

EconS 594 - Theory of Industrial Organization
Homework #2 - Due date: Thursday, September 17th.

1. **Vertical product differentiation and cost of quality.** A consumer with income m who consumes a product of quality s_i and pays p_i obtains the utility $\frac{s_i m}{6} - p_i$. If instead the consumer decides not buy the good, the resulting utility is zero. Consumer income m is uniformly distributed on the interval $[2, 8]$, so its density is $f(m) = \frac{1}{8-2} = \frac{1}{6}$. The total mass of consumers is equal to 1.

There are two firms in the market, 1 and 2, offering qualities s_1 and s_2 , respectively. Assume that $s_1, s_2 \in [1, 2]$. Label firms such that $s_1 \leq s_2$. Suppose that firm i has constant marginal cost equal to $c \times s_i$ where $c < 2$.

- (a) Derive the demand of firms 1 and 2, and calculate the best-response functions of the two firms presuming that first-order conditions hold with equality. Distinguish between full and partial market coverage.
 - (b) Calculate the Nash Equilibrium in prices and find the equilibrium profits as a function of s_1 and s_2 . Distinguish between full and partial market coverage.
 - (c) What are the equilibrium quality choices of the two firms? (Again distinguish between the full and market coverage cases using first-order conditions.)
 - (d) Which firm is more profitable? Consider the two cases mentioned above.
 - (e) How does in the partial coverage equilibrium an increase in cost c affect the profits of the two firms?
2. **Exercise 6.1**, from Belleflamme and Peitz's book (see page 159).
3. **Surplus-increasing advertising in the Hotelling model.** Consider a horizontally differentiated product market in which firms are located at the extreme points of the unit interval. Firms produce at marginal costs equal to zero. A continuum of consumers of mass 1 are uniformly distributed on the unit interval. They have unit demand and have an outside utility of $-\infty$. A consumer located at $x \in [0, 1]$ obtains indirect utility $v_1 = r_1 - tx - p_1$ if she buys one unit from firm 1 and $v_2 = r_2 - t(1 - x) - p_2$ if she buys from firm 2. Firms have marginal costs equal to zero.

- (a) Suppose that firms have set prices at p_1 and p_2 respectively. Determine the demand function for each firm for each admissible price pair.
- (b) Suppose that the social planner chooses first-best optimal prices. Which price pairs would be socially optimal?
- (c) Suppose that the two firms simultaneously set prices. Determine the market equilibrium for all possible combinations of (r_1, r_2) .
- (d) From now on consider the special case that $t = 1$. Suppose that each firm i can use advertising to increase the willingness to pay from $r_i = 1$ to $r_i = 2$. Consider the two-stage game in which firms choose advertising at the first stage and price at the second stage. Characterize the subgame-perfect equilibrium of the game depending on the advertising cost A . Evaluate your results when $A = \frac{2}{9}$, $A = \frac{3}{9}$, and $A = \frac{4}{9}$. What is the welfare ranking?

- (e) What are the equilibria for $A = \frac{5}{18}$ and $A = \frac{7}{18}$?
- (f) What are the welfare consequences of a reduction in the advertising cost from $A = \frac{5}{18} + \varepsilon$ to $A = \frac{5}{18} - \varepsilon$ where $\varepsilon \rightarrow 0$ (determine whether total surplus increases or decreases and by how much)? Comment on your result in one sentence.
- (g) What are the welfare consequences of a reduction in advertising the advertising cost from $A = \frac{7}{18} + \varepsilon$ to $A = \frac{7}{18} - \varepsilon$ where $\varepsilon \rightarrow 0$ (determine whether total surplus increases or decreases and by how much)? Comment on your result in one sentence.