Advanced Microeconomic Theory

Chapter 7: Monopoly
Outline

• Barriers to Entry
• Profit Maximization under Monopoly
• Welfare Loss of Monopoly
• Multiplant Monopolist
• Price Discrimination
• Advertising in Monopoly
• Regulation of Natural Monopolies
• Monopsony
Barriers to Entry
Barriers to Entry

• **Entry barriers**: elements that make the entry of potential competitors either impossible or very costly.

• Three main categories:

  1) **Legal**: the incumbent firm in an industry has the legal right to charge monopoly prices during the life of the patent

  – *Example*: newly discovered drugs
Barriers to Entry

2) **Structural**: the incumbent firm has a cost or demand advantage relative to potential entrants.
   - superior technology
   - a loyal group of customers
     - positive network externalities (Facebook, eBay)

3) **Strategic**: the incumbent monopolist has a reputation of fighting off newcomers, ultimately driving them off the market.
   - price wars
Profit Maximization under Monopoly
Profit Maximization

• Consider a demand function $x(p)$, which is continuous and strictly decreasing in $p$, i.e., $x'(p) < 0$.

• We assume that there is price $\bar{p} < \infty$ such that $x(p) = 0$ for all $p > \bar{p}$.

• Also, consider a general cost function $c(q)$, which is increasing and convex in $q$. 
Profit Maximization

- $\bar{p}$ is a “choke price”
- No consumers buy positive amounts of the good for $p > \bar{p}$.

Mathematically:

$x(p) = 0$ for all $p > \bar{p}$

$\frac{d}{dp}x(p) < 0$
Profit Maximization

• Monopolist’s decision problem is

$$\max_p px(p) - c(x(p))$$

• Alternatively, using \(x(p) = q\), and taking the inverse demand function \(p(q) = x^{-1}(q)\), we can rewrite the monopolist’s problem as

$$\max_{q \geq 0} p(q)q - c(q)$$
Profit Maximization

• Differentiating with respect to $q$,

$$p(q^m) + p'(q^m)q^m - c'(q^m) \leq 0$$

• Rearranging,

$$\underbrace{p(q^m) + p'(q^m)q^m}_{MR} \leq \underbrace{c'(q^m)}_{MC}$$

with equality if $q^m > 0$.

• Recall that total revenue is $TR(q) = p(q)q$
Profit Maximization

• In addition, we assume that $p(0) \geq c'(0)$.
  – That is, the inverse demand curve originates above the marginal cost curve.
  – Hence, the consumer with the highest willingness to pay for the good is willing to pay more than the variable costs of producing the first unit.

• Then, we must be at an interior solution $q_m^m > 0$, implying

$$p(q_m^m) + p'(q_m^m)q_m^m = c'(q_m^m)$$

\[\text{MR} \quad \text{MC}\]
Profit Maximization

• Note that

\[ p(q^m) + p'(q^m)q^m = c'(q^m) \]

• Then, \( p(q^m) > c'(q^m) \), i.e., monopoly price > \( MC \)

• Moreover, we know that in competitive equilibrium \( p(q^*) = c'(q^*) \).

• Then, \( p^m > p^* \) and \( q^m < q^* \).
Profit Maximization
Profit Maximization

• Marginal revenue in monopoly

\[ MR = p(q^m) + p'(q^m)q^m \]

MR describes two effects:

– A **direct (positive) effect**: an additional unit can be sold at \( p(q^m) \), thus increasing revenue by \( p(q^m) \).

– An **indirect (negative) effect**: selling an additional unit can only be done by reducing the market price of all units (the new and all previous units), ultimately reducing revenue by \( p'(q^m)q^m \).

- **Infframarginal units** – initial units before the marginal increase in output.
Profit Maximization

• Is the above FOC also sufficient?
  – Let’s take the FOC \( p(q^m) + p'(q^m)q^m - c'(q^m) \), and differentiate it wrt \( q \),
    \[
    p'(q) + p'(q) + p''(q)q - c''(q) \leq 0
    \]
    \[
    \frac{dMR}{dq} - \frac{dMC}{dq} \leq 0
    \]
  – That is, \( \frac{dMR}{dq} \leq \frac{dMC}{dq} \).
  – Since MR curve is decreasing and MC curve is weakly increasing, the second-order condition is satisfied for all \( q \).
Profit Maximization

![Diagram of profit maximization with functions MC(q), MR(q), x(p), and points p^m and q^m]
Profit Maximization

• What would happen if MC curve was decreasing in $q$ (e.g., concave technology given the presence of increasing returns to scale)?
  – Then, the slopes of MR and MC curves are both decreasing.
  – At the optimum, MR curve must be steeper MC curve.
Profit Maximization: Lerner Index

• Can we re-write the FOC in a more intuitive way? Yes.

  – Just take \( MR = p(q) + p'(q)q = p + \frac{\partial p}{\partial q} q \) and multiply by \( \frac{p}{p} \),

  \[
  MR = p \frac{p}{p} + \frac{\partial p}{\partial q} p = p + \frac{1}{\varepsilon_d} p
  \]

  – In equilibrium, \( MR(q) = MC(q) \). Hence, we can replace MR with MC in the above expression.
Profit Maximization: Lerner Index

• Rearranging yields
\[
\frac{p - MC(q)}{p} = - \frac{1}{\varepsilon_d}
\]

• This is the \textit{Lerner index} of market power
  – The price mark-up over marginal cost that a monopolist can charge is a function of the elasticity of demand.

• Note:
  – If \( \varepsilon_d \to \infty \), then \( \frac{p - MC(q)}{p} \to 0 \implies p = MC(q) \)
  – If \( \varepsilon_d \to 0 \), then \( \frac{p - MC(q)}{p} \to \infty \implies \) substantial mark-up
Profit Maximization: Lerner Index

• The Lerner index can also be written as

\[ p = \frac{MC(q)}{1 + \frac{1}{\varepsilon_d}} \]

which is referred to as the **Inverse Elasticity Pricing Rule** (IEPR).

