Advanced Microeconomic Theory

Chapter 7: Monopoly

Outline

- Barriers to Entry
- Profit Maximization under Monopoly
- Welfare Loss of Monopoly
- Multiplant Monopolist
- Price Discrimination
- Advertising in Monopoly
- Regulation of Natural Monopolies
- Monopsony

Barriers to Entry

Barriers to Entry

- ***Entry barriers***: elements that make the entry of potential competitors either impossible or very costly.

- Three main categories:
  1) ***Legal***: the incumbent firm in an industry has the legal right to charge monopoly prices during the life of the patent
     - *Example*: newly discovered drugs

Barriers to Entry

2) ***Structural***: the incumbent firm has a cost or demand advantage relative to potential entrants.
   - superior technology
   - a loyal group of customers
     - positive network externalities (Facebook, eBay)

3) ***Strategic***: the incumbent monopolist has a reputation of fighting off newcomers, ultimately driving them off the market.
   - price wars

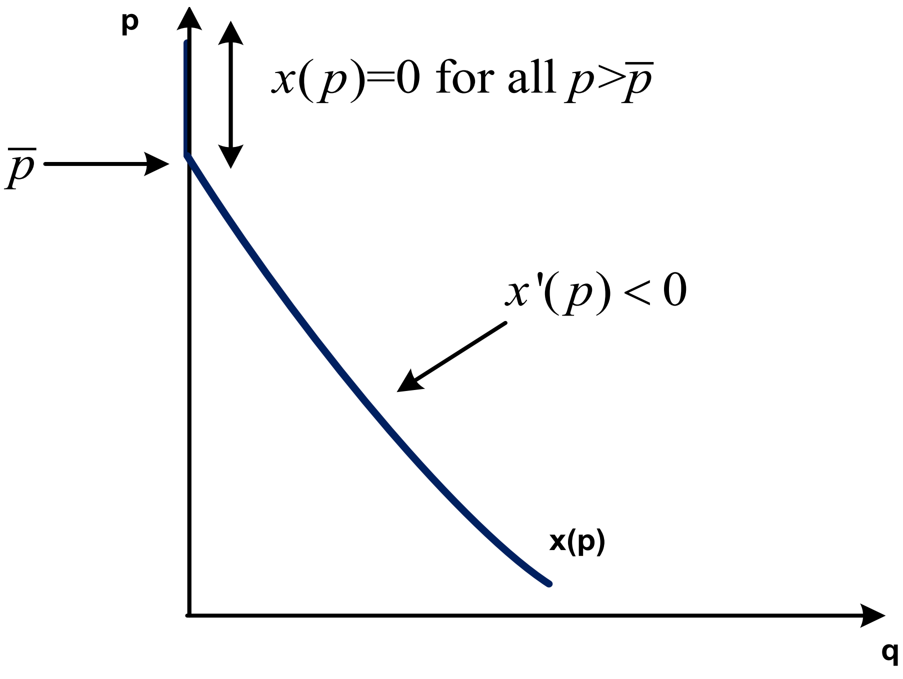
Profit Maximization under Monopoly

Profit Maximization

- Consider a demand function $x(p)$, which is continuous and strictly decreasing in p, i.e., $x'(p) < 0$.
- We assume that there is price $\bar{p} < \infty$ such that $x(p) = 0$ for all $p > \bar{p}$.
- Also, consider a general cost function $c(q)$, which is increasing and convex in q.

Profit Maximization

- $\bar{p}$ is a "choke price"
- No consumers buy positive amounts of the good for $p > \bar{p}$.



Profit Maximization

- Monopolist's decision problem is

$$\max_{p} \; px(p) - c(x(p))$$

- Alternatively, using $x(p) = q$, and taking the inverse demand function $p(q) = x^{-1}(q)$, we can rewrite the monopolist's problem as

$$\max_{q \geq 0} \; p(q)q \; - c(q)$$

Profit Maximization

- Differentiating with respect to q,

$$p(q^m) + p'(q^m)q^m - c'(q^m) \leq 0$$

- Rearranging,

$$\underbrace{p(q^m) + p'(q^m)q^m}_{MR=\frac{d[p(q)q]}{dq}} \leq \underbrace{c'(q^m)}_{MC}$$

with equality if $q^m > 0$.

- Recall that total revenue is $TR(q) = p(q)q$

Profit Maximization

- In addition, we assume that $p(0) \geq c'(0)$.
  - That is, the inverse demand curve originates above the marginal cost curve.
  - Hence, the consumer with the highest willingness to pay for the good is willing to pay more than the variable costs of producing the first unit.
- Then, we must be at an interior solution $q^m > 0$, implying

$$\underbrace{p(q^m) + p'(q^m)q^m}_{MR} = \underbrace{c'(q^m)}_{MC}$$

Profit Maximization

- Note that

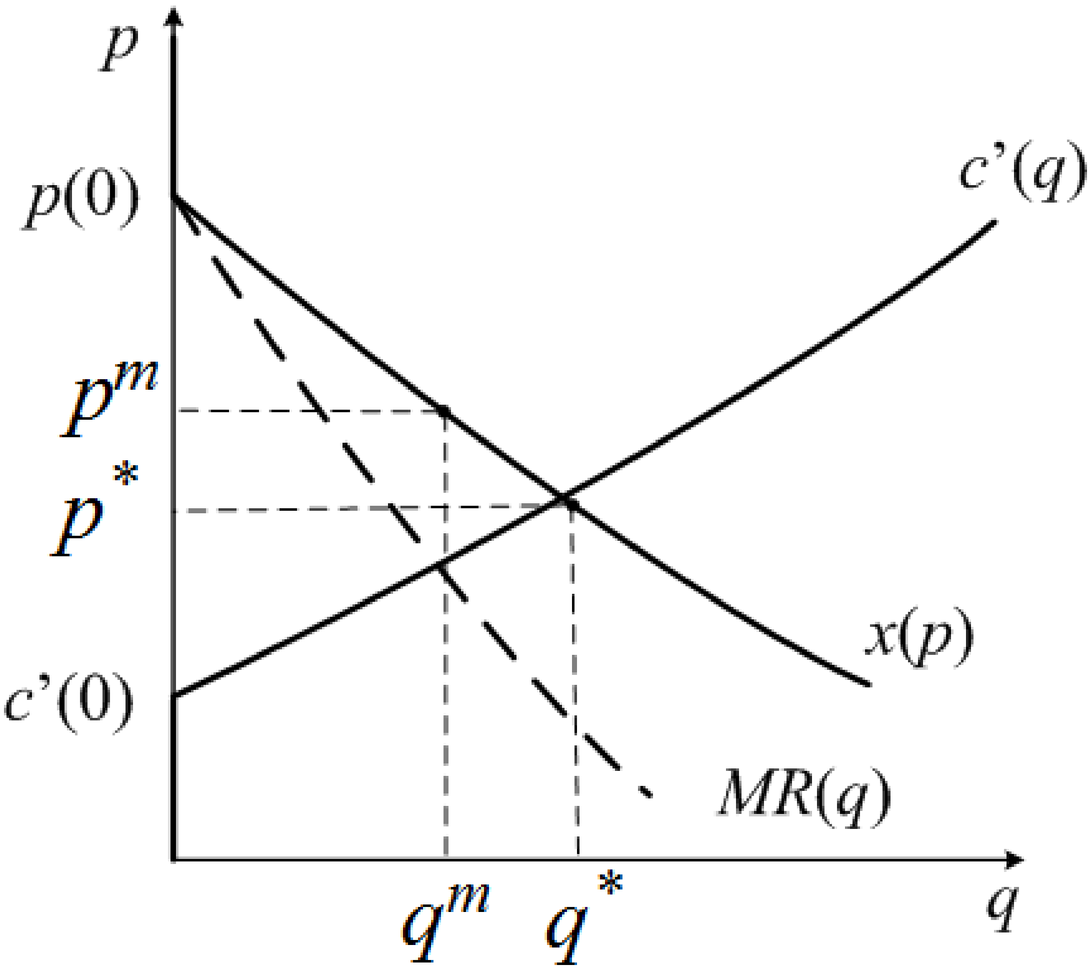
$$p(q^m) + \underbrace{p'(q^m)q^m}_{-} = c'(q^m)$$

- Then, $p(q^m) > c'(q^m)$, i.e.,

$$\text{monopoly price} > MC$$

- Moreover, we know that in competitive equilibrium $p(q^*) = c'(q^*)$.
- Then, $p^m > p^*$ and $q^m < q^*$.

Profit Maximization



Profit Maximization

- Marginal revenue in monopoly

$$MR = p(q^m) + p'(q^m)q^m$$

  MR describes two effects:
  - **A *direct* (positive) effect**: an additional unit can be sold at $p(q^m)$, thus increasing revenue by $p(q^m)$.
  - **An *indirect* (negative) effect**: selling an additional unit can only be done by reducing the market price of all units (the new and all previous units), ultimately reducing revenue by $p'(q^m)q^m$.
    - *Inframarginal units* – initial units before the marginal increase in output.

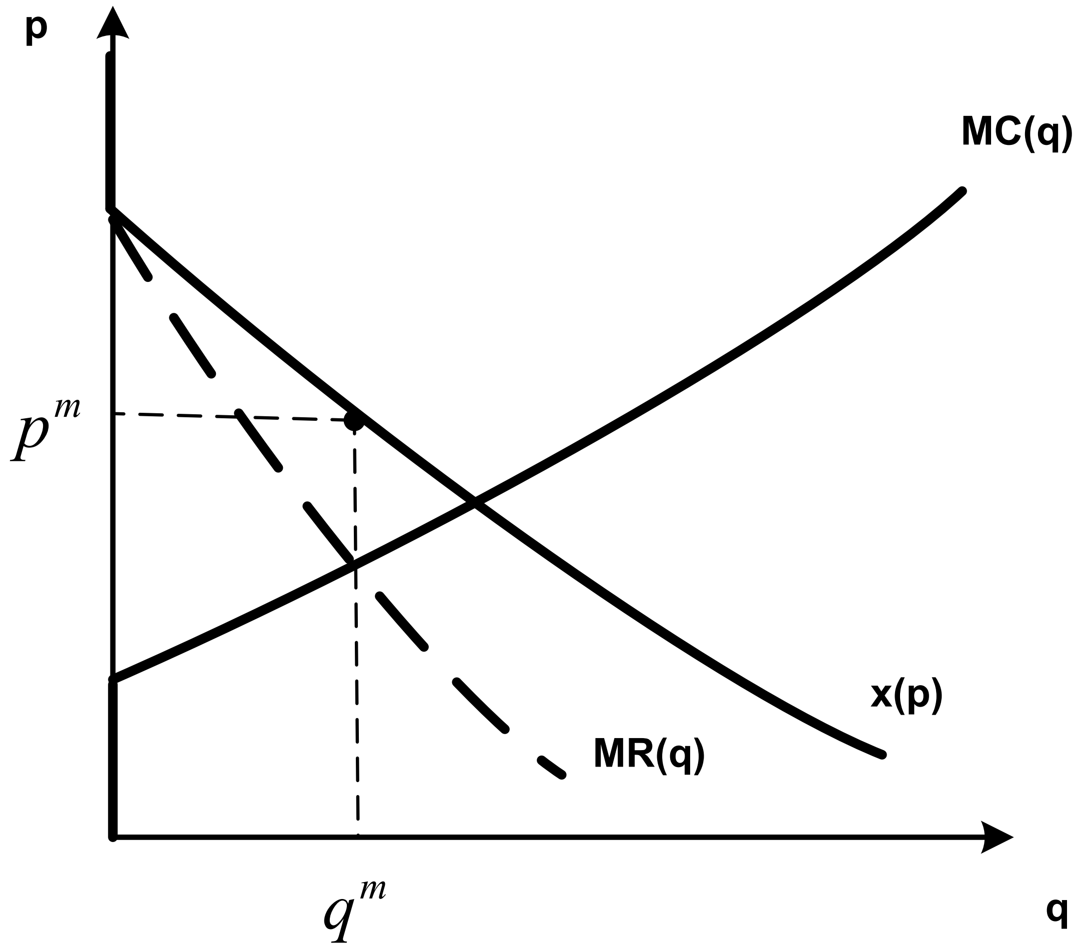
Profit Maximization

- Is the above FOC also sufficient?
  - Let's take the FOC $p(q^m) + p'(q^m)q^m - c'(q^m)$, and differentiate it wrt q,

$$\underbrace{p'(q) + p'(q) + p''(q)q}_{\frac{dMR}{dq}} - \underbrace{c''(q)}_{\frac{dMC}{dq}} \leq 0$$

  - That is, $\frac{dMR}{dq} \leq \frac{dMC}{dq}$.
  - Since MR curve is decreasing and MC curve is weakly increasing, the second-order condition is satisfied for all q.

