EconS 594 - Theory of Industrial Organization
Homework #1 - Due date: Thursday, September 10th.

1. **Cournot with asymmetric fixed costs.** Consider a Cournot duopoly, allowing firms to face fixed costs. In particular, assume that firm 1 faces a total cost function $TC_1(q_1) = F_1 + cq_1$, where $F_1 > 0$ denotes its fixed cost and $1 > c > 0$ represents its marginal cost. Firm 2's total cost function is similar, $TC_2(q_2) = F_2 + cq_2$, where $F_2 > 0$ denotes its fixed cost, and satisfies $F_2 > F_1$, and $c > 0$ is the same marginal cost as firm 1's. Firms face a linear inverse demand function $p(Q) = 1 - Q$, where Q denotes aggregate output.

   (a) Find the best response functions of each firm and the equilibrium output.

   (b) How are the equilibrium results affected? Interpret.

   (c) *Numerical example.* Evaluate your equilibrium results in part (b) at $c = 1/2$. What happens if c decreases to $c = 1/10$? Interpret.

2. **Can fewer firms decrease prices?** Consider an industry with $n \geq 2$ firms competing à la Cournot, facing an inverse demand function $p(Q) = 1 - Q$, where $Q \geq 0$ denotes aggregate output. Firms in this industry are asymmetric in their marginal costs. Specifically, a share $\alpha \in [0,1]$ of them have marginal cost c_H, which we regard as "inefficient," and the remaining share, $1 - \alpha$, are firms with marginal cost c_L, which we refer as "efficient," where $a > c_H > c_L$.

   (a) Find each type of firm's best response function. Interpret.

   (b) Find equilibrium output for every high-cost firm and every low-cost firm.

   (c) Find under which parameter conditions do high-cost firms produce a positive output level. Examine how this parameter condition is affected by the number of firms in the industry, n, and by the proportion of high-cost firms, α.

   (d) Find equilibrium output and prices if all high-cost firms exit the industry. Are consumers better off when all high-cost firms remain active or when they exit?

3. **Price competition with heterogeneous goods and asymmetric costs.** Consider two firms competing à la Bertrand selling heterogeneous goods. The demand function of firm i, where $i, j \in \{1, 2\}$, is

$$q_i(p_i, p_j) = 1 - \gamma p_i + p_j$$

   where $\gamma \geq 1$ represents the degree of product differentiation (homogeneous when $\gamma = 1$ but differentiated when $\gamma > 1$). Without loss of generality, assume that firm 1 has a lower marginal cost than firm 2 in producing every unit of the good, that is, $0 < c_1 < c_2 < 1$.

   (a) Characterize the firms' best response functions and graphically illustrate your results.

(b) What are the equilibrium price, output, and profit of each firm? Find the *sufficient* condition on γ in which both firms produce output, and the output level if every firm sets its price at the marginal cost.

(c) *Numerical example.* Evaluate equilibrium outcomes under $c_1 = 1/4$ and $c_2 = 1/2$ as a function of γ. Under which conditions of γ will both firms produce a positive output?

4. **Horizontal differentiation in two dimensions, based on Irmen and Thisse (1998).**[1] Consider the model of horizontally differentiated products discussed in class. However, assume now that, in the first stage, every firm i chooses its location, l_i, in the interval $[0, 1]$, where $i = \{1, 2\}$; and, similarly, its location h_i in the interval $[0, 1]$. Intuitively, this indicates that firms differentiate along two dimensions (e.g., sweetness and color), which implies that consumer preferences in this setting are uniformly distributed in a unit square (i.e., a square of side one). For simplicity, assume that consumer's per-unit disutility from purchasing a good that does not coincide with his ideal coincides across both dimensions.

(a) *Third stage - Finding demand.* For given locations from the first stage (l_1, l_2, h_1, h_2), and given prices from the second stage (p_1, p_2), find the demand that each firm has in the third stage.

(b) *Second stage - Prices.* Given locations from the first stage, find the price that each firm sets in the second stage.

(c) *First stage - Equilibrium location.* Anticipating equilibrium behavior in the second and third stages, find the equilibrium location choice of each firm in the first stage of the game.

(d) *Comparison.* Compare your equilibrium location and profit with those in the standard Hoteling model where firms differentiate their products in just one dimension.

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[1]Irmen, A. and J-F. Thisse (1998) "Competition in Multi-characteristics Spaces: Hotelling Was Almost Right," Journal of Economic Theory, 78, pp. 76–102.