• **Example** (Perloff, 2012):
  
  – Prilosec OTC: \( \varepsilon_d = -1.2 \). Then price should be \( p = \frac{MC(q)}{1 + \frac{1}{-1.2}} = 6MC \)
  
  – Designed jeans: \( \varepsilon_d = -2 \). Then price should be \( p = \frac{MC(q)}{1 + \frac{1}{-2}} = 2MC \)
Profit Maximization: Lerner Index

• **Example 1** (linear demand):
  
  – Market inverse demand function is
    \[ p(q) = a - bq \]
    where \( b > 0 \)
  
  – Monopolist’s cost function is \( c(q) = cq \)
  
  – We usually assume that \( a > c \geq 0 \)
    
    ▪ To guarantee \( p(0) > c'(0) \)
    
    ▪ That is, \( p(0) = a - b0 = a \) and \( c'(q) = c \), thus implying \( c'(0) = c \)
Profit Maximization: Lerner Index

• **Example 1** (continued):
  – Monopolist’s objective function
    \[
    \pi(q) = (a - bq)q - cq
    \]
  – FOC: \[a - 2bq - c = 0\]
  – SOC: \[-2b < 0 \text{ (concave)}\]
    ▪ Note that as long as \(b > 0\), i.e., negatively sloped demand function, profits will be concave in output.
    ▪ Otherwise (i.e., Giffen good, with positively sloped demand function) profits will be convex in output.
Profit Maximization: Lerner Index

• **Example 1** (continued):
  
  – Solving for the optimal $q_m$ in the FOC, we find monopoly output

  $$q_m = \frac{a - c}{2b}$$

  – Inserting $q_m = \frac{a-c}{2b}$ in the demand function, we obtain monopoly price

  $$p_m = a - b \left( \frac{a - c}{2b} \right) = \frac{a + c}{2}$$

  – Hence, monopoly profits are

  $$\pi_m = p_m q_m - c q_m = \frac{(a - c)^2}{4b}$$
Profit Maximization: Lerner Index

• Example 1 (continued):
Example 1 (continued):

- Non-constant marginal cost
- The cost function is convex in output $c(q) = cq^2$
- Marginal cost is $c'(q) = 2cq$
Profit Maximization: Lerner Index

**Example 2** (Constant elasticity demand):

– The demand function is

\[ q(p) = Ap^{-b} \]

– We can show that \( \varepsilon(q) = -b \) for all \( q \), i.e.,

\[
\varepsilon(q) = \frac{\partial q(p)}{\partial p} \frac{p}{q} = \left(\frac{(-b)Ap^{-b-1}}{\partial q(p)}\right) \frac{p}{Ap^{-b}} \frac{p}{q} = -b \]

\[
= -b \frac{p^{-b}}{p} \frac{p}{p^{-b}} = -b
\]
Profit Maximization: Lerner Index

• **Example 2** (continued):
  
  – We can now plug \( \varepsilon(q) = -b \) into the Lerner index,

  \[
  p^m = \frac{c}{1 + \frac{1}{\varepsilon(q)}} = \frac{c}{1 - \frac{1}{b}} = \frac{bc}{b - 1}
  \]

  – That is, price is a constant mark-up over marginal cost.
Welfare Loss of Monopoly
Welfare Loss of Monopoly

- Welfare comparison for perfect competition and monopoly.

\[
\int_{q^m}^{q^*} [p(s) - c'(s)] ds
\]

\[\text{MR} = p(q) + p'(q)q\]

Advanced Microeconomic Theory
Welfare Loss of Monopoly

• Consumer surplus
  – Perfect competition: $A+B+C$
  – Monopoly: $A$

• Producer surplus:
  – Perfect competition: $D+E$
  – Monopoly: $B+D$

• Deadweight loss of monopoly: $C+E$

\[
DWL = \int_{q_m}^{q^*} [p(s) - c'(s)]ds
\]

• DWL decreases as demand and/or supply become more elastic.
Welfare Loss of Monopoly

- Infinitely elastic demand
  \[ p'(q) = 0 \]
- The inverse demand curve becomes totally flat.
- Marginal revenue coincides with inverse demand:
  \[ MR(q) = p(q) + 0 \cdot q = p(q) \]
- Profit-maximizing \( q \)
  \[ MR(q) = MC(q) \implies p(q) = MC(q) \]
- Hence, \( q^m = q^* \) and \( DWL = 0 \).
Welfare Loss of Monopoly

- **Example** (Welfare losses and elasticity):
  - Consider a monopolist with constant marginal and average costs, \( c'(q) = c \), who faces a market demand with constant elasticity
    \[
    q(p) = p^e
    \]
    where \( e \) is the price elasticity of demand (\( e < -1 \))
  - Perfect competition: \( p_c = c \)
  - Monopoly: using the IEPR
    \[
    p^m = \frac{c}{1 + \frac{1}{e}}
    \]
Welfare Loss of Monopoly

• **Example** (continued):
  
  – The consumer surplus associated with any price \((p_0)\) can be computed as

  \[
  CS = \int_{p_0}^{\infty} Q(P) \, dp = \int_{p_0}^{\infty} p^e \, dp = \left. \frac{p^{e+1}}{e+1} \right|_{p_0}^{\infty} = -\frac{p_0^{e+1}}{e+1} > 0
  \]

  – Under perfect competition, \(p_c = c\),

  \[
  CS = -\frac{c^{e+1}}{e+1}
  \]

  – Under monopoly, \(p^m = \frac{c}{1+1/e'}\),

  \[
  CS_m = -\left(\frac{c}{1 + 1/e}\right)^{e+1} \frac{e+1}{e+1}
  \]
Welfare Loss of Monopoly

• Example (continued):
  – Taking the ratio of these two surpluses
    \[ \frac{CS_m}{CS} = \left( \frac{1}{1 + 1/e} \right)^{e+1} \]
  – If \( e = -2 \), this ratio is \( \frac{1}{2} \)
    ▪ CS under monopoly is half of that under perfectly competitive markets
Welfare Loss of Monopoly

• **Example** (continued):
  
  – The ratio \( \frac{CS_m}{CS} = \left( \frac{1}{1+1/e} \right)^{e+1} \) decreases as demand becomes more elastic (\( e \) increases in absolute value).
Welfare Loss of Monopoly

• Example (continued):
  – Monopoly profits are given by
    \[ \pi^m = p^m q^m - cq^m = \left( \frac{c}{1 + 1/e} - c \right) q^m \]
    where \( q^m(p) = p^e = \left( \frac{c}{1+1/e} \right)^e \).
  – Rearranging,
    \[ \pi^m = \left( \frac{-c/e}{1 + 1/e} \right) \left( \frac{c}{1 + 1/e} \right)^e \]
    \[ = - \left( \frac{c}{1 + 1/e} \right)^{e+1} \cdot \frac{1}{e} \]
Welfare Loss of Monopoly

• Example (continued):
  – To find the transfer from CS into monopoly profits that consumers experience when moving from perfect competition to a monopoly, divide monopoly profits by the competitive CS

\[
\frac{\pi^m}{CS} = \left(\frac{e+1}{e}\right) \left(\frac{1}{1+1/e}\right)^{e+1} = \left(\frac{e}{1+e}\right)^e
\]