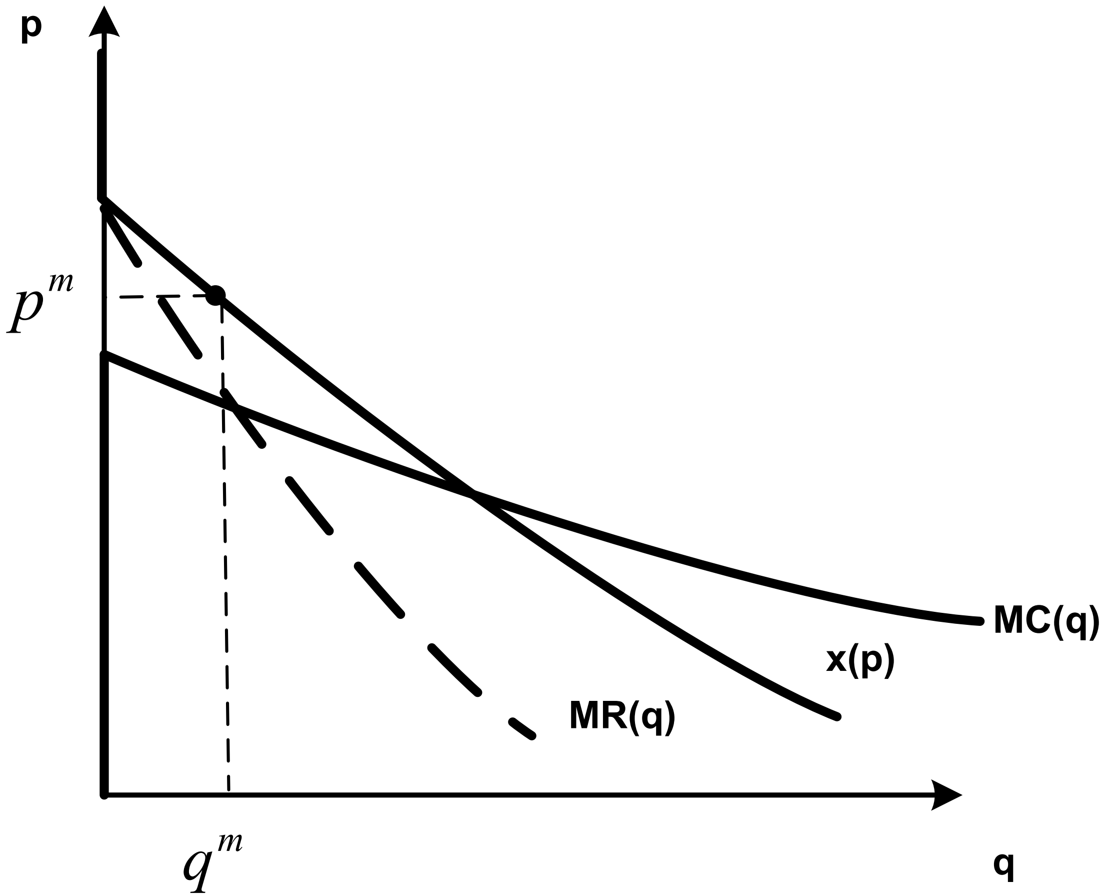
Profit Maximization



Profit Maximization

- What would happen if MC curve was decreasing in q (e.g., concave technology given the presence of increasing returns to scale)?
  - Then, the slopes of MR and MC curves are both decreasing.
  - At the optimum, MR curve must be steeper MC curve.

Profit Maximization



Profit Maximization: Lerner Index

- Can we re-write the FOC in a more intuitive way? Yes.
  – Just take $MR = p(q) + p'(q)q = p + \frac{\partial p}{\partial q} q$ and multiply by $\frac{p}{p}$,

$$MR = p\frac{p}{p} + \underbrace{\frac{\partial p}{\partial q}\frac{q}{p}}_{1/\varepsilon_d} p = p + \frac{1}{\varepsilon_d} p$$

  – In equilibrium, $MR(q) = MC(q)$. Hence, we can replace MR with MC in the above expression.

Profit Maximization: Lerner Index

- Rearranging yields

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_d}$$

- This is the ***Lerner index*** of market power
  - The price mark-up over marginal cost that a monopolist can charge is a function of the elasticity of demand.
- Note:
  - If $\varepsilon_d \to \infty$, then $\frac{p-MC(q)}{p} \to 0 \implies p = MC(q)$
  - If $\varepsilon_d \to 0$, then $\frac{p-MC(q)}{p} \to \infty \implies$ substantial mark-up

Profit Maximization: Lerner Index

- The Lerner index can also be written as

$$p = \frac{MC(q)}{1+\frac{1}{\varepsilon_d}}$$

which is referred to as the ***Inverse Elasticity Pricing Rule*** (IEPR).

- *Example* (Perloff, 2012):
  - Prilosec OTC: $\varepsilon_d = -1.2$. Then price should be $p = \frac{MC(q)}{1+\frac{1}{-1.2}} = 6MC$
  - Designed jeans: $\varepsilon_d = -2$. Then price should be $p = \frac{MC(q)}{1+\frac{1}{-2}} = 2MC$

Profit Maximization: Lerner Index

- ***Example 1*** (linear demand):
  - Market inverse demand function is
    $$p(q) = a - bq$$
    where $b > 0$
  - Monopolist's cost function is $c(q) = cq$
  - We usually assume that $a > c \geq 0$
    - To guarantee $p(0) > c'(0)$
    - That is, $p(0) = a - b0 = a$ and $c'(q) = c$, thus implying $c'(0) = c$

Profit Maximization: Lerner Index

- ***Example 1*** (continued):
  - Monopolist's objective function

$$\pi(q) = (a - bq)q - cq$$

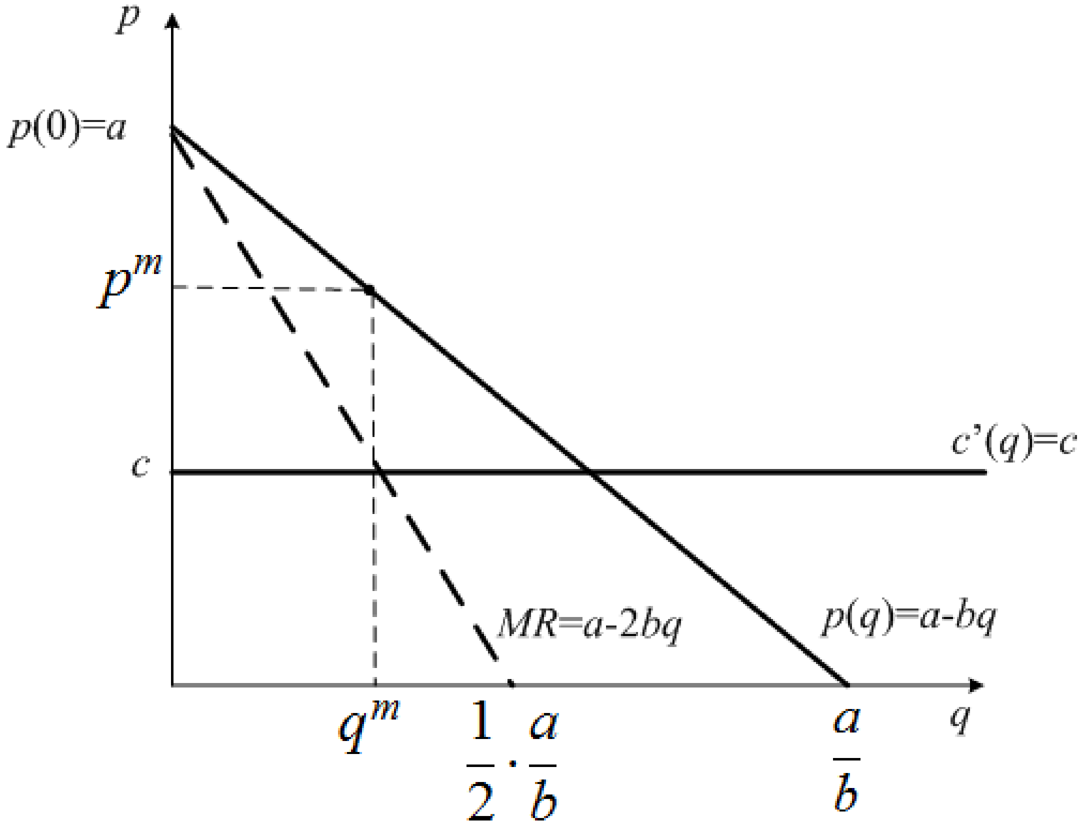
  - FOC: $a - 2bq - c = 0$
  - SOC: $-2b < 0$ (concave)
    - Note that as long as $b > 0$, i.e., negatively sloped demand function, profits will be concave in output.
    - Otherwise (i.e., Giffen good, with positively sloped demand function) profits will be convex in output.

Profit Maximization: Lerner Index

- ***Example 1*** (continued):
  - Solving for the optimal q^m in the FOC, we find monopoly output
$$q^m = \frac{a-c}{2b}$$
  - Inserting $q^m = \frac{a-c}{2b}$ in the demand function, we obtain monopoly price
$$p^m = a - b\left(\frac{a-c}{2b}\right) = \frac{a+c}{2}$$
  - Hence, monopoly profits are
$$\pi^m = p^m q^m - cq^m = \frac{(a-c)^2}{4b}$$

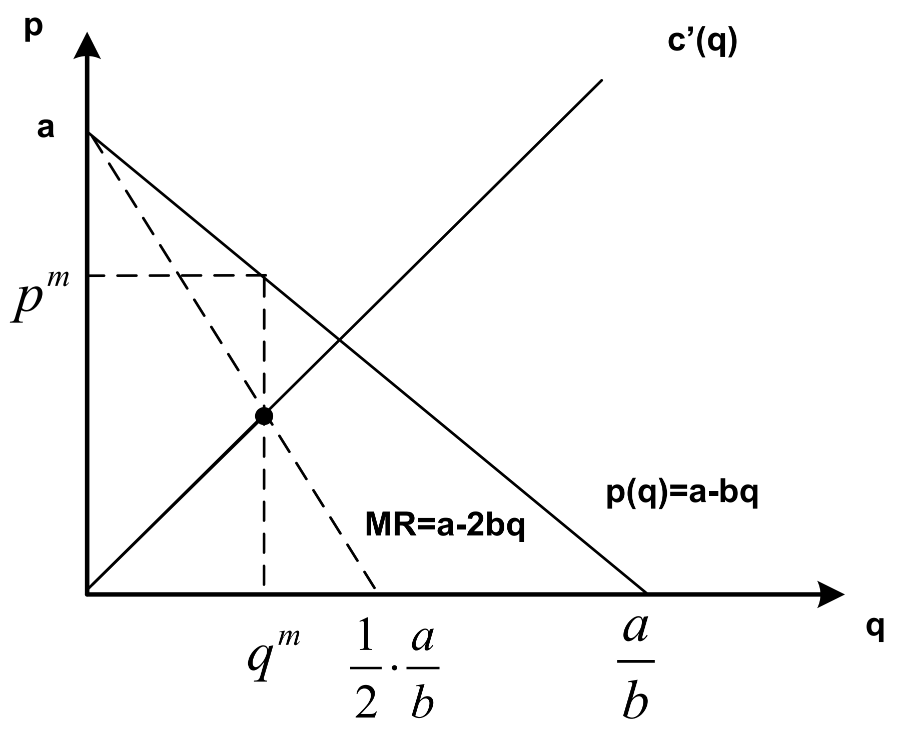
Profit Maximization: Lerner Index

- ***Example 1*** (continued):



Profit Maximization: Lerner Index

- ***Example 1*** (continued):
  - Non-constant marginal cost
  - The cost function is convex in output
    $$c(q) = cq^2$$
  - Marginal cost is
    $$c'(q) = 2cq$$



Profit Maximization: Lerner Index

- ***Example 2*** (Constant elasticity demand):
  - The demand function is

$$q(p) = Ap^{-b}$$

  - We can show that $\varepsilon(q) = -b$ for all q, i.e.,

$$\varepsilon(q) = \frac{\partial q(p)}{\partial p}\frac{p}{q} = \underbrace{(-b)Ap^{-b-1}}_{\frac{\partial q(p)}{\partial p}}\underbrace{\frac{p}{Ap^{-b}}}_{\frac{p}{q}}$$

$$= -b\frac{p^{-b}}{p}\frac{p}{p^{-b}} = -b$$

Profit Maximization: Lerner Index

- ***Example 2*** (continued):
  - We can now plug $\varepsilon(q) = -b$ into the Lerner index,

