EconS 594 - Industrial Organization
Menu pricing with two firms

In this handout, we explore two models of menu pricing in a context of two firms, one by Ellison (2005) and another by Amstrong and Vickers (2001, 2002). While the standard model of menu pricing analyzed in class is useful to explain second-degree price discrimination by a monopolist, we often see firms in a duopoly (or, more generally, in an oligopoly) offering several qualities for their products and targeting different consumer types (airlines is a typical example).

1 **Competitive quality, based on Ellison (2005).**[1] Consider an industry with two firms, 1 and 2, located in a Hoteling unit line, with firm 1 at 0 and firm 2 at 1. Every firm can offer a high-quality good and a low-quality good at prices p_{iH} and p_{iL} where $i = \{1, 2\}$ denotes the firm. If a consumer buys good $K = \{H, L\}$ from firm 1 at price p_{1K}, his utility is $s_K - \alpha_k p_{1K} - x$, where s_K denotes the intrinsic utility form the good, where $s_H > s_L$ so the high-quality good is more valuable. Parameter α_k represents the marginal utility of income, and $\alpha_h < \alpha_l$ so high-type consumers have a lower marginal utility of income (e.g., wealthy individuals) and suffer a smaller disutility from paying price p_{1K} for the good. Finally, this consumer must travel a distance x to purchase the good from firm 1. If, instead, the consumer buys from firm 2, his utility is $s_K - \alpha_k p_{2K} - (1-x)$, since he must travel $1-x$ to reach firm 2 (which is located at the other end of the unit interval). For simplicity assume that firms incur no production cost.

In this exercise, we find under which conditions we can support a "discriminatory equilibrium," where every firm i offers both the high- and low-quality goods, each at a different price, and only one consumer type buys each product. Afterwards, we study under which conditions we can sustain a "nondiscriminatory equilibrium," where every firm i offers a good of only one quality (e.g., high quality) and both types of consumers buy it.

(a) *Discriminatory equilibrium.* Consider a strategy profile where every firm i offers both the high- and low-quality goods at prices p_{iH} and p_{iL}, respectively. Upon observing these price pairs from each firm, high-type consumers (those with α_h) purchase a unit of the high-quality good, while low-type consumers (those with α_l) buy a unit of the low-quality good. In this and following parts of the exercise, we analyze the conditions that support this strategy profile in equilibrium.

Find the demand for each firm, and from each consumer type, in this strategy profile.

- The high-type consumer buys the high-quality good (H, for compactness), so he is indifferent between buying H from firm 1 and 2 (as both firms offer H) if

$$s_H - \alpha_h p_{1H} - x_h = s_H - \alpha_h p_{2H} - (1 - x_h)$$

[1] Ellison, G. (2005) "A model of add-on pricing," Quarterly Journal of Economics, 120, pp. 585-637.

which, solving for x_h, yields

$$x_h = \frac{1+\alpha_h(p_{2H}-p_{1H})}{2}.$$

Therefore, the demand of firm 1 coming from high-type consumers is x_h, which is increasing in α_h, and in the price premium of firm 2, $p_{2H}-p_{1H}$. As a result, the demand of firm 2 coming from high-type consumers is $1-x_h$.

- Similarly, the low-type consumer buys the low-quality good (L, for compactness), so he is indifferent between buying L from firm 1 and 2 (as both firms offer L) if

$$s_L - \alpha_l p_{1L} - x_l = s_L - \alpha_l p_{2L} - (1-x_l)$$

which, solving for x_l, yields

$$x_l = \frac{1+\alpha_l(p_{2L}-p_{1L})}{2}.$$

Therefore, the demand of firm 1 coming from low-type consumers is x_l, which is increasing in α_l, and in the price premium of firm 2, $p_{2L}-p_{1L}$. As a consequence, the demand of firm 2 coming from low-type consumers is $1-x_l$.

- Overall, these demands entail that the price profile is equivalent to two separate Hoteling models: (1) high-type consumers purchase H, dividing the unit line at x_h; and (2) low-type consumers buy L, dividing the unit line at x_l.

(b) Given the demands found in part (a), write down each firm's profit-maximization problem, and find their equilibrium prices, (p_{1H}, p_{1L}) and (p_{2H}, p_{2L}). Evaluate equilibrium demands and profits in this context.