– If \( e = -2 \), this ratio is \( \frac{1}{4} \)

  ▪ One-fourth of the consumer surplus under perfectly competitive markets is transferred to monopoly profits
Welfare Loss of Monopoly

• More social costs of monopoly:
  – Excessive R&D expenditure (patent race)
  – Persuasive (not informative) advertising
  – Lobbying costs (different from bribes)
  – Resources to avoid entry of potential firms in the industry
Comparative Statics
Comparative Statics

• We want to understand how $q^m$ varies as a function of the monopolist’s marginal cost
Comparative Statics

- Formally, we know that at the optimum, $q^m(c)$, the monopolist maximizes its profits:

$$\frac{\partial \pi(q^m(c), c)}{\partial q^m} = 0$$

- Differentiating wrt $c$, and using the chain rule,

$$\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2} \frac{dq^m(c)}{dc} + \frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = 0$$

- Solving for $\frac{dq^m(c)}{dc}$, we have

$$\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}}$$
Comparative Statics

**Example:**
- Assume linear demand curve \( p(q) = a - bq \)
- Then, the cross-derivative is

\[
\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = \frac{\partial}{\partial c} \left( \frac{\partial [(a - bq)q - cq]}{\partial q} \right) = \frac{\partial [a - 2bq - c]}{\partial c} = -1
\]

and

\[
\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}} = -\frac{-1}{-2b} < 0
\]
Comparative Statics

• **Example** (continued):
  – That is, an increase in marginal cost, $c$, decreases monopoly output, $q^m$.
  – Similarly for any other demand.
  – Even if we don’t know the accurate demand function, but know the sign of

\[
\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}
\]
Comparative Statics

• **Example** (continued):
  
  – Marginal costs are increasing in $q$
  
  – For convex cost curve $c(q) = cq^2$, monopoly output is

  \[
  q^m(c) = \frac{a}{2(b + c)}
  \]

  – Here,

  \[
  \frac{dq^m(c)}{dc} = -\frac{a}{2(b + c)^2} < 0
  \]
Comparative Statics

• Example (continued):

– Constant marginal cost
– For the constant-elasticity demand curve \( q(p) = p^e \), we have
  \[ p^m = \frac{c}{1+1/e} \]
  \[ q^m(c) = \left( \frac{ec}{1+e} \right)^e \]
– Here,
  \[ \frac{dq^m(c)}{dc} = \frac{e}{c} \left( \frac{ec}{1+e} \right)^e \]
  \[ = \frac{e}{c} q^m < 0 \]
Multiplant Monopolist
Multiplant Monopolist

• Monopolist produces output $q_1, q_2, \ldots, q_N$ across $N$ plants it operates, with total costs $TC_i(q_i)$ at each plant $i = \{1,2, \ldots, N\}$.

• Profits-maximization problem

$$\max_{q_1, \ldots, q_N} [a - b \sum_{i=1}^{N} q_i] \sum_{i=1}^{N} q_i - \sum_{i=1}^{N} TC_i(q_i)$$

• FOCs wrt production level at every plant $j$

$$a - 2b \sum_{i=1}^{N} q_i - MC_j(q_j) = 0$$

$$\iff MR(Q) = MC_j(q_j)$$

for all $j$. 

Multiplant Monopolist

- Multiplant monopolist operating two plants with marginal costs $MC_1$ and $MC_2$. 

![Diagram showing marginal cost curves and unique price]$MC_1$ $MC_2$ $MC_{Total}$ $p(q)$ $q$ $q_1$ $q_2$ $Q_{Total}$ Unique Price $p^m$
Multiplant Monopolist

• Total marginal cost is $MC_{total} = MC_1 + MC_2$ (i.e., horizontal sum)

• $Q_{total}$ is determined by $MR = MC_{total}$ (i.e., point A)

• Mapping $Q_{total}$ in the demand curve, we obtain price $p^m$ (both plants selling at the same price)

• At the MC level for which $MR = MC_{total}$ (i.e., point A), extend a line to the left crossing $MC_1$ and $MC_2$.

• This will give us output levels $q_1$ and $q_2$ that plants 1 and 2 produce, respectively.
Multiplant Monopolist

**Example 1** (symmetric plants):

- Consider a monopolist operating $N$ plants, where all plants have the same cost function $TC_i(q_i) = F + cq_i^2$. Hence, all plants produce the same output level $q_1 = q_2 = \cdots = q_N = q$ and $Q = Nq$. The linear demand function is given by $p = a - bQ$.

- FOCs:

\[
a - 2b \sum_{j=1}^{N} q_j = 2cq_j \quad \text{or} \quad a - 2bNq_j = 2cq_j
\]

\[
q_j = \frac{a}{2(bN + c)}
\]
Multiplant Monopolist

- **Example 1** (continued):
  - Total output produced by the monopolist is
    \[ Q = Nq_j = \frac{Na}{2(bN + c)} \]
    and market price is
    \[ p = a - bQ = a - b \frac{Na}{2(bN + c)} = \frac{a(bN + 2c)}{2(bN + c)} \]
  - Hence, the profits of every plant \( j \) are
    \[ \pi_j = \left( \frac{a(bN + 2c)}{2(bN + c)} \right) \frac{a}{2(bN + c)} - c \left( \frac{a}{2(bN + c)} \right)^2 = \frac{a^2}{4(bN + c)} - F \]
  - Total profits become
    \[ \pi_{total} = \frac{Na^2}{4(bN + c)} - NF \]
Multiplant Monopolist

• **Example 1** (continued):
  
  – The optimal number of plants $N^*$ is determined by
    
    $$\frac{d\pi_{total}}{dN} = \frac{a^2}{4} \frac{c}{(bN + c)^2} - F = 0$$
    
    and solving for $N$
    
    $$N^* = \frac{1}{b} \left( \frac{a}{2} \sqrt{\frac{c}{F}} - c \right)$$
    
    – $N^*$ is decreasing in the fixed costs $F$
    
    – $N^*$ is decreasing in $c$ as long as $a < 4\sqrt{cF}$, since
    
    $$\frac{dN^*}{dc} = \frac{1}{b} \left( \frac{a - 4\sqrt{cF}}{4\sqrt{cF}} \right)$$
Multiplant Monopolist

• **Example 1** (continued):
  
  – Note that when $N = 1$, $Q = q^m$ and $p = p^m$.
  