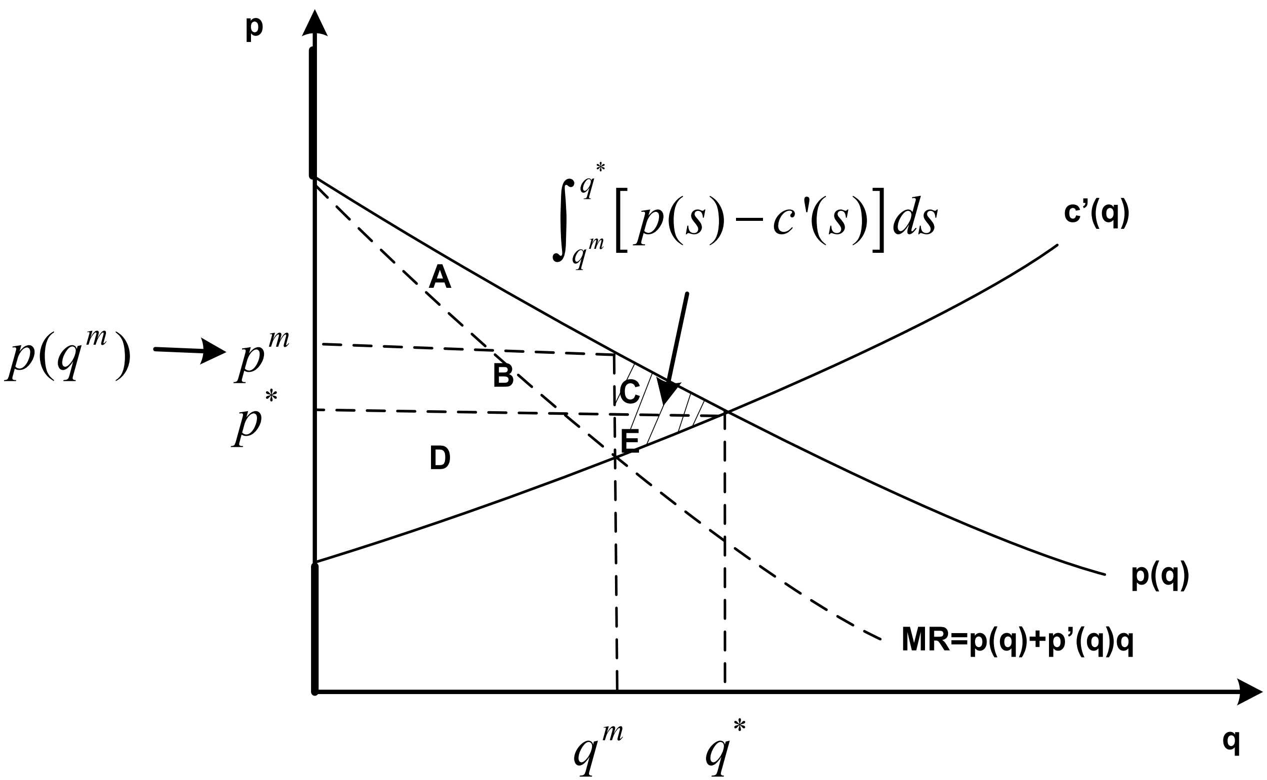
$$p^m = \frac{c}{1 + \frac{1}{\varepsilon(q)}} = \frac{c}{1 - \frac{1}{b}} = \frac{bc}{b - 1}$$

  - That is, price is a constant mark-up over marginal cost.

Welfare Loss of Monopoly

Welfare Loss of Monopoly

- Welfare comparison for perfect competition and monopoly.



Welfare Loss of Monopoly

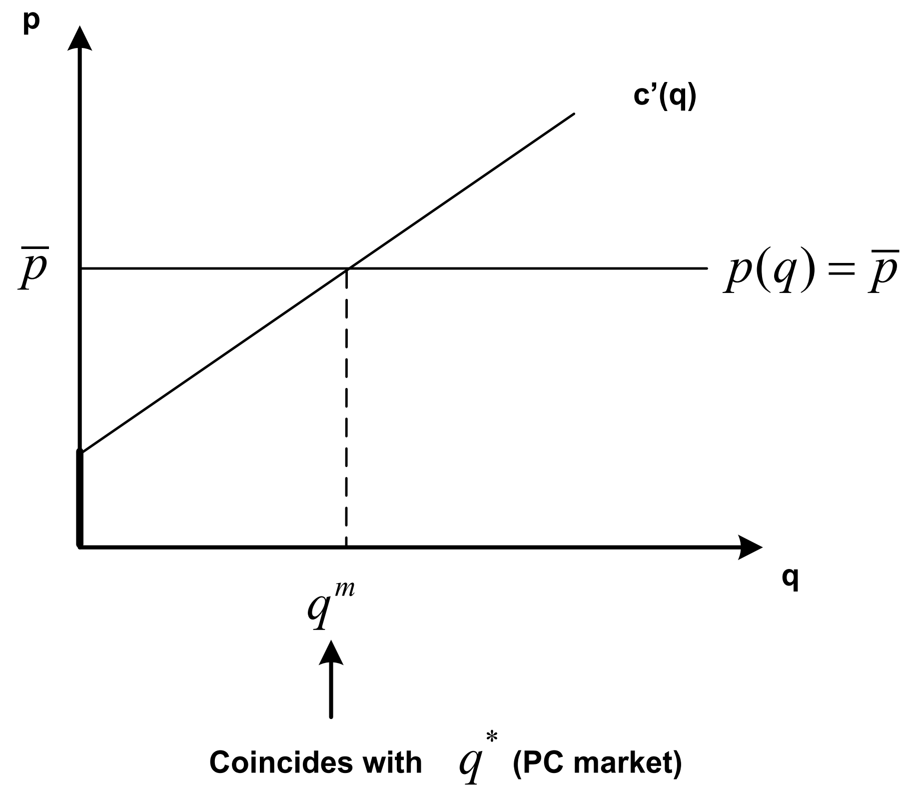
- Consumer surplus
  - Perfect competition: *A*+*B*+*C*
  - Monopoly: *A*
- Producer surplus:
  - Perfect competition: *D*+*E*
  - Monopoly: *B*+*D*
- Deadweight loss of monopoly: *C*+*E*

$$DWL = \int_{q^m}^{q^*} [p(s) - c'(s)]ds$$

- DWL decreases as demand and/or supply become more elastic.

Welfare Loss of Monopoly

- Infinitely elastic demand
$$p'(q) = 0$$
- The inverse demand curve becomes totally flat.
- Marginal revenue coincides with inverse demand:
$$MR(q) = p(q) + 0 \cdot q$$
$$= p(q)$$
- Profit-maximizing q
$$MR(q) = MC(q) \Longrightarrow$$
$$p(q) = MC(q)$$
- Hence, $q^m = q^*$ and $DWL = 0$.



Welfare Loss of Monopoly

- ***Example*** (Welfare losses and elasticity):
  - Consider a monopolist with constant marginal and average costs, $c'(q) = c$, who faces a market demand with constant elasticity
    $$q(p) = p^e$$
    where e is the price elasticity of demand ($e < -1$)
  - Perfect competition: $p_c = c$
  - Monopoly: using the IEPR
    $$p^m = \frac{c}{1 + \frac{1}{e}}$$

Welfare Loss of Monopoly

- ***Example*** (continued):
  - The consumer surplus associated with any price (p_0) can be computed as

$$CS = \int_{p_0}^{\infty} Q(P)dp = \int_{p_0}^{\infty} p^e dp = \left.\frac{p^{e+1}}{e+1}\right|_{p_0}^{\infty} = -\frac{p_0^{e+1}}{\underbrace{e+1}_{<0}} > 0$$

  - Under perfect competition, $p_c = c$,

$$CS = -\frac{c^{e+1}}{e+1}$$

  - Under monopoly, $p^m = \frac{c}{1+1/e}$,

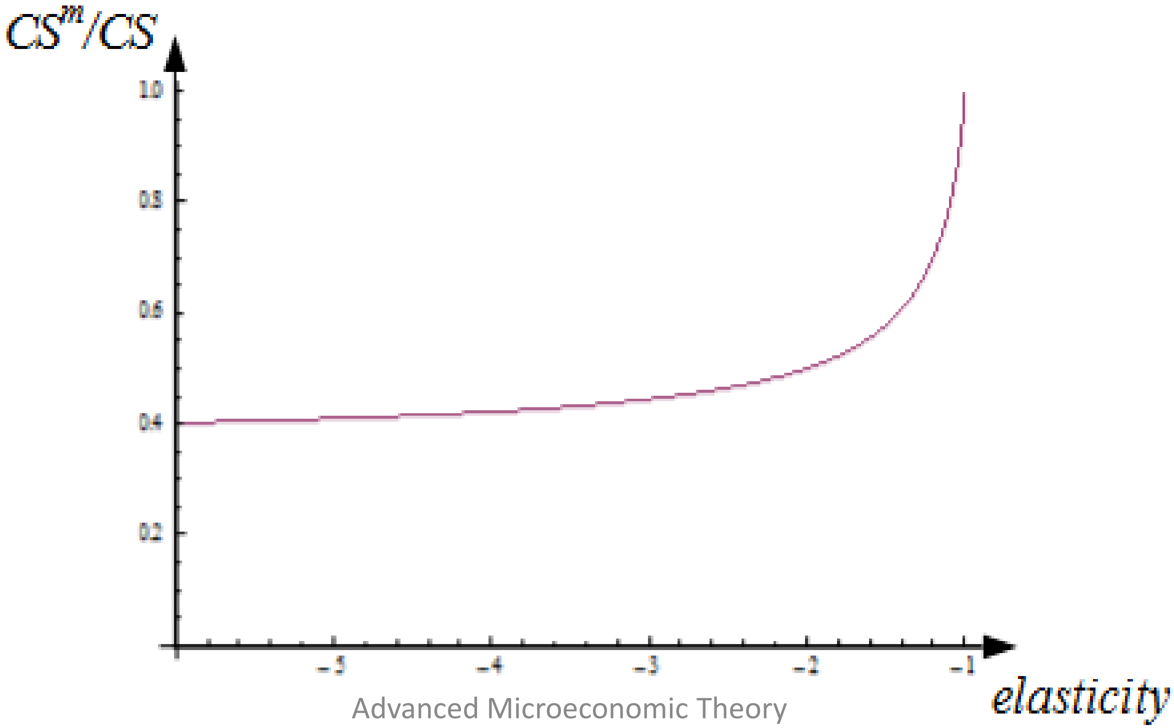
$$CS_m = -\frac{\left(\frac{c}{1+1/e}\right)^{e+1}}{e+1}$$

Welfare Loss of Monopoly

- ***Example*** (continued):
  - Taking the ratio of these two surpluses
  $$\frac{CS_m}{CS} = \left(\frac{1}{1 + 1/e}\right)^{e+1}$$
  - If $e = -2$, this ratio is ½
    - CS under monopoly is half of that under perfectly competitive markets

Welfare Loss of Monopoly

- ***Example*** (continued):
  - The ratio $\frac{CS_m}{CS} = \left(\frac{1}{1+1/e}\right)^{e+1}$ decreases as demand becomes more elastic (e increases in absolute value).



Welfare Loss of Monopoly

- ***Example*** (continued):
  - Monopoly profits are given by

$$\pi^m = p^m q^m - cq^m = \left(\frac{c}{1+1/e} - c\right) q^m$$

where $q^m(p) = p^e = \left(\frac{c}{1+1/e}\right)^e$.