- Firm 1 solves

$$\max_{p_{1H},p_{1L}} \pi_1 = p_{1H}\underbrace{\left[\frac{1+\alpha_h(p_{2H}-p_{1H})}{2}\right]}_{x_h} + p_{1L}\underbrace{\left[\frac{1+\alpha_l(p_{2L}-p_{1L})}{2}\right]}_{x_l}$$

since x_h (x_l) denotes the sales to high-type (low-type) consumers and the firm incurs no production cost. Differentiating with respect to p_{1H} yields

$$\frac{\partial \pi_1}{\partial p_{1H}} = \frac{1+\alpha_h(p_{2H}-2p_{1H})}{2} = 0$$

which, solving for p_{1H}, gives us the best response function

$$p_{1H}(p_{2H}) = \frac{1}{2\alpha_h} + \frac{1}{2}p_{2H}$$

which originates at $\frac{1}{2\alpha_h}$, and increases in firm 2's price, p_{2H}. Intuitively, when high-type consumers have a larger marginal utility of income (higher α_h), the best response function shifts downward, in a parallel fashion, yielding lower prices. Differentiating with respect to p_{1L}, we obtain

$$\frac{\partial \pi_1}{\partial p_{1L}} = \frac{1+\alpha_l(p_{2L}-2p_{1L})}{2} = 0$$

which, solving for p_{1L}, gives us the best response function

$$p_{1L}(p_{2L}) = \frac{1}{2\alpha_l} + \frac{1}{2}p_{2L}$$

which exhibits the same properties as $p_{1H}(p_{2H})$, described above.

- Firm 2 solves

$$\max_{p_{2H},p_{2L}} \pi_2 = p_{2H}\underbrace{\left[1 - \frac{1+\alpha_h(p_{2H}-p_{1H})}{2}\right]}_{1-x_h} + p_{2L}\underbrace{\left[1 - \frac{1+\alpha_l(p_{2L}-p_{1L})}{2}\right]}_{1-x_l}$$

since $1-x_h$ $(1-x_l)$ denotes the sales to high-type (low-type) consumers, and the firm incurs no production cost. Differentiating with respect to p_{2H} yields

$$\frac{\partial \pi_2}{\partial p_{2H}} = \frac{1+\alpha_h(p_{1H}-2p_{2H})}{2} = 0$$

which, solving for p_{2H}, gives us the best response function

$$p_{2H}(p_{1H}) = \frac{1}{2\alpha_h} + \frac{1}{2}p_{1H}$$

which is symmetric to that of firm 1. Differentiating with respect to p_{2L}, we obtain

$$\frac{\partial \pi_2}{\partial p_{2L}} = \frac{1+\alpha_l(p_{1L}-2p_{2L})}{2} = 0$$

which, solving for p_{2L}, gives us the best response function

$$p_{2L}(p_{1L}) = \frac{1}{2\alpha_l} + \frac{1}{2}p_{1L}$$

which is also symmetric to that of firm 1.

- Invokying symmetry in the prices of the high-quality good, $p_{1H} = p_{2H} = p_H$, we obtain

$$p_H = \frac{1}{2\alpha_h} + \frac{1}{2}p_H$$

which yields $p_H^* = \frac{1}{\alpha_h}$. Similarly, invokying symmetry in the prices of the low-quality good, $p_{1L} = p_{2L} = p_L$, we have that

$$p_L = \frac{1}{2\alpha_l} + \frac{1}{2}p_L$$

which yields $p_L^* = \frac{1}{\alpha_l}$.

- Equilibrium demands for firm 1 are

$$\begin{aligned} x_h^* &= \frac{1+\alpha_h(p_H^*-p_H^*)}{2} = \frac{1}{2}, \text{ and} \\ x_l^* &= \frac{1+\alpha_l(p_L^*-p_L^*)}{2} = \frac{1}{2}. \end{aligned}$$

Therefore, the equilibrium demands of firm 2 are $1-x_h^* = 1-\frac{1}{2} = \frac{1}{2}$ and, similarly, $1-x_l^* = 1-\frac{1}{2} = \frac{1}{2}$.