  – Note that an increase in $N$ decreases $q_j$ and $\pi_j$, as
    
    \[
    \frac{dq_j}{dN} = -\frac{ab}{2(bN + c)^2} < 0
    \]
    
    \[
    \frac{d\pi_j}{dN} = -\frac{a^2 b}{4(bN + c)^2} < 0
    \]
Example 2 (asymmetric plants):

Consider a monopolist operating two plants with marginal costs $MC_1(q_1) = 10 + 20q_1$ and $MC_2(q_2) = 60 + 5q_2$, respectively. A linear demand function is given by $p(Q) = 120 - 3Q$.

Note that $MC_{total} \neq MC_1(q_1) + MC_2(q_2)$

This is a vertical (not a horizontal) sum.

Instead, first invert the marginal cost functions

$MC_1(q_1) = 10 + 20q_1 \iff q_1 = \frac{MC_1}{20} - \frac{1}{2}$

$MC_2(q_2) = 60 + 5q_2 \iff q_2 = \frac{MC_2}{5} - 12$
Multiplant Monopolist

- **Example 2** (continued):
  - Second,
    \[
    Q_{\text{total}} = q_1 + q_2 = \frac{MC_{\text{total}}}{20} - \frac{1}{2} + \frac{MC_{\text{total}}}{5} - 12
    \]
    \[
    = \frac{1}{4} MC_{\text{total}} - 12.5
    \]
  - Hence, \( MC_{\text{total}} = 50 + 4Q_{\text{total}} \)
  - Setting \( MR(Q) = 120 - 6Q = 50 + 4Q = MC_{\text{total}} \), we obtain \( Q_{\text{total}} = 7 \) and \( p = 120 - 3 \cdot 7 = 99 \).
  - Since \( MR(Q_{\text{total}}) = 120 - 6 \cdot 7 = 78 \), then
    \[
    MR(Q_{\text{total}}) = MC_1(q_1) \implies 78 = 10 + 20q_1 \implies q_1 = 3.4
    \]
    \[
    MR(Q_{\text{total}}) = MC_2(q_2) \implies 78 = 60 + 5q_2 \implies q_2 = 3.6
    \]
Price Discrimination
Price Discrimination

• Can the monopolist capture an even larger surplus?
  
  – Charge $p > p^m$ to those who buy the product at $p^m$ and are willing to pay more
  
  – Charge $c < p < p^m$ to those who do not buy the product at $p^m$, but whose willingness to pay for the good is still higher than the marginal cost of production, $c$.
  
  – With $p^m$ for all units, the monopolist does not capture the surplus of neither of these segments.
Price Discrimination: First-degree

• **First-degree (perfect) price discrimination:**
  – The monopolist charges to every customer his/her maximum willingness to pay for the object.

  – *Personalized price:* The first buyer pays $p_1$ for the $q_1$ units, the second buyer pays $p_2$ for $q_2 - q_1$ units, etc.
Price Discrimination: First-degree

- The monopolist continues doing so until the last buyer is willing to pay the marginal cost of production.
- In the limit, the monopolist captures all the area below the demand curve and above the marginal cost (i.e., consumer surplus).
Price Discrimination: First-degree

• Suppose that the monopolist can charge a fixed fee, \( r^* \), and an amount of the good, \( q^* \), that maximizes profits.

• PMP:

\[
\max_{r,q} \quad r - cq \\
\text{s.t.} \quad u(q) \geq r
\]

• Note that the monopolist raises the fee \( r \) until \( u(q) = r \). Hence we can reduce the set of choice variables

\[
\max_q \quad u(q) - cq
\]

• FOC: \( u'(q^*) - c = 0 \) or \( u'(q^*) = c \).

  – **Intuition:** the monopolist increases output until the marginal utility that consumers obtain from additional units coincides with the marginal cost of production.
Price Discrimination: First-degree

- Given the level of production $q^*$, the optimal fee is $r^* = u(q^*)$

- **Intuition**: the monopolist charges a fee $r^*$ that coincides with the utility that the consumer obtains from consuming $q^*$. 

\[
\text{Profits} = \int_0^{q^*} p(q) dq - c(q^*)
\]
Price Discrimination: First-degree

• **Example:**

  – A monopolist faces inverse demand curve \( p(q) = 20 - q \) and constant marginal costs \( c = $2 \).

  – No price discrimination:

    \[
    MR = MC \implies 20 - 2q = 2 \implies q^m = 9
    \]

    \[
    p^m = $11, \quad \pi^m = $81
    \]

  – Price discrimination:

    \[
    p(Q) = MC \implies 20 - Q = 2 \implies Q = 18
    \]

    \[
    \pi = \frac{18 \times (20 - 2)}{2} = $162
    \]
Price Discrimination: First-degree

- **Example** (continued):

![Graph showing first-degree pricing and profits comparison]

- No Price Disc. Profits = $81
- First-degree pricing disc. Profits = $162
- \( p^m = 11 \)
- \( c^3(q) \)
- \( p(q) = 20 - Q \)
- \( q^m = 9 \)
- \( q^* = 18 \)
Price Discrimination: First-degree

- Summary:
  - Total output coincides with that in perfect competition
  - Unlike in perfect competition, the consumer does not capture any surplus
  - The producer captures all the surplus
  - Due to information requirements, we do not see many examples of it in real applications
    - Financial aid in undergraduate education ("tuition discrimination")
Price Discrimination: First-degree

**Example** (two-block pricing):

– A monopolist faces a inverse demand curve $p(q) = a - bq$, with constant marginal costs $c < a$.

– Under two-block pricing, the monopolist sells the first $q_1$ units at a price $p(q_1) = p_1$ and the remaining $q_2 - q_1$ units at a price $p(q_2) = p_2$. 

![Diagram](image.png)
Price Discrimination: First-degree

- **Example** (continued):
  - Profits from the first $q_1$ units
    \[ \pi_1 = p(q_1) \cdot q_1 - cq_1 = (a - bq_1 - c)q_1 \]
  while from the remaining $q_2 - q_1$ units
    \[ \pi_2 = p(q_2) \cdot (q_2 - q_1) - c \cdot (q_2 - q_1) \]
    \[ = (a - bq_2 - c)(q_2 - q_1) \]
  - Hence total profits are
    \[ \pi = \pi_1 + \pi_2 \]
    \[ = (a - bq_1 - c)q_1 + (a - bq_2 - c)(q_2 - q_1) \]
Price Discrimination: First-degree

• **Example** (continued):
  
  – FOCs:
  
  \[
  \frac{\partial \pi}{\partial q_1} = a - 2bq_1 - c - a + bq_2 + c = 0
  \]
  
  \[
  \frac{\partial \pi}{\partial q_2} = -b(q_2 - q_1) + a - bq_2 - c = 0
  \]
  