  - Rearranging,

$$\pi^m = \left(\frac{-c/e}{1+1/e}\right)\left(\frac{c}{1+1/e}\right)^e$$

$$= -\left(\frac{c}{1+1/e}\right)^{e+1} \cdot \frac{1}{e}$$

Welfare Loss of Monopoly

- ***Example*** (continued):
  - To find the transfer from CS into monopoly profits that consumers experience when moving from perfect competition to a monopoly, divide monopoly profits by the competitive CS

$$\frac{\pi^m}{CS} = \left(\frac{e+1}{e}\right)\left(\frac{1}{1+1/e}\right)^{e+1} = \left(\frac{e}{1+e}\right)^{e}$$

  - If $e = -2$, this ratio is ¼
    - One-fourth of the consumer surplus under perfectly competitive markets is transferred to monopoly profits

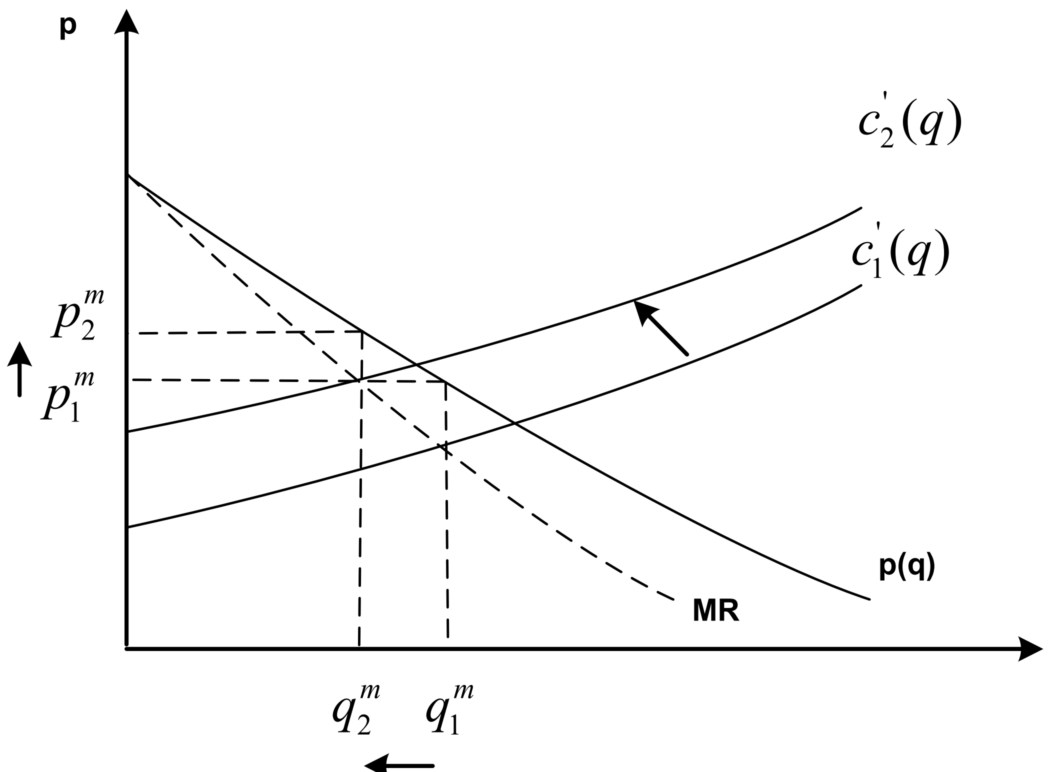
Welfare Loss of Monopoly

- More social costs of monopoly:
  - Excessive R&D expenditure (patent race)
  - Persuasive (not informative) advertising
  - Lobbying costs (different from bribes)
  - Resources to avoid entry of potential firms in the industry

Comparative Statics

Comparative Statics

- We want to understand how q^m varies as a function of the monopolist's marginal cost



Comparative Statics

- Formally, we know that at the optimum, $q^m(c)$, the monopolist maximizes its profits

$$\frac{\partial \pi(q^m(c), c)}{\partial q^m} = 0$$

- Differentiating wrt c, and using the chain rule,

$$\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2} \frac{dq^m(c)}{dc} + \frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = 0$$

- Solving for $\frac{dq^m(c)}{dc}$, we have

$$\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}}$$

Comparative Statics

- ***Example***:
  - Assume linear demand curve $p(q) = a - bq$
  - Then, the cross-derivative is

$$\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = \frac{\partial \left(\frac{\partial[(a - bq)q - cq]}{\partial q} \right)}{\partial c}$$

$$= \frac{\partial[a - 2bq - c]}{\partial c} = -1$$

and

$$\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}} = -\frac{-1}{-2b} < 0$$

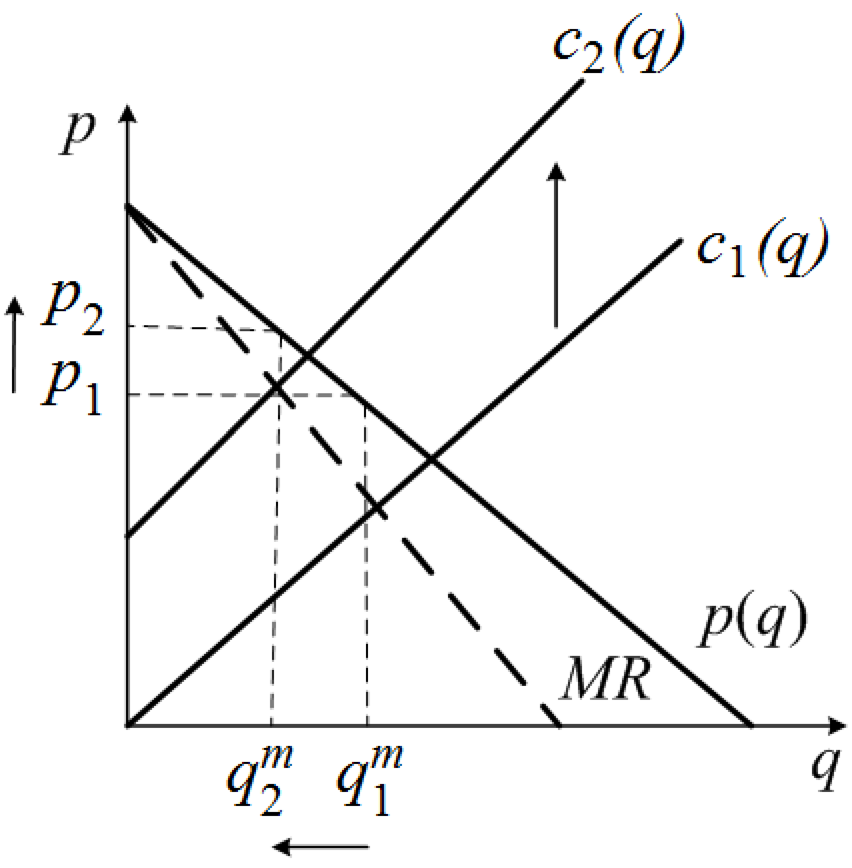
Comparative Statics

- ***Example*** (continued):
  - That is, an increase in marginal cost, c, decreases monopoly output, q^m.
  - Similarly for any other demand.
  - Even if we don't know the accurate demand function, but know the sign of

$$\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}$$

Comparative Statics

- ***Example*** (continued):
  - Marginal costs are increasing in q
  - For convex cost curve $c(q) = cq^2$, monopoly output is
    $$q^m(c) = \frac{a}{2(b+c)}$$
  - Here,
    $$\frac{dq^m(c)}{dc} = -\frac{a}{2(b+c)^2} < 0$$

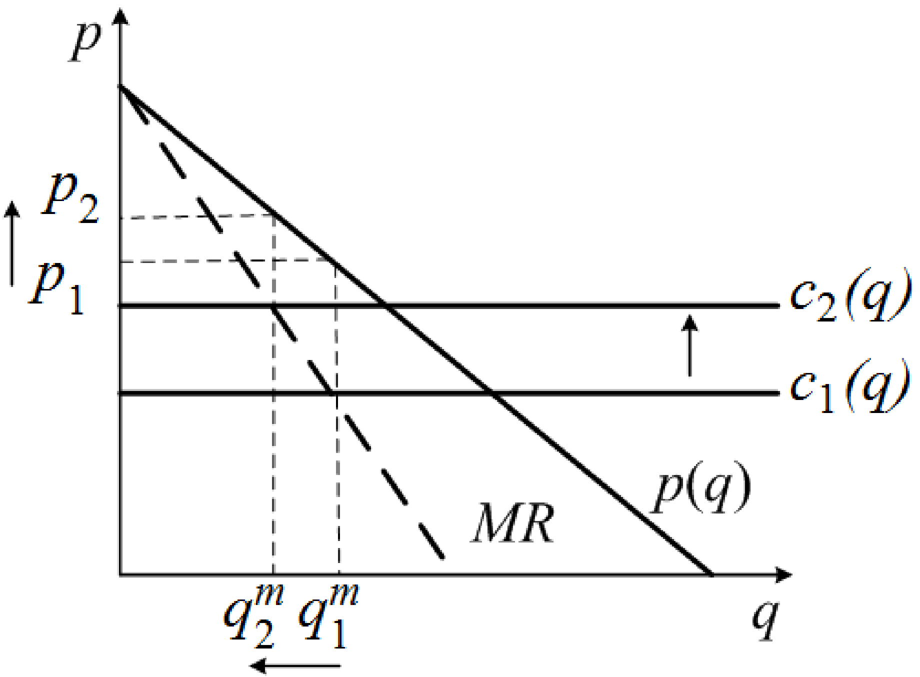


Comparative Statics

- ***Example*** (continued):
  - Constant marginal cost
  - For the constant-elasticity demand curve $q(p) = p^e$, we have $p^m = \frac{c}{1+1/e}$ and $q^m(c) = \left(\frac{ec}{1+e}\right)^e$
  - Here,

$$\frac{dq^m(c)}{dc} = \frac{e}{c}\left(\frac{ec}{1+e}\right)^e$$

$$= \frac{e}{c}q^m < 0$$



Multiplant Monopolist

Multiplant Monopolist

- Monopolist produces output $q_1, q_2, \ldots, q_N$ across N plants it operates, with total costs $TC_i(q_i)$ at each plant $i = \{1,2, \ldots, N\}$.
- Profits-maximization problem

$$\max_{q_1,\ldots,q_N} \left[a - b\sum_{i=1}^{N} q_i\right] \sum_{i=1}^{N} q_i - \sum_{i=1}^{N} TC_i(q_i)$$

- FOCs wrt production level at every plant j

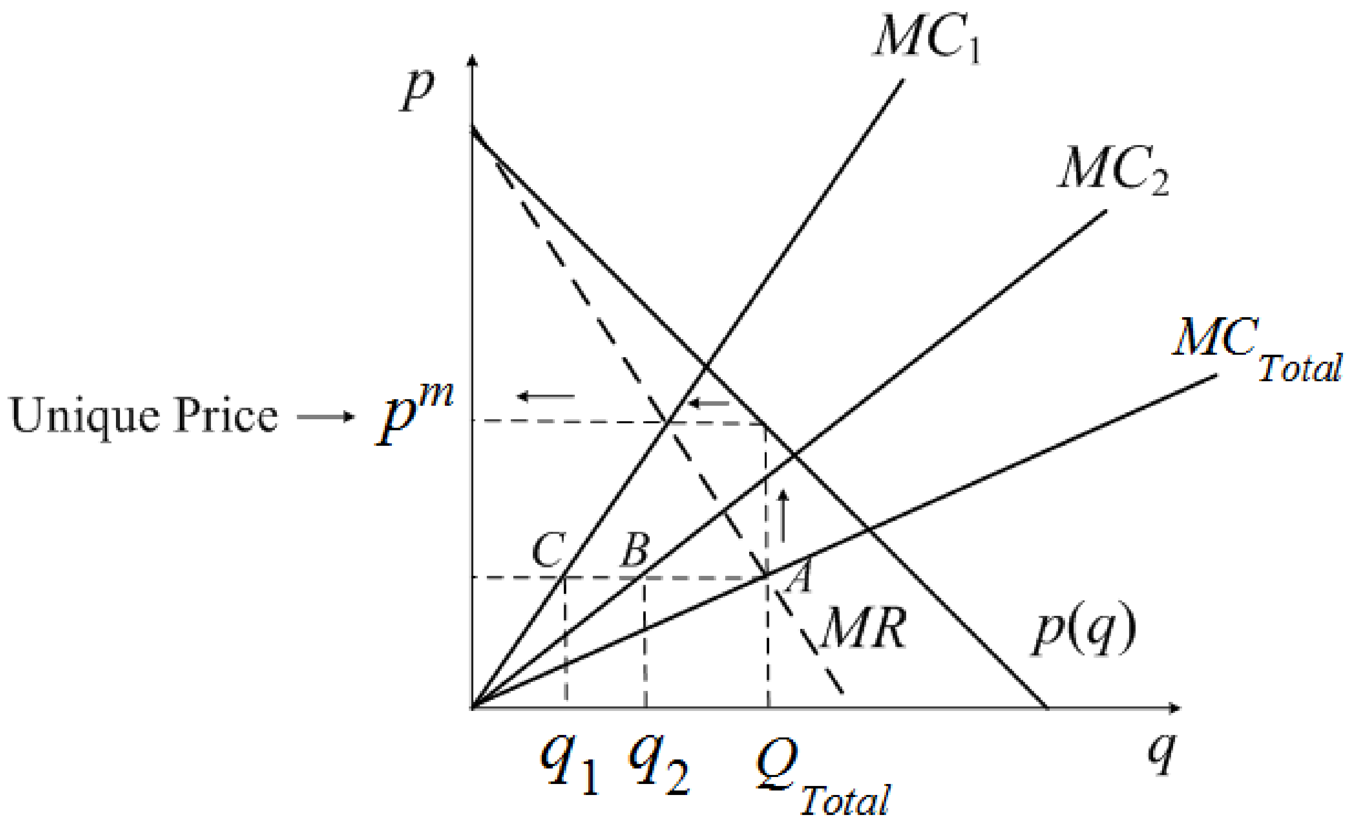
$$a - 2b\sum_{i=1}^{N} q_i - MC_j(q_j) = 0$$

$$\Leftrightarrow MR(Q) = MC_j(q_j)$$

for all j.

