{aligned}
& \frac{t_At_B}{t_A+t_B}\left[\left(\frac{1}{2}+\frac{1}{4}\frac{t_A-t_B}{t_A+t_B}\right)+\left(\frac{1}{2}+\frac{1}{4}\frac{t_B-t_A}{t_A+t_B}\right)\right]\\
= & \frac{t_At_B}{t_A+t_B}\left[1+\frac{1}{4}\frac{t_A-t_B}{t_A+t_B}+\frac{1}{4}\frac{t_B-t_A}{t_A+t_B}\right]\\
= & \frac{t_At_B}{t_A+t_B}
\end{aligned}$$

5. Denote equilibrium profits as a function of the regimes U or D for each of the two firms. Using parameter values $t_A = 2$ and $t_B = 1$, we have

$$\begin{aligned}
\pi_1^*(U,U) &= \pi_2^*(U,U) = t_At_B/(t_A+t_B) = 2/3\\
\pi_1^*(D,D) &= \pi_2^*(D,D) = (t_A+t_B)/4 - C = 3/4 - C\\
\pi_1^*(D,U) &= \pi_2^*(U,D) = \frac{1}{16(t_A+t_B)^2}(t_A(t_A+3t_B)^2+t_B(t_B+3t_A)^2) - C\\
&= \frac{1}{16\times 9}(50+49) - C = \frac{11}{16} - C\\
\pi_1^*(U,D) &= \pi_2^*(D,U) = t_At_B/(t_A+t_B) = 2/3
\end{aligned}$$

We note that (U,U) is an equilibrium if $\pi_1^*(U,U) \geq \pi_1^*(D,U)$ which is equivalent to $C \geq 1/48$. Furthermore, (D,D) is an equilibrium if $\pi_1^*(D,D) \geq \pi_1^*(U,D)$ which is equivalent to $C \leq 1/12$. Hence, there is a unique equilibrium (D,D) for $C < 1/48$, (U,U) and (D,D) are equilibria for $C \in [1/48, 1/12]$, and (U,U) is the unique equilibrium if $C > 1/12$.

6. If it is not too costly, firms will opt to acquire information on consumer types t. With discrimination their profits will be larger than using a uniform price that applies to both consumer groups.

3. Consumer surplus for agents of type 2 at any price $p \leq 2$ is computed as $CS_2(p) = (1/4)(2-p)^2$. We recall from (2) that $CS_1(p) = (1/2)(1-p)^2$ for any $p \leq 1$. For any price where the two types of consumers buy (i.e., $p \leq 1$), we check that $CS_2(p) \geq CS_1(p)$. One option for the monopolist is to make sure that consumers of both types buy; for this, $m = CS_1(p)$ and $p \leq 1$. We have then the same problem as in (2), with $c = 1/2$: $(m,p) = (1/8, 1/2)$ and $\pi_1 = 1/8$. The alternative option is to sell only to type-2 consumers with $m = CS_2(p)$ and $p \leq 2$. The monopolist's problem is then to choose p to maximize $\pi_2 = (1/2)((1/4)(2-p)^2 + (p-1/2)(1-p/2)$. The first-order condition yields $p = 1/2$. As $p = 1/2$ (i.e., marginal cost pricing violates the constraint), the monopolist chooses $p = 1$, so that $m = 1/4$. Theresulting profit is $\pi_2 = (1/2)(1/2) = 1/4$. We see that $\pi_2 > \pi_1$: the monopolist prefers to sell to type-2 consumers only by setting a price above marginal cost.

Exercise 12 *Multi-stop shopping*[2] *[included in 2nd edition of the book]*
#3, CHAPTER 9

Suppose that a supermarket offers a product selection consisting of of products A and B. Consumers are willing to pay 10 Euro for one unit of product A and 10 Euro for product B. Consumers have heterogeneous shopping cost z. This shopping cost is uniformly distributed over the interval $[0, 10]$. Consumers are of mass 10. The firm has marginal cost of 6 for product A and 0 for product B.