  – Solving for \( q_1 \) and \( q_2 \)
  
  \[
  q_1 = \frac{a - c}{3b} \quad q_2 = \frac{2(a - c)}{3b}
  \]
  
  which entails prices of
  
  \[
  p(q_1) = a - b \cdot \frac{a - c}{3b} = \frac{2a + c}{3} \quad p(q_2) = \frac{a + 2c}{3}
  \]
  
  where \( p(q_1) > p(q_2) \) since \( a > c \).
Price Discrimination: First-degree

• **Example** (continued):

  – The monopolist’s profits from each block are

    \[ \pi_1 = (p(q_1) - c) \cdot q_1 \]

    \[ = \left( \frac{2a + c}{3} - c \right) \cdot \frac{a - c}{3b} = \frac{2}{b} \left( \frac{a - c}{3} \right)^2 \]

    \[ \pi_2 = (p(q_2) - c)(q_2 - q_1) \]

    \[ = \left( \frac{a + 2c}{3} - c \right) \cdot \left( \frac{2(a - c)}{3b} - \frac{a - c}{3b} \right) = \frac{1}{b} \left( \frac{a - c}{3} \right)^2 \]

  – Thus, \( \pi = \pi_1 + \pi_2 = \frac{(a-c)^2}{3b} \), which is larger than those arising from uniform pricing, \( \pi^u = \frac{(a-c)^2}{4b} \).
Price Discrimination: Third-degree

- **Third degree price discrimination:**
  - The monopolist charges different prices to two or more groups of customers (each group must be easily recognized by the seller).
    - *Example:* youth vs. adult at the movies, airline tickets
  - Firm’s PMP:
    \[
    \max_{x_1,x_2} p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2
    \]
  - FOCs:
    \[
    p_1(x_1) + p'_1(x_1)x_1 - c = 0 \implies MR_1 = MC
    \]
    \[
    p_2(x_2) + p'_2(x_2)x_2 - c = 0 \implies MR_2 = MC
    \]
  - FOCs coincides with those of a regular monopolist who serves two completely separated markets practicing uniform pricing.
Price Discrimination: Third-degree

**Example:** $p_1(x_1) = 38 - x_1$ for adults and $p_2(x_2) = 14 - \frac{1}{4}x_2$ for seniors, with $MC = $10 for both markets.

\[
MR_1(x_1) = MC \implies 38 - 2x_1 = 10 \implies x_1 = 14 \quad p_1 = $24
\]

\[
MR_2(x_2) = MC \implies 14 - \frac{1}{2}x_2 = 10 \implies x_2 = 8 \quad p_2 = $12
\]
Price Discrimination: Third-degree

• Using the Inverse Elasticity Pricing Rule (IERP), we can obtain the prices

\[ p_1(x_1) = \frac{c}{1 - 1/\varepsilon_1} \quad \text{and} \quad p_2(x_2) = \frac{c}{1 - 1/\varepsilon_2} \]

where \( c \) is the common marginal cost.

• Then, \( p_1(x_1) > p_2(x_2) \) if and only if

\[ \frac{1}{1 - 1/\varepsilon_1} > \frac{1}{1 - 1/\varepsilon_2} \quad \Rightarrow \quad 1 - \frac{1}{\varepsilon_2} < 1 - \frac{1}{\varepsilon_1} \]

\[ \Rightarrow \quad \frac{1}{\varepsilon_2} > \frac{1}{\varepsilon_1} \quad \Rightarrow \quad \varepsilon_2 < \varepsilon_1 \]

• *Intuition*: the monopolist charges lower price in the market with more elastic demand.
Price Discrimination: Third-degree

• **Example** (Pullman-Seattle route):
  – The price-elasticity of demand for business-class seats is -1.15, while that for economy seats is -1.52
  – From the IEPR,
    \[
    \begin{align*}
    p_B &= \frac{MC}{1 - 1/1.15} \quad \Rightarrow \quad 0.13p_B = MC \\
    p_E &= \frac{MC}{1 - 1/1.52} \quad \Rightarrow \quad 0.34p_E = MC
    \end{align*}
    \]
  – Hence, \(0.13p_B = 0.34p_E\) or \(p_B = 2.62p_E\)
    - Airline maximizes its profits by charging business-class seats a price 2.62 times higher than that of economy-class seats
Second-degree price discrimination:

- The monopolist cannot observe the type of each consumer (e.g., his willingness to pay).
- Hence the monopolist offers a menu of two-part tariffs, \((F_L, q_L)\) and \((F_H, q_H)\), with the property that the consumer with type \(i = \{L, H\}\) has the incentive to self-select the two-part tariff \((F_i, q_i)\) meant for him.
Price Discrimination: Second-degree

• Assume the utility function of type $i$ consumer

$$U_i(q_i, F_i) = \theta_i u(q_i) - F_i$$

where

– $q_i$ is the quantity of a good consumed
– $F_i$ is the fixed fee paid to the monopolist for $q_i$
– $\theta_i$ measures the valuation consumer assigns to the good, where $\theta_H > \theta_L$, with corresponding probabilities $p$ and $1 - p$.

• The monopolist’s constant marginal cost $c$ satisfies $\theta_i > c$ for all $i = \{L, H\}$.
Price Discrimination: Second-degree

• The monopolist must guarantee that
  1) both types of customers are willing to participate ("participation constraint")
     ▪ the two-part tariff meant for each type of customer provides him with a weakly positive utility level
  2) customers do not have incentives to choose the two-part tariff meant for the other type of customer ("incentive compatibility")
     ▪ type $i$ customer prefers $(F_i, q_i)$ over $(F_j, q_j)$ where $j \neq i$
Price Discrimination: Second-degree

• The participation constraints (PC) are

\[ \theta_L u(q_L) - F_L \geq 0 \quad PC_L \]
\[ \theta_H u(q_H) - F_H \geq 0 \quad PC_H \]

• The incentive compatibility conditions are

\[ \theta_L u(q_L) - F_L \geq \theta_L u(q_H) - F_H \quad IC_L \]
\[ \theta_H u(q_H) - F_H \geq \theta_H u(q_L) - F_L \quad IC_H \]
Price Discrimination: Second-degree

• Rearranging the four inequalities, the monopolist’s profit maximization problem becomes:

\[
\max_{F_L, q_L, F_H, q_H} \quad p[F_H - cq_H] + (1 - p)[F_L - cq_L]
\]

\[
\theta_L u(q_L) \geq F_L
\]

\[
\theta_H u(q_H) \geq F_H
\]

\[
\theta_L [u(q_L) - u(q_H)] + F_H \geq F_L
\]

\[
\theta_H [u(q_H) - u(q_L)] + F_L \geq F_H
\]
Price Discrimination: Second-degree