Multiplant Monopolist

- Multiplant monopolist operating two plants with marginal costs MC_1 and MC_2.



Multiplant Monopolist

- Total marginal cost is $MC_{total} = MC_1 + MC_2$ (i.e., horizontal sum)
- Q_{total} is determined by $MR = MC_{total}$ (i.e., point *A*)
- Mapping Q_{total} in the demand curve, we obtain price p^m (both plants selling at the same price)
- At the MC level for which $MR = MC_{total}$ (i.e., point *A*), extend a line to the left crossing MC_1 and MC_2.
- This will give us output levels q_1 and q_2 that plants 1 and 2 produce, respectively.

Multiplant Monopolist

- ***Example 1*** (symmetric plants):
  - Consider a monopolist operating N plants, where all plants have the *same* cost function $TC_i(q_i) = F + cq_i^2$. Hence, all plants produce the same output level $q_1 = q_2 = \cdots = q_N = q$ and $Q = Nq_j$. The linear demand function is given by $p = a - bQ$.
  - FOCs:

$$a - 2b\sum_{j=1}^{N} q_j = 2cq_j \quad \text{or} \quad a - 2bNq_j = 2cq_j$$

$$q_j = \frac{a}{2(bN + c)}$$

Multiplant Monopolist

- ***Example 1*** (continued):
  - Total output produced by the monopolist is

$$Q = Nq_j = \frac{Na}{2(bN + c)}$$

    and market price is

$$p = a - bQ = a - b\frac{Na}{2(bN + c)} = \frac{a(bN + 2c)}{2(bN + c)}$$

  - Hence, the profits of every plant *j* are

$$\pi_j = \left(\frac{a(bN + 2c)}{2(bN + c)}\right)\frac{a}{2(bN + c)} - c\left(\frac{a}{2(bN + c)}\right)^2 = \frac{a^2}{4(bN + c)} - F$$

  - Total profits become

$$\pi_{total} = \frac{Na^2}{4(bN + c)} - NF$$

Multiplant Monopolist

- ***Example 1*** (continued):
  - The optimal number of plants N^* is determined by

$$\frac{d\pi_{total}}{dN} = \frac{a^2}{4}\frac{c}{(bN + c)^2} - F = 0$$

  and solving for N

$$N^* = \frac{1}{b}\left(\frac{a}{2}\sqrt{\frac{c}{F}} - c\right)$$

  - N^* is decreasing in the fixed costs F
  - N^* is decreasing in c as long as $a < 4\sqrt{cF}$, since

$$\frac{dN^*}{dc} = \frac{1}{b}\left(\frac{a - 4\sqrt{CF}}{4\sqrt{CF}}\right)$$

Multiplant Monopolist

- ***Example 1*** (continued):
  - Note that when $N = 1$, $Q = q^m$ and $p = p^m$.
  - Note that an increase in N decreases q_j and π_j, as

$$\frac{dq_j}{dN} = -\frac{ab}{2(bN + c)^2} < 0$$

$$\frac{d\pi_j}{dN} = -\frac{a^2 b}{4(bN + c)^2} < 0$$

Multiplant Monopolist

- ***Example 2*** (asymmetric plants):
  - Consider a monopolist operating two plants with marginal costs $MC_1(q_1) = 10 + 20q_1$ and $MC_2(q_2) = 60 + 5q_2$, respectively. A linear demand function is give by $p(Q) = 120 - 3Q$.
  - Note that $MC_{total} \neq MC_1(q_1) + MC_2(q_2)$
    - This is a vertical (not a horizontal) sum.
  - Instead, first invert the marginal cost functions

$$MC_1(q_1) = 10 + 20q_1 \Leftrightarrow q_1 = \frac{MC_1}{20} - \frac{1}{2}$$

$$MC_2(q_2) = 60 + 5q_2 \Leftrightarrow q_2 = \frac{MC_2}{5} - 12$$

Multiplant Monopolist

- ***Example 2*** (continued):
  - Second,

$$Q_{total} = q_1 + q_2 = \frac{MC_{total}}{20} - \frac{1}{2} + \frac{MC_{total}}{5} - 12$$
$$= \frac{1}{4} MC_{total} - 12.5$$

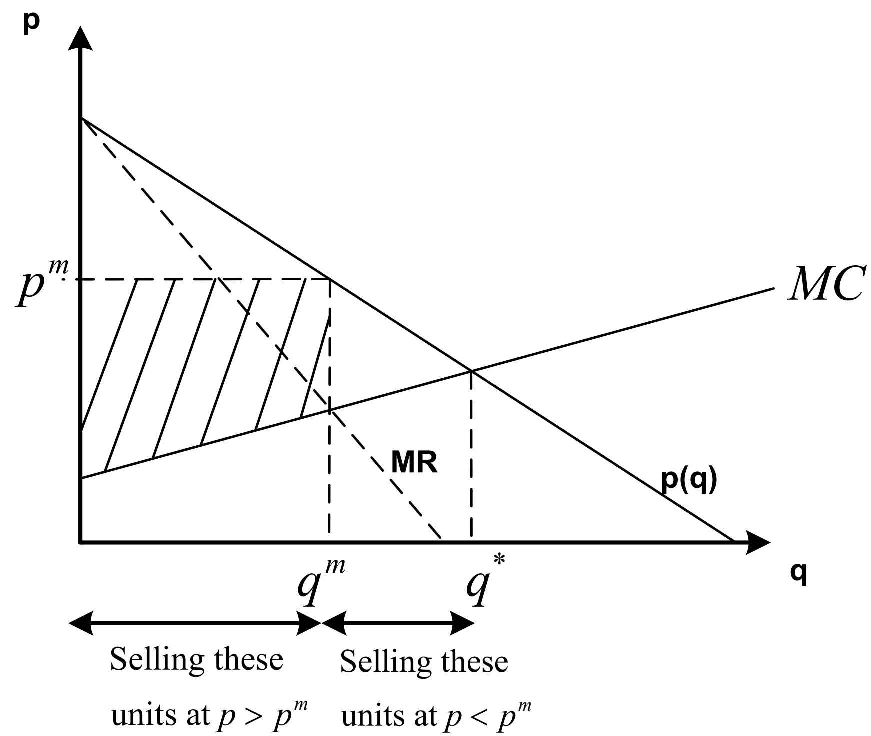
  - Hence, $MC_{total} = 50 + 4Q_{total}$
  - Setting $MR(Q) = 120 - 6Q = 50 + 4Q = MC_{total}$, we obtain $Q_{total} = 7$ and $p = 120 - 3 \cdot 7 = 99$.
  - Since $MR(Q_{total}) = 120 - 6 \cdot 7 = 78$, then

$$MR(Q_{total}) = MC_1(q_1) \Longrightarrow 78 = 10 + 20q_1 \Longrightarrow q_1 = 3.4$$
$$MR(Q_{total}) = MC_2(q_2) \Longrightarrow 78 = 60 + 5q_2 \Longrightarrow q_2 = 3.6$$

Price Discrimination

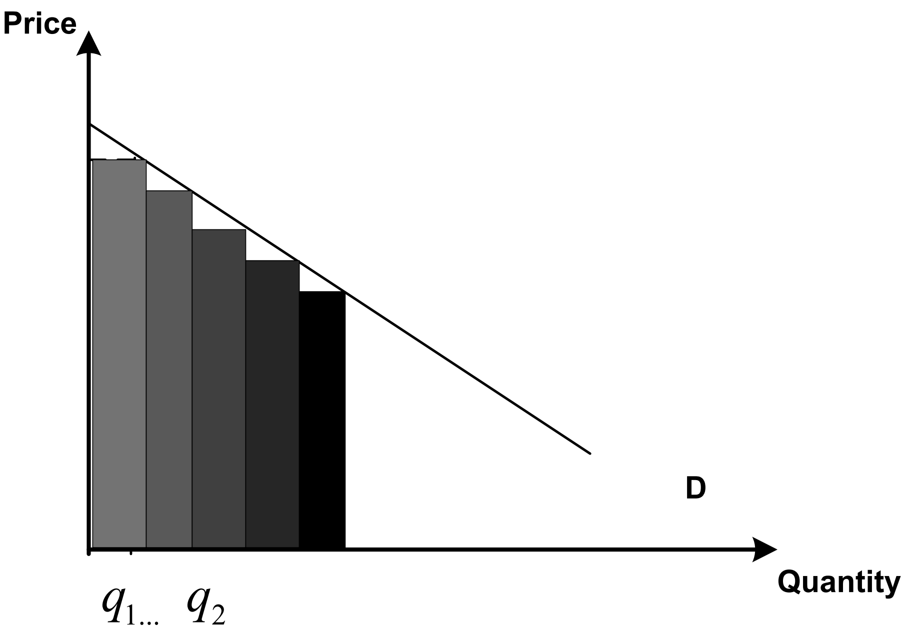
Price Discrimination

- Can the monopolist capture an even larger surplus?
  - Charge $p > p^m$ to those who buy the product at p^m and are willing to pay more
  - Charge $c < p < p^m$ to those who do not buy the product at p^m, but whose willingness to pay for the good is still higher than the marginal cost of production, c.
  - With p^m for all units, the monopolist does not capture the surplus of neither of these segments.



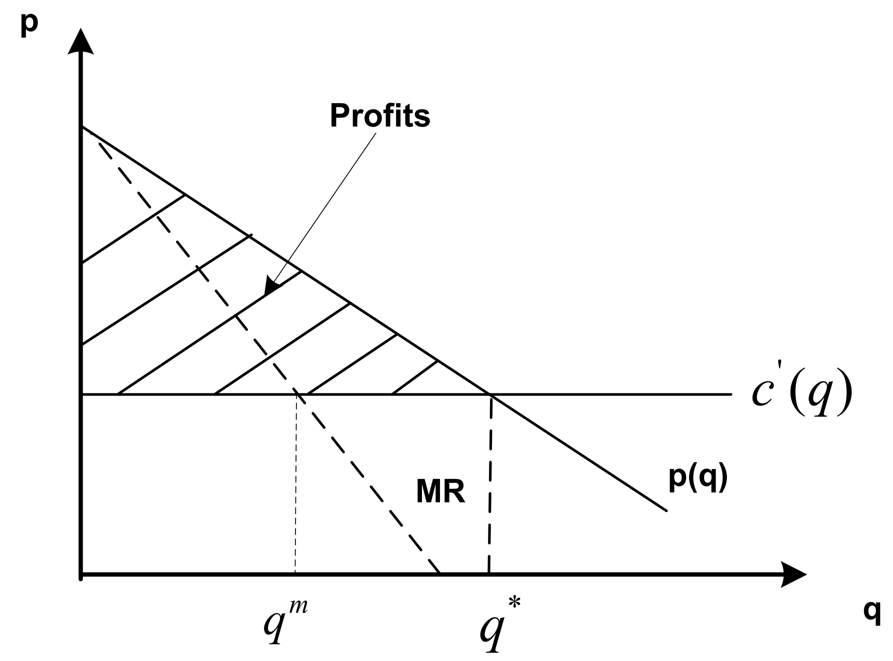
Price Discrimination: First-degree

- ***First-degree (perfect) price discrimination***:
  - The monopolist charges to every customer his/her maximum willingness to pay for the object.
  - *Personalized price*: The first buyer pays p_1 for the q_1 units, the second buyer pays p_2 for $q_2 - q_1$ units, etc.



Price Discrimination: First-degree

- The monopolist continues doing so until the last buyer is willing to pay the marginal cost of production.
- In the limit, the monopolist captures all the area below the demand curve and above the marginal cost (i.e., consumer surplus).