- Finally, equilibrium profits are

$$\begin{aligned}
\pi_1^* &= p_H^* x_h^* + p_L^* x_l^* = \frac{1}{\alpha_h}\frac{1}{2} + \frac{1}{\alpha_l}\frac{1}{2} = \frac{1}{2}\left(\frac{1}{\alpha_h} + \frac{1}{\alpha_l}\right), \text{ and} \\
\pi_2^* &= p_H^* (1 - x_h^*) + p_L^*(1 - x_l^*) = \frac{1}{\alpha_h}\frac{1}{2} + \frac{1}{\alpha_l}\frac{1}{2} = \frac{1}{2}\left(\frac{1}{\alpha_h} + \frac{1}{\alpha_l}\right).
\end{aligned}$$

(c) Assume, for simplicity, that $\alpha_h = 1$ and $\alpha_l = 5$ (so $\alpha_h < \alpha_l$, as required), $s_H = 3$ and $s_L = 2$. Evaluate the equilibrium prices, demands, and profits found in part (b) at these parameter values. Then, argue that these prices are incentive compatible for both consumer types; and that no firm has unilateral incentives to offer the high-quality good alone.

- If $\alpha_h = 1$ and $\alpha_l = 5$, equilibrium prices become $p_H^* = \frac{1}{1} = 1$ and $p_L^* = \frac{1}{5}$, equilibrium demands are still $\frac{1}{2}$ for every firm $i = \{1, 2\}$ and every consumer type (h and l), and equilibrium profits simplify to

$$\pi_1^* = \pi_2^* = \frac{1}{2}\left(\frac{1}{1} + \frac{1}{5}\right) = \frac{3}{5} = 0.6.$$

- *Incentive compatible prices.* The high-type consumer enjoys a higher utility buying the H good than deviating to purchase the L good since

$$\begin{aligned}
s_H - \alpha_h p_H^* &\geq s_L - \alpha_h p_L^* \\
3 - (1 \times 1) &\geq 2 - \left(1 \times \frac{1}{5}\right) \\
2 &\geq \frac{9}{5}
\end{aligned}$$

which holds. Similarly, the low-type consumer buys the L good rather than purchasing the H good given that

$$\begin{aligned}
s_L - \alpha_l p_L^* &\geq s_H - \alpha_l p_H^* \\
2 - \left(5 \times \frac{1}{5}\right) &\geq 3 - (5 \times 1) \\
1 &\geq -2
\end{aligned}$$

which also holds.

- *Price deviations by the firm.* Assume that firm 2 sticks to offering both quality goods, H and L, at equilibrium prices p_H^* and p_L^*, but firm 1 unilaterally deviates, selling only the H good at a price p_{1H} (which we need to find next). In this context, the high-type consumers can purchase H from firm 1 and 2 (as both firms offer this good), so the indifferent consumer satisfies

$$s_H - \alpha_h p_{1H} - x_h = s_H - \alpha_h p_H^* - (1 - x_h)$$

which, solving for x_h, yields

$$x_h = \frac{1 + \alpha_h(p_H^* - p_{1H})}{2}$$

which is symmetric to the indifferent consumer that we found in part (a). Low-type consumers, however, can only purchase L from firm 2 in this setting, as firm 1 only offers the H good, so the indifferent consumer now satisfies

$$s_H - \alpha_l p_{1H} - \widetilde{x}_l = s_L - \alpha_l p_L^* - (1 - \widetilde{x}_l),$$

that is, he either buys H from firm 1 at p_{1H} or L from firm 2 at p_L^*. Solving for $\widetilde{x}_l$, yields

$$\widetilde{x}_l = \frac{1 + (s_H - s_L) + \alpha_l(p_L^* - p_{1H})}{2}.$$

Given these demands, firm 1 chooses the price p_{1H} that solves

$$\max_{p_{1H}} \widetilde{\pi}_1 = p_{1H} \underbrace{\left[\frac{1 + \alpha_h(p_H^* - p_{1H})}{2}\right]}_{x_h} + p_{1H} \underbrace{\left[\frac{1 + (s_H - s_L) + \alpha_l(p_L^* - p_{1H})}{2}\right]}_{\widetilde{x}_l}$$

since both consumer types pay p_{1H} for the H good. We can simplify this profit since $\alpha_h = 1$ and $\alpha_l = 5$, $s_H = 3$ and $s_L = 2$, and prices $p_H^* = 1$ and $p_L^* = \frac{1}{5}$. Rearranging, yields

$$\max_{p_{1H}} \widetilde{\pi}_1 = p_{1H} \frac{5 - 6p_{1H}}{2}.$$

Differentiating with respect to p_{1H}, we obtain $\frac{5}{2} - 6p_{1H} = 0$, so $p_{1H} = \frac{5}{12}$. Inserting this price back in the profit function, yields

$$\widetilde{\pi}_1 = \frac{5}{12} \frac{5 - 6\frac{5}{12}}{2} \simeq 0.521$$

which is less than the equilibrium profits that firm 1 earns in the discriminatory equilibrium, $\pi_1^* = \frac{3}{5} = 0.6$. Therefore, no firm has unilateral incentives to deviate from discriminatory equilibrium prices.