• Both $PC_H$ and $IC_H$ are expressed in terms of the fee $F_H$
  – The monopolist increases $F_H$ until such fee coincides with the lowest of $\theta_H u(q_H)$ and $\theta_H [u(q_H) - u(q_L)] + F_L$ for all $i = \{L, H\}$
  – Otherwise, one (or both) constraints will be violated, leading the high-demand customer to not participate
Price Discrimination: Second-degree

\[ \text{Maximal } F_i \text{ that achieves participation and self-selection} \]

\[ PC_i \text{ is binding} \]

\[ \theta_i u(q_i) \quad \theta_i [u(q_i) - u(q_j)] + F_j \]

\[ F_i \]

\[ \text{Maximal } F_i \text{ that achieves participation and self-selection} \]

\[ IC_i \text{ is binding} \]

\[ \theta_i [u(q_i) - u(q_j)] + F_j \quad \theta_i u(q_i) \]

\[ F_i \]
Price Discrimination: Second-degree

• **High-demand customer:**
  – Let us show that $IC_H$ is binding
  – An indirect way to show that
    \[
    F_H = \theta_H [u(q_H) - u(q_L)] + F_L
    \]
    is to demonstrate that $F_H < \theta_H u(q_H)$
  – Proving this by contradiction, assume that
    \[
    F_H = \theta_H u(q_H)
    \]
Price Discrimination: Second-degree

– Then, \( IC_H \) can be written as

\[
F_H - \theta_H u(q_L) + F_L \geq F_H
\]

\[
\Rightarrow F_L \geq \theta_H u(q_L)
\]

– Combining this result with the fact that \( \theta_H > \theta_L \),

\[
F_L \geq \theta_H u(q_L) > \theta_L u(q_L)
\]

which implies \( F_L > \theta_L u(q_L) \)

– However, this violates \( PC_L \)
  - We then reached a contradiction
  - Thus, \( F_H < \theta_H u(q_H) \)
  - \( IC_H \) is binding but \( PC_H \) is not.
Price Discrimination: Second-degree

- **Low-demand customer**: 
  - Let us show that $PC_L$ binding
  - Similarly as for the high-demand customer, an indirect way to show that
    \[ F_L = \theta_L u(q_L) \]
    is to demonstrate that
    \[ F_L < \theta_L [u(q_L) - u(q_H)] + F_H \]
  - Proving this by contradiction, assume that
    \[ F_L = \theta_L [u(q_L) - u(q_H)] + F_H \]
Price Discrimination: Second-degree

Then, \( IC_H \) can be written as

\[
\theta_H [u(q_H) - u(q_L)] + \theta_L [u(q_L) - u(q_H)] + F_H = F_H
\]

\[
\Rightarrow \theta_H [u(q_H) - u(q_L)] = \theta_L [u(q_H) - u(q_L)]
\]

\[
\Rightarrow \theta_H = \theta_L
\]

which violates the initial assumption \( \theta_H > \theta_L \)

- We reached a contradiction
- Thus, \( F_L < \theta_L [u(q_L) - u(q_H)] + F_H \)
- \( PC_L \) is binding but \( IC_L \) is not
Price Discrimination: Second-degree

• In summary:
  – From $PC_L$ binding we have
    $$\theta_L u(q_L) = F_L$$
  – From $IC_H$ binding we have
    $$\theta_H [u(q_H) - u(q_L)] + F_L = F_H$$
  – In addition,
    • $PC_L$ binding implies that $IC_L$ holds, and
    • $IC_H$ binding entails that $PC_H$ is also satisfied,
    • That is, all four constraints hold.
Price Discrimination: Second-degree

• The monopolist’s expected PMP can then be written as unconstrained problem, as follows,

\[
\max_{q_L, q_H \geq 0} \quad p \left[ F_H - c q_H \right] + (1 - p) \left[ F_L - c q_L \right]
= p \left\{ \theta_H \left[ u(q_H) - u(q_L) \right] + F_L - c q_H \right\}
+ (1 - p) \left\{ \theta_L u(q_L) - c q_L \right\}
= p \left\{ \theta_H \left[ u(q_H) - u(q_L) \right] + \theta_L u(q_L) - c q_H \right\}
+ (1 - p) \left\{ \theta_L u(q_L) - c q_L \right\}
= p \left[ \theta_H u(q_H) - (\theta_H - \theta_L) u(q_L) - c q_H \right]
+ (1 - p) \left[ \theta_L u(q_L) - c q_L \right]
\]
Price Discrimination: Second-degree

• FOC with respect to $q_H$:
  \[ p[\theta_H u'(q_H) - c] = 0 \implies \theta_H u'(q_H) = c \]
  – which coincides with that under complete information.
  – That is, there is not output distortion for high-demand buyer
  – Informally, we say that there is “no distortion at the top”.

• FOC with respect to $q_L$:
  \[ p[-(\theta_H - \theta_L)u'(q_L)] + (1 - p)[\theta_L u'(q_L) - c] = 0 \]
  which can be re-written as
  \[ u'(q_L)[\theta_L - p\theta_H] = (1 - p)c \]
Price Discrimination: Second-degree

• Dividing both sides by \((1 - p)\), we obtain

\[
u'(q_L) \left[ \frac{\theta_L - p\theta_H}{1 - p} \right] = c
\]

• The above expression can alternatively be written as

\[
u'(q_L) \left[ \theta_L - \frac{p}{1 - p} (\theta_H - \theta_L) \right] = c
\]
Price Discrimination: Second-degree

- \( u'(q_L) \cdot \theta_L \) depicts the socially optimal output \( q_{LSO} \), i.e., that arising under complete information.
- The output offered to high-demand customers is socially efficient due to the absence of output distortion for high-type agents.
- The output offered to low-demand customers entails a distortion, i.e., \( q_L < q_{LSO} \).
- Per-unit price for high-type and low-type differs, i.e., \( F_H \neq F_L \).
  - Monopolist practices price discrimination among the two types of customers.
Price Discrimination: Second-degree

• Since constraint $PC_L$ binds while $PC_H$ does not, then only the high-demand customer retains a positive utility level, i.e., $\theta_H u(q_H) - F_H > 0$.