Price Discrimination: First-degree

- Suppose that the monopolist can charge a fixed fee, r^*, and an amount of the good, q^*, that maximizes profits.
- PMP:

$$\max_{r,q} \; r - cq$$
$$s.t. \; u(q) \geq r$$

- Note that the monopolist raises the fee r until $u(q) = r$. Hence we can reduce the set of choice variables

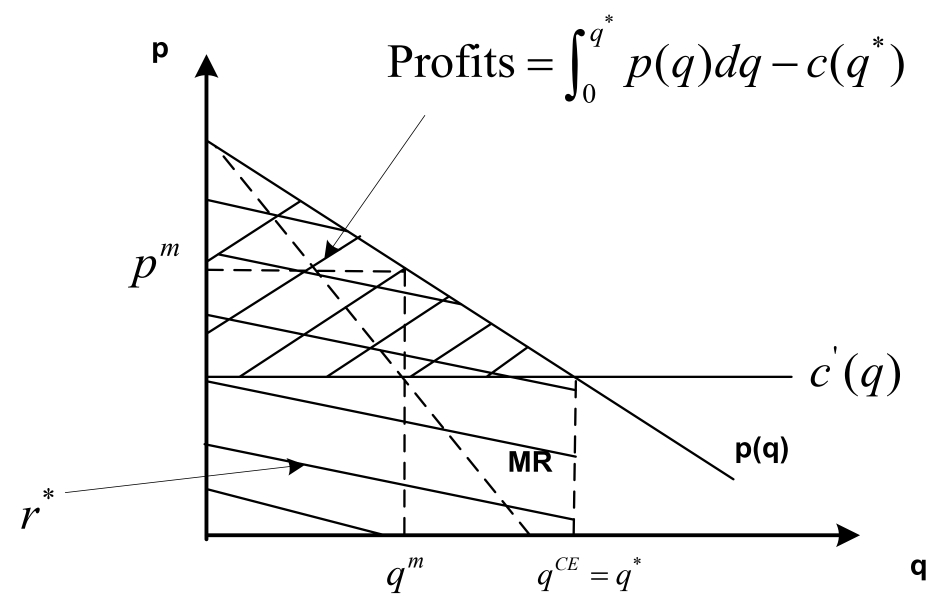
$$\max_{q} \; u(q) - cq$$

- FOC: $u'(q^*) - c = 0$ or $u'(q^*) = c$.
  - *Intuition*: the monopolist increases output until the marginal utility that consumers obtain from additional units coincides with the marginal cost of production

Price Discrimination: First-degree

- Given the level of production q^*, the optimal fee is
$$r^* = u(q^*)$$

- *Intuition*: the monopolist charges a fee r^* that coincides with the utility that the consumer obtains from consuming q^*.



Price Discrimination: First-degree

- ***Example***:
  - A monopolist faces inverse demand curve $p(q) = 20 - q$ and constant marginal costs $c = \$2$.
  - No price discrimination:

$$MR = MC \implies 20 - 2q = 2 \implies q^m = 9$$

$$p^m = \$11, \qquad \pi^m = \$81$$

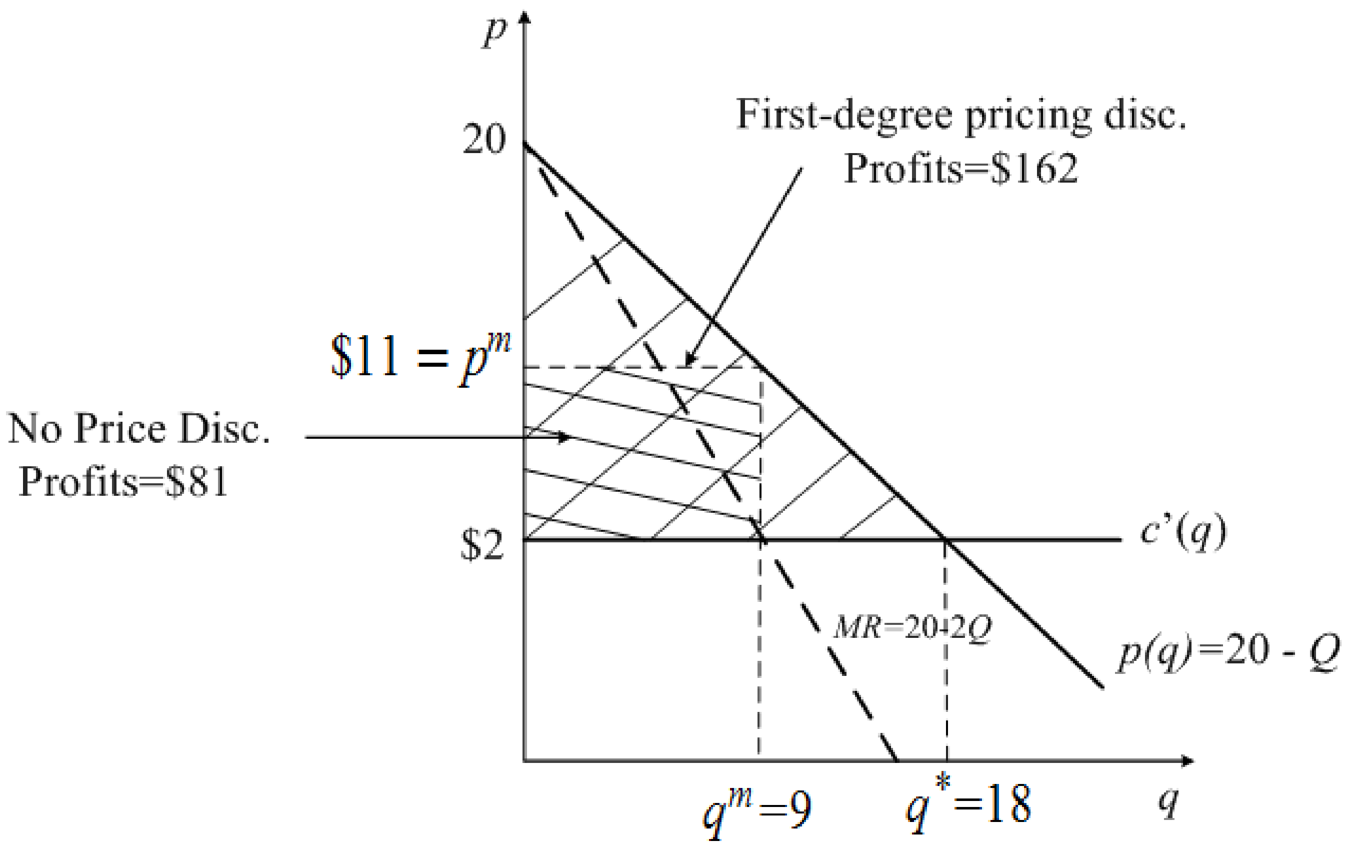
  - Price discrimination:

$$p(Q) = MC \implies 20 - Q = 2 \implies Q = 18$$

$$\pi = \frac{18 \times (20 - 2)}{2} = \$162$$

Price Discrimination: First-degree

- ***Example*** (continued):

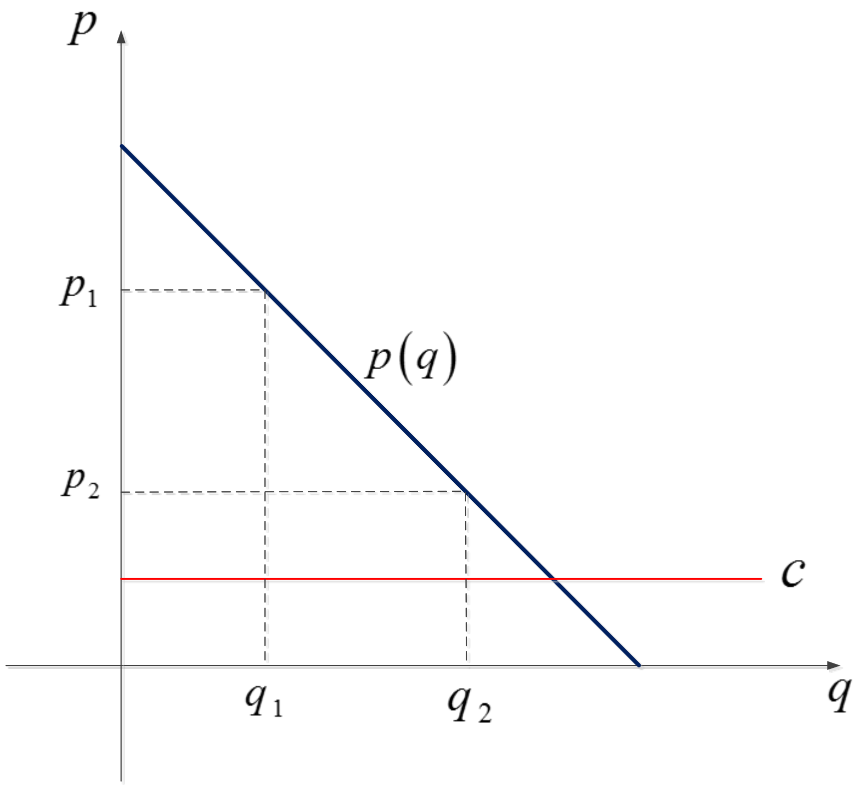


Price Discrimination: First-degree

- Summary:
  - Total output coincides with that in perfect competition
  - Unlike in perfect competition, the consumer does not capture any surplus
  - The producer captures all the surplus
  - Due to information requirements, we do not see many examples of it in real applications
    - Financial aid in undergraduate education (“tuition discrimination”)

Price Discrimination: First-degree

- ***Example*** (two-block pricing):
  - A monopolist faces a inverse demand curve $p(q) = a - bq$, with constant marginal costs $c < a$.
  - Under two-block pricing, the monopolist sells the first q_1 units at a price $p(q_1) = p_1$ and the remaining $q_2 - q_1$units at a price $p(q_2) = p_2$.



Price Discrimination: First-degree

- ***Example*** (continued):
  - Profits from the first q_1 units
    $$\pi_1 = p(q_1) \cdot q_1 - cq_1 = (a - bq_1 - c)q_1$$
    while from the remaining $q_2 - q_1$ units
    $$\pi_2 = p(q_2) \cdot (q_2 - q_1) - c \cdot (q_2 - q_1)$$
    $$= (a - bq_2 - c)(q_2 - q_1)$$
  - Hence total profits are
    $$\pi = \pi_1 + \pi_2$$
    $$= (a - bq_1 - c)q_1 + (a - bq_2 - c)(q_2 - q_1)$$

Price Discrimination: First-degree

- ***Example*** (continued):
  - FOCs:

$$\frac{\partial \pi}{\partial q_1} = a - 2bq_1 - c - a + bq_2 + c = 0$$

$$\frac{\partial \pi}{\partial q_2} = -b(q_2 - q_1) + a - bq_2 - c = 0$$

  - Solving for q_1 and q_2

$$q_1 = \frac{a-c}{3b} \qquad q_2 = \frac{2(a-c)}{3b}$$

which entails prices of

$$p(q_1) = a - b \cdot \frac{a-c}{3b} = \frac{2a+c}{3} \qquad p(q_2) = \frac{a+2c}{3}$$

where $p(q_1) > p(q_2)$ since $a > c$.

Price Discrimination: First-degree

- ***Example*** (continued):
  - The monopolist's profits from each block are

$$\pi_1 = (p(q_1) - c) \cdot q_1$$
$$= \left(\frac{2a + c}{3} - c\right) \cdot \frac{a - c}{3b} = \frac{2}{b}\left(\frac{a - c}{3}\right)^2$$

$$\pi_2 = (p(q_2) - c)(q_2 - q_1)$$
$$= \left(\frac{a + 2c}{3} - c\right) \cdot \left(\frac{2(a - c)}{3b} - \frac{a - c}{3b}\right) = \frac{1}{b}\left(\frac{a - c}{3}\right)^2$$

  - Thus, $\pi = \pi_1 + \pi_2 = \frac{(a-c)^2}{3b}$, which is larger than those arising from uniform pricing , $\pi^u = \frac{(a-c)^2}{4b}$.