(d) *Non-discriminatory equilibrium.* Consider now a price profile where every firm i only offers the high-quality good, and all types of consumers buy it. In this and following parts of the exercise, we analyze the conditions that support this strategy profile in equilibrium.

Find the demand for each firm, and from each consumer type, in this strategy profile.

- The high-type consumer buys the high-quality good (H, for compactness), so he is indifferent between buying H from firm 1 and 2 (as both firms offer H) if

$$s_H - \alpha_h p_{1H} - x_h = s_H - \alpha_h p_{2H} - (1 - x_h)$$

which, solving for x_h, yields

$$x_h = \frac{1 + \alpha_h(p_{2H} - p_{1H})}{2}$$

which coincides with the expression in part (a). Similarly, low-type consumers buy the H good, being indifferent between firm 1 and 2 if

$$s_H - \alpha_l p_{1H} - \widehat{x}_l = s_H - \alpha_l p_{2H} - (1 - \widehat{x}_l)$$

which, solving for x_l, yields

$$\widehat{x}_l = \frac{1 + \alpha_l(p_{2H} - p_{1H})}{2}$$

which only differs from x_h in parameter α_l.

- Therefore, firm 1's demand from high-type consumers is x_h and that from low-type consumers is $\widehat{x}_l$. As a result, firm 2's demand form the high-type consumers is $1 - x_h$ and that from low-type consumers is $1 - \widehat{x}_l$.

(e) Given the demands found in part (d), write down each firm's profit-maximization problem, and find their equilibrium prices, (p_{1H}, p_{2L}). Evaluate equilibrium demands and profits in this context at parameter values $\alpha_h = 1$ and $\alpha_l = 5$, $s_H = 3$ and $s_L = 2$.

- Firm 1 solves

$$\begin{aligned}
\max_{p_{1H}} \pi_1 &= p_{1H} \underbrace{\left[\frac{1 + \alpha_h(p_{2H} - p_{1H})}{2}\right]}_{x_h} + p_{1H} \underbrace{\left[\frac{1 + \alpha_l(p_{2H} - p_{1H})}{2}\right]}_{\widehat{x}_l} \\
&= p_{1H} \left[\frac{1 + \alpha_h(p_{2H} - p_{1H})}{2} + \frac{1 + \alpha_l(p_{2H} - p_{1H})}{2}\right] \\
&= p_{1H} \left[1 + \frac{\alpha_h + \alpha_l}{2}(p_{2H} - p_{1H})\right] \\
&= p_{1H} \left[1 + \widehat{\alpha}(p_{2H} - p_{1H})\right]
\end{aligned}$$

where, for compactness, $\widehat{\alpha} \equiv \frac{\alpha_h + \alpha_l}{2}$ denotes the average α. Note that firm 1 sells H to both consumer types at the same price, p_{1H}. Differentiating with respect to p_{1H} yields

$$\frac{\partial \pi_1}{\partial p_{1H}} = 1 + \widehat{\alpha}(p_{2H} - 2p_{1H}) = 0$$

which, solving for p_{1H}, gives us the best response function

$$p_{1H}(p_{2H}) = \frac{1}{2\widehat{\alpha}} + \frac{1}{2}p_{2H}.$$

- Similarly, firm 2 solves

$$\max_{p_{2H}} \pi_2 = p_{2H} \underbrace{\left[1 - \frac{1 + \alpha_h(p_{2H} - p_{1H})}{2}\right]}_{1 - x_h} + p_{2H} \underbrace{\left[1 - \frac{1 + \alpha_l(p_{2H} - p_{1H})}{2}\right]}_{1 - \widehat{x}_l}$$

Differentiating with respect to p_{2H} yields

$$\frac{\partial \pi_2}{\partial p_{2H}} = 1 + \widehat{\alpha}(p_{1H} - 2p_{2H}) = 0$$

which, solving for p_{2H}, gives us the best response function

$$p_{2H}(p_{1H}) = \frac{1}{2\widehat{\alpha}} + \frac{1}{2}p_{1H}$$

which is symmetric to the best response function of firm 1.