• The firm’s lack of information provides the high-demand customer with an “information rent.”
  – Intuitively, the information rent emerges from the seller’s attempt to reduce the incentives of the high-type customer to select the contract meant for the low type.
  – The seller also achieves self-selection by setting an attractive output for the low-type buyer, i.e., $q_L$ is lower than under complete information.
Price Discrimination: Second-degree

• **Example:**
  – Consider a monopolist selling a textbook to two types of graduate students, low- and high-demand, with utility function

  \[ U_i(q_i, F_i) = \frac{q_i^2}{2} - \theta_i q_i - F_i \]

  where \( i = \{L, H\} \) and \( \theta_H > \theta_L \).
  – Hence, the UMP of type \( i \) student is

  \[
  \max_{q_i} \quad \frac{q_i^2}{2} - \theta_i q_i - F_i \quad \text{s.t.} \quad pq_i + F_i \leq w_i
  \]

  where \( w_i > 0 \) denotes the student’s wealth.
• **Example** (continued):
  
  – By Walras’ law, the constraint binds
  \[ F_i = w_i - pq_i \]
  
  – Then, the UMP can be expressed as
  \[
  \max_{q_i} \frac{q_i^2}{2} - \theta_i q_i - (w_i - pq_i)
  \]
  – FOCs wrt \( q_i \) yields the direct demand function:
  \[ q_i - \theta_i + p = 0 \text{ or } q_i = \theta_i - p \]
• **Example** (continued):

– Assume that the proportion of high-demand (low-demand) students is \( \gamma \) \((1 - \gamma)\), respectively.

– The monopolist’s constant marginal cost is \( c > 0 \), which satisfies \( \theta_i > c \) for all \( i = \{L, H\} \).

– Consider for simplicity that \( \theta_L > \frac{\theta_H + c}{2} \).

– This implies that each type of student would buy the textbook, both when the firm practices uniform pricing and when it sets two-part tariffs

  ▪ Exercise.
Advertising in Monopoly
Advertising in Monopoly

• **Advertising**: non-price strategy to capture surplus

• The monopolist must balance the additional demand that advertising entails and its associated costs ($A$ dollars)

• The monopolist solves

$$\max_A p \cdot q(p, A) - TC(q(p, A)) - A$$

where the demand function $q(p, A)$ depends on price and advertising.
Advertising in Monopoly

• Taking FOCs with respect to $A$,

$$p \cdot \frac{\partial q(p,A)}{\partial A} - \frac{\partial TC}{\partial q} \cdot \frac{\partial q(p,A)}{\partial A} - 1 = 0$$

Rearranging, we obtain

$$(p - MC) \frac{\partial q(p,A)}{\partial A} = 1$$

• Let us define the advertising elasticity of demand

$$\varepsilon_{q,A} = \frac{\% \text{ increase in } q}{\% \text{ increase in } A} = \frac{\partial q(p,A)}{\partial A} \cdot \frac{A}{q}$$

Or, rearranging,

$$\varepsilon_{q,A} \cdot \frac{q}{A} = \frac{\partial q(p,A)}{\partial A}$$
Advertising in Monopoly

• We can then rewrite the above FOC as

\[
(p - MC) \varepsilon_{q,A} \cdot \frac{q}{A} = 1
\]

\[
\frac{\partial q(p,A)}{\partial A}
\]

• Dividing both sides by \( \varepsilon_{q,A} \) and rearranging

\[
p - MC = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{q}
\]

• Dividing both sides by \( p \)

\[
\frac{p - MC}{p} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{p \cdot q}
\]
Advertising in Monopoly

• From the Lerner index, we know that $\frac{p - MC}{p} = -\frac{1}{\epsilon_{q,p}}$. Hence,

$$-\frac{1}{\epsilon_{q,p}} = \frac{1}{\epsilon_{q,A}} \cdot \frac{A}{p \cdot q}$$

• And rearranging

$$-\frac{\epsilon_{q,A}}{\epsilon_{q,p}} = \frac{A}{p \cdot q}$$

– The right-hand side represents the advertising-to-sales ratio.

– For two markets with the same $\epsilon_{q,p}$, the advertising-to-sales ratio must be larger in the market where demand is more sensitive to advertising (higher $\epsilon_{q,A}$).
Advertising in Monopoly

• Example:

  – If the price-elasticity in a given monopoly market is $\varepsilon_{q,p} = -1.5$ and the advertising-elasticity is $\varepsilon_{q,A} = 0.1$, the advertising-to-sales ratio should be

  $$\frac{A}{p \cdot q} = -\frac{0.1}{-1.5} = 0.067$$

  – Advertising should account for 6.7% of this monopolist’s revenue.
Regulation of Natural Monopolies
Regulation of Natural Monopolies

- **Natural monopolies**: Monopolies that exhibit decreasing cost structures, with the MC curve lying below the AC curve.
- Hence, having a single firm serving the entire market is cheaper than having multiple firms, as aggregate average costs for the entire industry would be lower.
Regulation of Natural Monopolies

- Unregulated natural monopolist maximizes profits at the point where MR=MC, producing $Q_1$ units and selling them at a price $p_1$.
- Regulated natural monopolist will charge $p_2$ (where demand crosses MC) and produce $Q_2$ units.
- The production level $Q_2$ implies a loss of $p_2 - c_2$ per unit.
Regulation of Natural Monopolies

• Dilemma with natural monopolies:
  – abandon the policy of setting prices equal to marginal cost, OR
  – continue applying marginal cost pricing but subsidize the monopolist for his losses

• Solution to the dilemma:
  – A multi-price system that allows for price discrimination
  – Charging some users a high price while maintaining a low price to other users
Regulation of Natural Monopolies

• Multi-price system:
  – a high price $p_1$
  – a low price $p_2$

• Benefit: $(p_1 - c_1)$ per unit in the interval from $0$ to $Q_1$

• Loss: $(c_2 - p_2)$ per unit in the interval $(Q_2 - Q_1)$

• The monopolist price discriminates iff

$$ (p_1 - c_1)Q_1 > (c_2 - p_2)(Q_2 - Q_1) $$
Regulation of Natural Monopolies

• An alternative regulation:
  – allow the monopolist to charge a price above marginal cost that is sufficient to earn a “fair” rate of return on capital investments

• Two difficulties:
  – what is a “fair” rate of return
  – overcapitalization
Regulation of Natural Monopolies

• *Overcapitalization of natural monopolies:*
  
  – Suppose a production function of the form $q = f(k, l)$. An unregulated monopoly with profit function $pf(k, l) - wl - rk$ has a rate of return on capital, $r$. Suppose furthermore that the rate of return on capital investments, $r$, is constrained by a regulatory agency to be equal to $r_0$. 
Regulation of Natural Monopolies

• PMP:

\[ L = pf(k, l) - wl - rk + \lambda[wl + r_0k - pf(k, l)] \]

where \(0 < \lambda < 1\).