1. Calculate the profit-maximizing prices p_A and p_B. How much profit can the supermarket make.

2. Suppose that a discounter has entered the market who sells product A at its (lower) marginal costs of 4 Euro. Now consumers can opt for one-stop shopping at the supermarket, two-stop shopping at the supermarket and the discounter, or not to shop at all. The shopping cost z applies to each stop. Determine the profit maximizing prices of the supermarket. Determine the supermarket's profit.

3. Compare your results in (2) to those in (1). Interpret your findings.

Solutions to Exercise 12 #3, CH. 9

1. Profit-maximizing prices satisfy $p_A \leq 10$ and $p_B \leq 10$. Then consumers buy either both products or do not buy at all. Denote the consumer who is indifferent between not buying at all and buying both products by z_2: $z_2 = 20 - p_A - p_B$. Thus demand is $Q(p_A, p_B) = 20 - p_A - p_B$. Denote the total price as $p = p_A + p_B$. The maximization problem is $\max_p (p-6)(20-p)$. Thus $p^m = 13$ and $\pi^m = 49$. Any prices p_A, p_B with $p_A \leq 10$, $p_B \leq 10$, and $p_A + p_B = 13$ maximize monopoly profits.

[2] This exercise is inspired by Patrick Rey and Zhijun Chen (2012), "Loss Leading as an Exploitative Practice", American Economic Review 102, 3462-3482.

2. Consumers with low shopping cost buy product A from the discounter and product B from the supermarket. Denote the consumer who is indifferent between this two-stop shopping and one-stop shopping at the supermarket by z_1. This consumer satisfies $(10-4)+(10-p_B)-2z_1 = 20-p_A-p_B-z_1$ which is equivalent to $z_1 = p_A - 4$. Hence, consumers between 0 and z_1 buy product A at 4 Euro from the discounter and product B at price p_B. Consumers between z_1 and z_2 buy both products at the supermarket and consumers with shopping costs above z_2 do not buy.

   The profit-maximization problem is $\max_{p_A,p_B} p_B(p_A-4)+(p_A+p_B-6)(24-2p_A-p_B)$. Rewriting the first-order conditions gives $p_A+p_B = 13$ and $2p_A + p_B = 18$. Thus, profit-maximizing prices are $p_A = 5$ and $p_B = 8$. These prices are lower than the willingness-to-pay of consumers and involve product A being sold below costs. Given these prices, z_1 is located at 1 and z_2 at 7. Profits under competition from the discounter are $\pi^c = 1 \cdot 8 + (7-1)(13-6) = 50$.

3. The supermarket makes higher profit under competition from a more efficient discounter than under monopoly. Product A can be seen as a loss leader since it is sold below marginal costs. Under competition the supermarket is better able to price-discriminate between different consumer types. Setting the price for product A below marginal costs allows the supermarket to raise the price of product B, which it sells to all consumers including those with low shopping costs. By contrast, under monopoly, the supermarket would sell product B at a loss to all consumers if it mimicked the price strategy under competition (which is one of the many solutions to the monopoly problem).

Exercise 13 *Ticket sales*

Consider a monopoly which can sell up to 50 concert tickets in a small town (at zero marginal costs); i.e., it may sell fewer tickets but cannot exceed the capacity of 50. Consumers have unit demand. The inverse demand of all consumers in this town is either $100-q$ or $160-q$. Each of the corresponding states of the world occurs with probability 1/2. Consumers know the state of the world when making purchasing decisions.

1. Suppose that the firm knows the local demand conditions (i.e., the state of the world) when it chooses its selling strategy. Determine the optimal strategy to sell tickets at a uniform price. Calculate profit-maximizing price and profits.

2. Suppose that the firm does not know local demand conditions (i.e., the state of the world) when it chooses its selling strategy. Determine the optimal strategy to sell tickets at a uniform price. Calculate profit-maximizing price and profits.

3. Suppose that the firm does not know local demand conditions (i.e., the state of the world), but consumers do and that the firm can sell tickets over