- Invokyng symmetry, $p_{1H} = p_{2H} = p_H$, we obtain

$$p_H = \frac{1}{2\widehat{\alpha}} + \frac{1}{2}p_H$$

which yields $p_H^* = \frac{1}{\widehat{\alpha}}$. In our above setting, where $\alpha_h = 1$ and $\alpha_l = 5$, this yields

$$p_H^* = \frac{1}{\widehat{\alpha}} = \frac{1}{\frac{\alpha_h+\alpha_l}{2}} = \frac{1}{\frac{1+5}{2}} = \frac{1}{3},$$

demands are $x_h = \widehat{x}_l = \frac{1}{2}$, entailing that equilibrium profits are

$$\pi_1^* = p_{1H}x_h + p_{1H}\widehat{x}_l = \frac{1}{3}\frac{1}{2} + \frac{1}{3}\frac{1}{2} = \frac{1}{3}.$$

(f) Argue that the non-discriminatory prices are incentive compatible for both consumer types; and that no firm has unilateral incentives to offer both the high- and low-quality goods.

- *Incentive compatible prices.* Since only a high-quality good is offered by both firms, every consumer cannot choose to purchase the low-quality good. In addition, all consumer types enjoy a positive utility level, thus implying that buying the good is preferred to not buying it. In particular, the high-type consumer's utility is

$$s_H - \alpha_h p_H^* = 3 - \left(1 \times \frac{1}{3}\right) = \frac{8}{3}$$

Similarly, the low-type consumer's utility is also positive since

$$s_H - \alpha_l p_H^* = 3 - \left(5 \times \frac{1}{3}\right) = \frac{4}{3}.$$

- *Price deviations by the firm.* Assume that firm 2 sticks to only offering the high-quality good, H, at equilibrium price $p_H^* = \frac{1}{3}$, but firm 1 unilaterally deviates, selling the H good only to the high-type consumers at a price p_{1H}. As we next show, this price is higher than the equilibrium price $p_H^* = \frac{1}{3}$, but firm 1 sells to fewer customers, ultimately earning a lower profit than in equilibrium. First, we need to characterize under which conditions for price p_{1H} the low-type consumers won't buy from firm 1. This occurs when a low-type consumer, despite being located at the same position as firm 1 (zero transportation cost), he does not buy from firm 1, that is,

$$s_H - \alpha_l p_H^* - 1 > s_H - \alpha_l p_{1H} - 0$$

which, solving for p_{1H}, yields

$$p_{1H} > p_H^* + \frac{1}{\alpha_l} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15} \simeq 0.53.$$

Therefore, when finding the profit-maximizing price for firm 1, p_{1H}, we need to guarantee that $p_{1H} > 0.53$. In this context, firm 1 solves

$$\max_{p_{1H}} \pi_{1H} = p_{1H} \underbrace{\left[\frac{1 + \alpha_h(p_H^* - p_{1H})}{2}\right]}_{x_h}$$

since firm 1 only sells the good to high-type consumers. We can simplify this profit since $\alpha_h = 1$ and $\alpha_l = 5$, and the equilibrium price from firm 2, $p_H^* = \frac{1}{3}$, to obtain

$$\max_{p_{1H}} \pi_{1H} = p_{1H} \frac{4 - 3p_{1H}}{6}.$$

Differentiating with respect to p_{1H}, we obtain $\frac{2}{3} - p_{1H} = 0$, so $p_{1H} = \frac{2}{3}$. (This price satisfies the initial condition, $p_{1H} > 0.53$, so firm 1's deviation only attracts high-type consumers.) Inserting this price back in the profit function, yields

$$\pi_{1H} = \frac{2}{3} \frac{4 - 3\frac{2}{3}}{6} = \frac{2}{9} \simeq 0.22$$

which is less than the equilibrium profits that firm 1 earns in the non-discriminatory equilibrium, $\pi_1^* = \frac{1}{3} = 0.33$. Therefore, no firm has unilateral incentives to deviate from non-discriminatory equilibrium prices.