

EconS 594 - Theory of Industrial Organization  
Homework #3 - Due date: Thursday, October 2nd.

1. Exercise 3 from Chapter 8 in Belleflamme and Peitz's book (page 219).
2. **Price discrimination in duopoly.** Consider a duopoly market with two firms and a continuum of consumers. Each firm  $i \in \{1, 2\}$  sells its product at price  $p_i$  and incurs marginal costs equal to zero. Consumers are of measure 1 and have unit demand. When buying one unit of product  $i$  a consumer of type  $(t; x)$  obtains utility  $r - t|x - l_i| - p_i$  where  $l_i$  is the location of firm  $i$  and  $p_i$  is its price; if she does not buy her utility is set equal to  $-\infty$ . (This assumption is equivalent to setting the utility of not buying the product at zero, but assuming that  $r$  is high enough.) Half of consumers belong to the group with type  $t_A$  and half of consumers to the other group with type  $t_B$ ; where  $t_A \geq t_B$ . Within each group, consumers are uniformly distributed on the unit interval,  $x \in [0, 1]$ . Firms are located at the endpoints of the line, 0 and 1, respectively.
  - (a) Suppose that  $t_A = t_B$ . Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Report equilibrium prices, outputs, and profits.
  - (b) Suppose that  $t_A > t_B$ . Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Report equilibrium prices, outputs, and profits.
  - (c) Suppose that  $t_A > t_B$ . Suppose furthermore that both firms observe consumer type  $t$  and that they can condition their price on this type; i.e., firm  $i$  set  $p_i(t)$ . (Consumers are assumed not to be able to trade among each other). Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game. Compare your result to the setting in part (b). Discuss whether firms benefit from regulation that requires them to set uniform prices.
  - (d) Suppose that  $t_A > t_B$ . Suppose now that only firm 1 observes consumer type  $t$  and that it can condition its price on this type –i.e., firm 1 set  $p_1(t)$ – whereas firm 2 has to charge the same price to all consumers. (Consumers are assumed not to be able to trade among each other). Determine the demand function faced by the two firms. Determine the equilibrium in the simultaneous-move price game.
  - (e) Suppose that  $t_A = 2 > 1 = t_B$ . Consider the two-stage game in which, in the first stage, firms acquire the ability to identify a consumer's type  $t$  at cost  $C$  and, in the second stage, they compete in prices. Using your insights from parts (b) to (d), characterize the subgame-perfect equilibria as a function of cost  $C$ .
  - (f) Discuss your findings.
3. Exercise 3 from Chapter 9 in Belleflamme and Peitz's book (page 241).