Price Discrimination: Third-degree

- ***Third degree price discrimination***:
  - The monopolist charges different prices to two or more groups of customers (each group must be easily recognized by the seller).
    - *Example*: youth vs. adult at the movies, airline tickets
  - Firm's PMP:

$$\max_{x_1, x_2} \; p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2$$

  - FOCs:

$$p_1(x_1) + p_1'(x_1)x_1 - c = 0 \;\Longrightarrow\; MR_1 = MC$$
$$p_2(x_2) + p_2'(x_2)x_2 - c = 0 \;\Longrightarrow\; MR_2 = MC$$

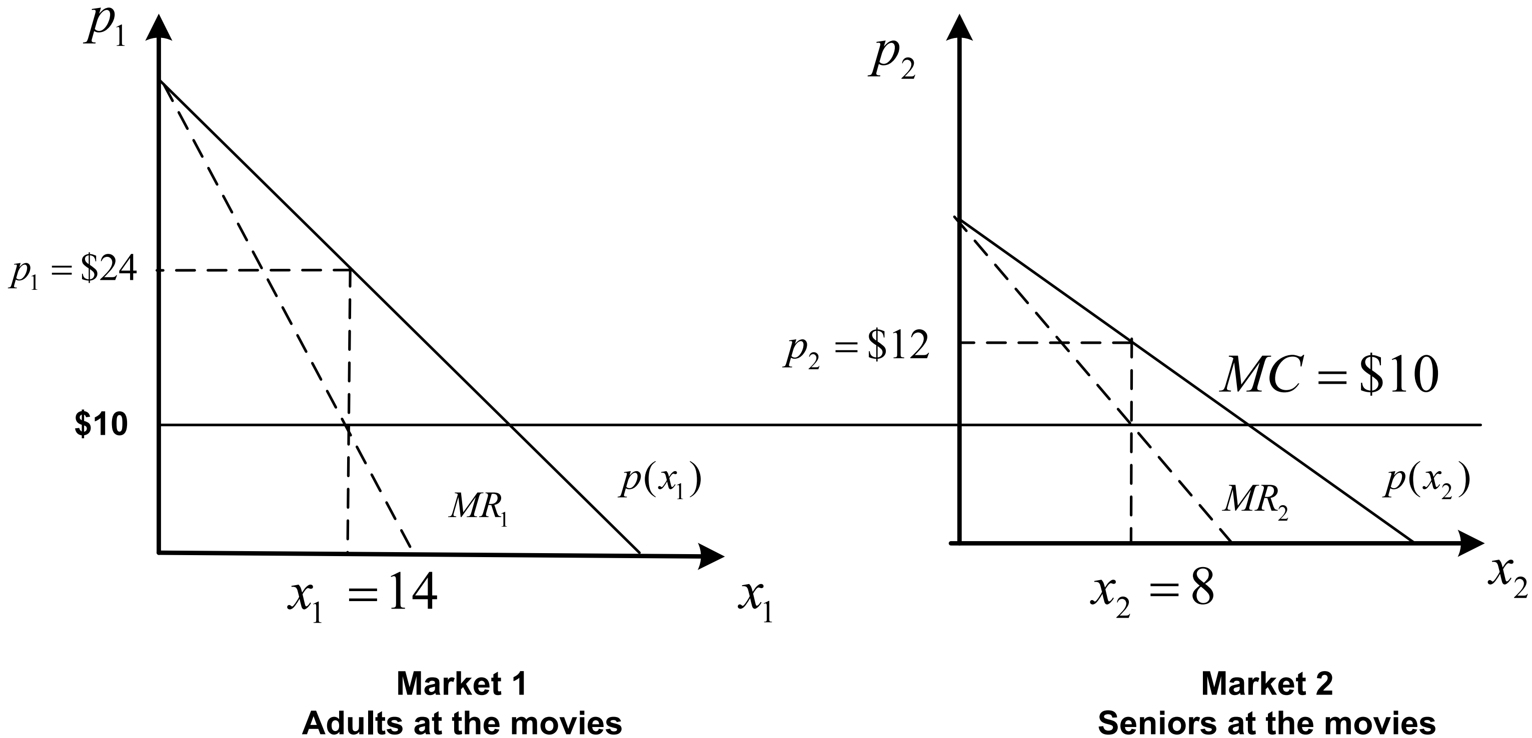
  - FOCs coincides with those of a regular monopolist who serves two completely separated markets practicing uniform pricing .

Price Discrimination: Third-degree

– ***Example***: $p_1(x_1) = 38 - x_1$ for adults and $p_2(x_2) = 14 - 1/4x_2$ for seniors, with $MC = \$10$ for both markets.

$$MR_1(x_1) = MC \implies 38 - 2x_1 = 10 \implies x_1 = 14 \quad p_1 = \$24$$

$$MR_2(x_2) = MC \implies 14 - 1/2x_2 = 10 \implies x_2 = 8 \quad p_2 = \$12$$



Price Discrimination: Third-degree

- Using the Inverse Elasticity Pricing Rule (IERP), we can obtain the prices

$$p_1(x_1) = \frac{c}{1-1/\varepsilon_1} \text{ and } p_2(x_2) = \frac{c}{1-1/\varepsilon_2}$$

  where c is the common marginal cost
- Then, $p_1(x_1) > p_2(x_2)$ if and only if

$$\frac{c}{1-1/\varepsilon_1} > \frac{c}{1-1/\varepsilon_2} \Longrightarrow 1 - \frac{1}{\varepsilon_2} < 1 - \frac{1}{\varepsilon_1}$$

$$\Longrightarrow \frac{1}{\varepsilon_2} > \frac{1}{\varepsilon_1} \Longrightarrow \varepsilon_2 < \varepsilon_1$$

- *Intuition*: the monopolist charges lower price in the market with more elastic demand.

Price Discrimination: Third-degree

- ***Example*** (Pullman-Seattle route):
  - The price-elasticity of demand for business-class seats is -1.15, while that for economy seats is -1.52
  - From the IEPR,

$$p_B = \frac{MC}{1 - 1/1.15} \implies 0.13 p_B = MC$$
$$p_E = \frac{MC}{1 - 1/1.52} \implies 0.34 p_E = MC$$

  - Hence, $0.13 p_B = 0.34 p_E$ or $p_B = 2.62 p_E$
    - Airline maximizes its profits by charging business-class seats a price 2.62 times higher than that of economy-class seats

Price Discrimination: Second-degree

- ***Second-degree price discrimination***:
  - The monopolist cannot observe the type of each consumer (e.g., his willingness to pay).
  - Hence the monopolist offers a menu of two-part tariffs, (F_L, q_L) and (F_H, q_H), with the property that the consumer with type $i = \{L, H\}$ has the incentive to self-select the two-part tariff (F_i, q_i) meant for him.

Price Discrimination: Second-degree

- Assume the utility function of type i consumer
$$U_i(q_i, F_i) = \theta_i u(q_i) - F_i$$
where
  - q_i is the quantity of a good consumed
  - F_i is the fixed fee paid to the monopolist for q_i
  - θ_i measures the valuation consumer assigns to the good, where $\theta_H > \theta_L$, with corresponding probabilities p and $1 - p$.
- The monopolist's constant marginal cost c satisfies $\theta_i > c$ for all $i = \{L, H\}$.

Price Discrimination: Second-degree

- The monopolist must guarantee that
  1) both types of customers are willing to participate ("***participation constraint***")
     - the two-part tariff meant for each type of customer provides him with a weakly positive utility level
  2) customers do not have incentives to choose the two-part tariff meant for the other type of customer ("***incentive compatibility***")
     - type i customer prefers (F_i, q_i) over (F_j, q_j) where $j \neq i$

Price Discrimination: Second-degree

- The participation constraints (PC) are

$$\theta_L u(q_L) - F_L \geq 0 \qquad PC_L$$

$$\theta_H u(q_H) - F_H \geq 0 \qquad PC_H$$

- The incentive compatibility conditions are

$$\theta_L u(q_L) - F_L \geq \theta_L u(q_H) - F_H \qquad IC_L$$

$$\theta_H u(q_H) - F_H \geq \theta_H u(q_L) - F_L \qquad IC_H$$

Price Discrimination: Second-degree

- Rearranging the four inequalities, the monopolist's profit maximization problem becomes:

$$\max_{F_L, q_L, F_H, q_H} \quad p[F_H - cq_H] + (1-p)[F_L - cq_L]$$

$$\theta_L u(q_L) \geq F_L$$

$$\theta_H u(q_H) \geq F_H$$

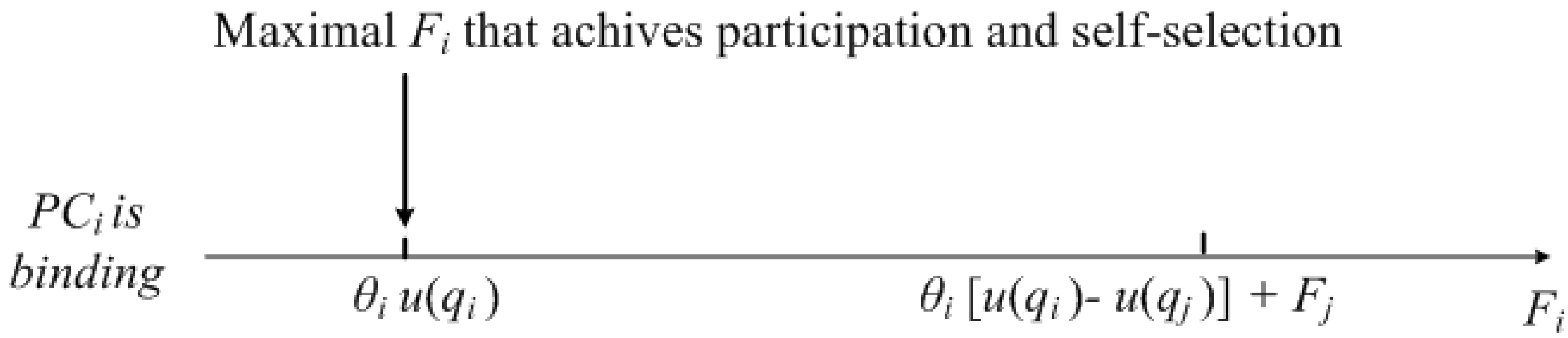
$$\theta_L[u(q_L) - u(q_H)] + F_H \geq F_L$$

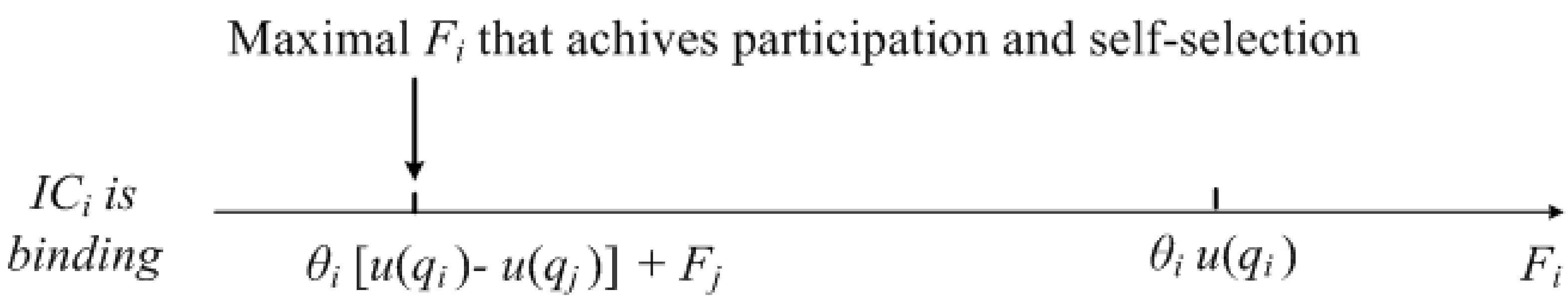
$$\theta_H[u(q_H) - u(q_L)] + F_L \geq F_H$$

Price Discrimination: Second-degree

- Both PC_H and IC_H are expressed in terms of the fee F_H
  - The monopolist increases F_H until such fee coincides with the lowest of $\theta_H u(q_H)$ and $\theta_H[u(q_H) - u(q_L)] + F_L$ for all $i = \{L, H\}$
  - Otherwise, one (or both) constraints will be violated, leading the high-demand customer to not participate

Price Discrimination: Second-degree





Price Discrimination: Second-degree

- ***High-demand customer***:
  - Let us show that IC_H is binding
  - An indirect way to show that
    $$F_H = \theta_H[u(q_H) - u(q_L)] + F_L$$
    is to demonstrate that $F_H < \theta_H u(q_H)$
  - Proving this by contradiction, assume that
    $$F_H = \theta_H u(q_H)$$

Price Discrimination: Second-degree

– Then, IC_H can be written as

$$F_H - \theta_H u(q_L) + F_L \geq F_H$$
$$\Longrightarrow F_L \geq \theta_H u(q_L)$$

– Combining this result with the fact that $\theta_H > \theta_L$,

$$F_L \geq \theta_H u(q_L) > \theta_L u(q_L)$$

which implies $F_L > \theta_L u(q_L)$

– However, this violates PC_L

- We then reached a contradiction
- Thus, $F_H < \theta_H u(q_H)$
- IC_H is binding but PC_H is not.