• FOCs:

\[ \frac{\partial L}{\partial l} = pf_l - w + \lambda(w - pf_l) = 0 \]
\[ \frac{\partial L}{\partial k} = pf_k - r + \lambda(r_0 - pf_k) = 0 \]
\[ \frac{\partial L}{\partial \lambda} = wl + r_0k - pf(k, l) = 0 \]
Regulation of Natural Monopolies

• From the first FOC:

\[ pf_l = w \]

• From the second FOC:

\[ (1 - \lambda)pf_k = r - \lambda r_0 \]

and rearranging

\[ pf_k = \frac{r - \lambda r_0}{1 - \lambda} = r - \frac{\lambda(r_0 - r)}{1 - \lambda} \]

– Since \( r_0 > r \) and \( 0 < \lambda < 1 \), then \( pf_k < r \).

– Hence, the firm would hire *more capital* than under unregulated condition, where \( pf_k = r \).
Regulation of Natural Monopolies

- \( pf_k \) is the value of the marginal product of capital
  - It is decreasing in \( k \) (due to diminishing marginal return, i.e., \( f_{kk} < 0 \))
- \( r \) and \( r - \frac{\lambda(r_0-r)}{1-\lambda} \) are the marginal cost of additional units of capital in the unregulated and regulated monopoly, respectively
  \[ r > r - \frac{\lambda(r_0-r)}{1-\lambda} \]
- Example: electricity and water suppliers
Regulation of Natural Monopolies

• An alternative illustration of the overcapitalization (Averch-Johnson effect)

• Before regulation, the firm selects \((L^{BR}, K^{BR})\)

• After regulation, the firm selects \((L^{AR}, K^{AR})\), where \(K^{AR} > K^{BR}\) but \(L^{AR} < L^{BR}\)

• The overcapitalization result only captures the substitution effect of a cheaper input.
  – Output effect?
Monopsony
Monopsony

• **Monopsony**: A single buyer of goods and services exercises “buying power” by paying prices below those that would prevail in a perfectly competitive context.

• Monopsony (single buyer) is analogous to that of a monopoly (single seller).

• *Examples*: a coal mine, Walmart Superstore in a small town, etc.
Monopsony

• Consider that the monopsony faces competition in the product market, where prices are given at $p > 0$, but is a monopsony in the input market (e.g., labor services).

• Assume an increasing and concave production function, i.e., $f'(x) > 0$ and $f''(x) \leq 0$.
  – This yields a total revenue of $pf(x)$.

• Consider a cost function $w(x) \cdot x$, where $w(x)$ denotes the inverse supply function of labor $x$.
  – Assume that $w'(x) > 0$ for all $x$.
  – This indicates that, as the firm hires more workers, labor becomes scarce, thus increasing the wages of additional workers.
Monopsony

• The monopsony PMP is
  \[ \max_x pf(x) - w(x)x \]

• FOC wrt the amount of labor services \( (x) \) yields
  \[ pf'(x^*) - w(x^*) - w'(x^*)x^* = 0 \]
  \[ \Rightarrow pf'(x^*) = w(x^*) + w'(x^*)x^* \]
  \[ A \quad B \]

  – \( A \): “marginal revenue product” of labor.
  – \( B \): “marginal expenditure” (ME) on labor.

  ▪ The additional worker entails a monetary outlay of \( w(x^*) \).
  ▪ Hiring more workers make labor become more scarce, ultimately forcing the monopsony to raise the prevailing wage on all inframarginal workers, as captured by \( w'(x^*)x^* \).
Monopsony

• Monopsonist hiring and salary decisions.

- The marginal revenue product of labor, $p f'(x)$, is decreasing in $x$ given that $f''(x) \leq 0$.
- The labor supply, $w(x)$, is increasing in $x$ since $w'(x) > 0$.
- The marginal expenditure (ME) on labor lies above the supply function $w(x)$ since $w'(x) > 0$.
- The monopsony hires $x^*$ workers at a salary of $w(x^*)$. 
Monopsony

• A deadweight loss from monopsony is

\[ DWL = \int_{x^*}^{x^{PC}} [pf'(x) - w(x)]dx \]

• That is, the area below the marginal revenue product and above the supply curve, between \( x^* \) and \( x^{PC} \) workers.
Monopsony

• We can write the monopsony profit-maximizing condition, i.e., \( pf'(x^*) = w(x^*) + w'(x^*)x^* \), in terms of labor supply elasticity, using the following steps:

\[
pf'(x^*) = w(x^*) + \frac{\partial w(x^*)}{\partial x^*} x^* \\
= w(x^*) \left( 1 + \frac{\partial w(x^*)}{\partial x^*} \frac{x^*}{w(x^*)} \right)
\]

• And rearranging,

\[
pf'(x^*) = w(x^*) \left( 1 + \frac{1}{\frac{\partial x^* w(x^*)}{\partial w x^*}} \right)
\]
Monopsony

- Since \( \frac{\partial x^*}{\partial w} \frac{w(x^*)}{x^*} \) represents the elasticity of labor supply \( \varepsilon \), then

\[
p f'(x^*) = w(x^*) \left( 1 + \frac{1}{\varepsilon} \right)
\]

- Intuitively, as \( \varepsilon \to \infty \) (labor supply becoming perfectly elastic), the behavior of the monopsonist approaches that of a pure competitor.
Monopsony

- The equilibrium condition above is also sufficient as long as
  \[ pf''(x^*) - 2w'(x^*) - w''(x^*)x^* < 0 \]

- Since \( f''(x^*) < 0 \), \( w'(x^*) > 0 \) (by assumption), we only need that either:
  a) the supply function is convex, i.e., \( w''(x^*) > 0 \); or
  b) if it is concave, i.e., \( w''(x^*) < 0 \), its concavity is not very strong, that is
  \[ pf''(x^*) - 2w'(x^*) < w''(x^*)x^* \]
Monopsony

• Example:

– Consider a monopsonist with production function $f(x) = ax$, where $a > 0$, and facing a given market price $p > 0$ per unit of output.

– Labor supply is $w(x) = bx$, where $b > 0$.

– The marginal revenue product of hiring an additional worker is $pf'(x) = pa$

– The marginal expenditure on labor is $w(x) + w'(x)x = bx + bx = 2bx$
Monopsony

• **Example** (continued):

  – Setting them equal to each other, \( ap = 2bx^* \), yields a profit-maximizing amount of labor:
    \[
    x^* = \frac{ap}{2b}
    \]

  – \( x^* \) increases in the price of output, \( p \), and in the marginal productivity of labor, \( a \); but decreases in the slope of labor supply, \( b \).

  – Sufficiency holds since
    \[
    pf''(x^*) - 2w'(x^*) = p0 - 2b < 0 = w''(x^*)x^*
    \]