Price Discrimination: Second-degree

- ***Low-demand customer***:
  - Let us show that PC_L binding
  - Similarly as for the high-demand customer, an indirect way to show that
  $$F_L = \theta_L u(q_L)$$
  is to demonstrate that $F_L < \theta_L[u(q_L) - u(q_H)] + F_H$
  - Proving this by contradiction, assume that
  $$F_L = \theta_L[u(q_L) - u(q_H)] + F_H$$

Price Discrimination: Second-degree

– Then, IC_H can be written as

$$\theta_H[u(q_H) - u(q_L)] + \theta_L[u(q_L) - u(q_H)] + F_H = F_H$$
$$\Longrightarrow \theta_H[u(q_H) - u(q_L)] = \theta_L[u(q_H) - u(q_L)]$$
$$\Longrightarrow \theta_H = \theta_L$$

which violates the initial assumption $\theta_H > \theta_L$

- We reached a contradiction
- Thus, $F_L < \theta_L[u(q_L) - u(q_H)] + F_H$
- PC_L is binding but IC_L is not

Price Discrimination: Second-degree

- In summary:
  - From PC_L binding we have
  $$\theta_L u(q_L) = F_L$$
  - From IC_H binding we have
  $$\theta_H[u(q_H) - u(q_L)] + F_L = F_H$$
  - In addition,
    - PC_L binding implies that IC_L holds, and
    - IC_H binding entails that PC_H is also satisfied,
    - That is, all four constraints hold.

Price Discrimination: Second-degree

- The monopolist's expected PMP can then be written as unconstrained problem, as follows,

$$\max_{q_L, q_H \geq 0} \; p\,[F_H - cq_H] + (1-p)[F_L - cq_L]$$

$$= p\left\{\underbrace{\theta_H[u(q_H) - u(q_L)] + F_L}_{F_H} - cq_H\right\} + (1-p)\left\{\underbrace{\theta_L u(q_L)}_{F_L} - cq_L\right\}$$

$$= p\left\{\theta_H[u(q_H) - u(q_L)] + \underbrace{\theta_L u(q_L)}_{F_L} - cq_H\right\} + (1-p)\{\theta_L u(q_L) - cq_L\}$$

$$= p[\theta_H u(q_H) - (\theta_H - \theta_L)u(q_L) - cq_H] + (1-p)[\theta_L u(q_L) - cq_L]$$

Price Discrimination: Second-degree

- FOC with respect to q_H:

$$p[\theta_H u'(q_H) - c] = 0 \quad \Longrightarrow \quad \theta_H u'(q_H) = c$$

  - which coincides with that under complete information.
  - That is, there is not output distortion for high-demand buyer
  - Informally, we say that there is "**no distortion at the top**".

- FOC with respect to q_L:

$$p[-(\theta_H - \theta_L)u'(q_L)] + (1-p)[\theta_L u'(q_L) - c] = 0$$

which can be re-written as

$$u'(q_L)[\theta_L - p\theta_H] = (1-p)c$$

Price Discrimination: Second-degree

- Dividing both sides by $(1-p)$, we obtain

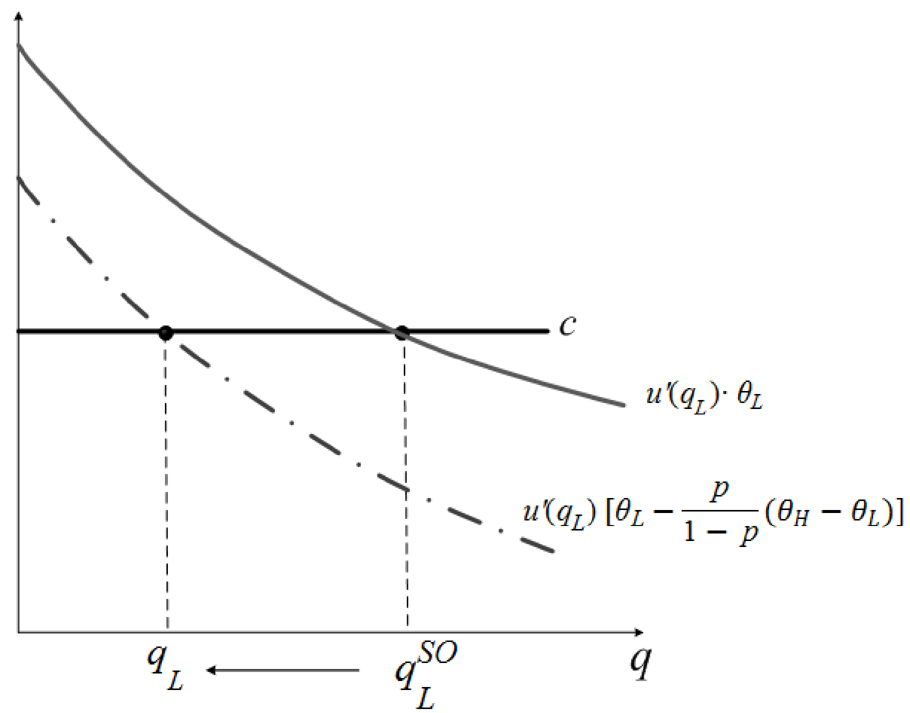
$$u'(q_L)\left[\frac{\theta_L - p\theta_H}{1-p}\right] = c$$

- The above expression can alternatively be written as

$$u'(q_L)\left[\theta_L - \frac{p}{1-p}(\theta_H - \theta_L)\right] = c$$

Price Discrimination: Second-degree

- $u'(q_L) \cdot \theta_L$ depicts the socially optimal output q_L^{so}, i.e., that arising under complete information
- The output offered to high-demand customers is socially efficient due to the absence of output distortion for high-type agents
- The output offered to low-demand customers entails a distortion, i.e., $q_L < q_L^{so}$
- Per-unit price for high-type and low-type differs, i.e., $F_H \neq F_L$
  - Monopolist practices price discrimination among the two types of customers.



Price Discrimination: Second-degree

- Since constraint PC_L binds while PC_H does not, then only the high-demand customer retains a positive utility level, i.e., $\theta_H u(q_H) - F_H > 0$.
- The firm's lack of information provides the high-demand customer with an "information rent."
  - Intuitively, the information rent emerges from the seller's attempt to reduce the incentives of the high-type customer to select the contract meant for the low type.
  - The seller also achieves self-selection by setting an attractive output for the low-type buyer, i.e., q_L is lower than under complete information.

Price Discrimination: Second-degree

- ***Example***:
  - Consider a monopolist selling a textbook to two types of graduate students, low- and high-demand, with utility function

$$U_i(q_i, F_i) = \frac{q_i^2}{2} - \theta_i q_i - F_i$$

  where $i = \{L, H\}$ and $\theta_H > \theta_L$.
  - Hence, the UMP of type i student is

$$\max_{q_i} \quad \frac{q_i^2}{2} - \theta_i q_i - F_i \quad \text{s.t.} \;\; pq_i + F_i \leq w_i$$

  where $w_i > 0$ denotes the student's wealth.

Price Discrimination: Second-degree

- ***Example*** (continued):
  - By Walras' law, the constraint binds
$$F_i = w_i - pq_i$$
  - Then, the UMP can be expressed as
$$\max_{q_i} \quad \frac{{q_i}^2}{2} - \theta_i q_i - (w_i - pq_i)$$
  - FOCs wrt q_i yields the direct demand function:
$$q_i - \theta_i + p = 0 \quad \text{or} \quad q_i = \theta_i - p$$

Price Discrimination: Second-degree

- ***Example*** (continued):
  - Assume that the proportion of high-demand (low-demand) students is γ ($1-\gamma$, respectively).
  - The monopolist's constant marginal cost is $c > 0$, which satisfies $\theta_i > c$ for all $i = \{L, H\}$.
  - Consider for simplicity that $\theta_L > \frac{\theta_H + c}{2}$.
  - This implies that each type of student would buy the textbook, both when the firm practices uniform pricing and when it sets two-part tariffs
    - Exercise.

Advertising in Monopoly

Advertising in Monopoly

- ***Advertising***: non-price strategy to capture surplus
- The monopolist must balance the additional demand that advertising entails and its associated costs (A dollars)
- The monopolist solves

$$\max_{A} \quad p \cdot q(p, A) - TC\big(q(p, A)\big) - A$$

where the demand function $q(p, A)$ depends on price and advertising.

Advertising in Monopoly

- Taking FOCs with respect to A,

$$p \cdot \frac{\partial q(p,A)}{\partial A} - \underbrace{\frac{\partial TC}{\partial q}}_{MC} \cdot \frac{\partial q(p,A)}{\partial A} - 1 = 0$$

Rearranging, we obtain

$$(p - MC)\frac{\partial q(p,A)}{\partial A} = 1$$

- Let us define the advertising elasticity of demand

$$\varepsilon_{q,A} = \frac{\%\text{ increse in } q}{\%\text{ increse in } A} = \frac{\partial q(p,A)}{\partial A} \cdot \frac{A}{q}$$

  Or, rearranging,

$$\varepsilon_{q,A} \cdot \frac{q}{A} = \frac{\partial q(p,A)}{\partial A}$$

Advertising in Monopoly

- We can then rewrite the above FOC as

$$(p - MC)\underbrace{\varepsilon_{q,A} \cdot \frac{q}{A}}_{\frac{\partial q(p,A)}{\partial A}} = 1$$

- Dividing both sides by $\varepsilon_{q,A}$ and rearranging

$$p - MC = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{q}$$

- Dividing both sides by p

$$\frac{p - MC}{p} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{p \cdot q}$$

Advertising in Monopoly

- From the Lerner index, we know that $\frac{p-MC}{p} = -\frac{1}{\varepsilon_{q,p}}$. Hence,

$$-\frac{1}{\varepsilon_{q,p}} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{p \cdot q}$$

- And rearranging

$$-\frac{\varepsilon_{q,A}}{\varepsilon_{q,p}} = \frac{A}{p \cdot q}$$

  – The right-hand side represents the *advertising-to-sales ratio*.
  – For two markets with the same $\varepsilon_{q,p}$, the advertising-to-sales ratio must be larger in the market where demand is more sensitive to advertising (higher $\varepsilon_{q,A}$).

Advertising in Monopoly

- ***Example***:
  - If the price-elasticity in a given monopoly market is $\varepsilon_{q,p} = -1.5$ and the advertising-elasticity is $\varepsilon_{q,A} = 0.1$, the advertising-to-sales ratio should be

$$\frac{A}{p \cdot q} = -\frac{0.1}{-1.5} = 0.067$$

  - Advertising should account for 6.7% of this monopolist's revenue.

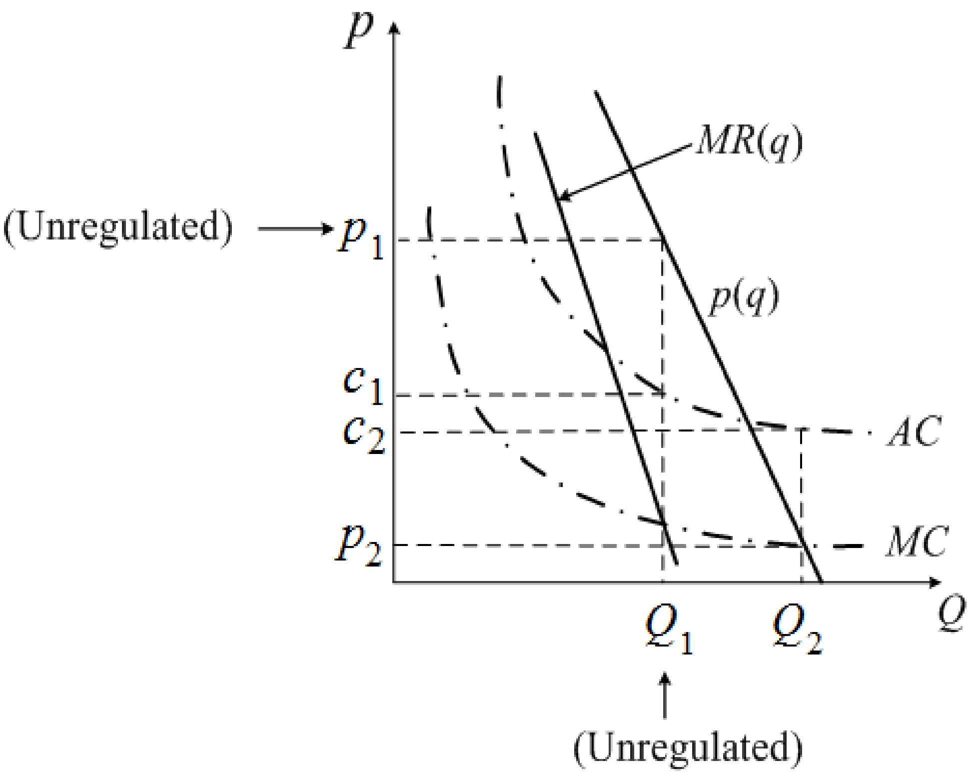
Regulation of Natural Monopolies

Regulation of Natural Monopolies

- ***Natural monopolies***: Monopolies that exhibit *decreasing* cost structures, with the MC curve lying below the AC curve.
- Hence, having a single firm serving the entire market is cheaper than having multiple firms, as aggregate average costs for the entire industry would be lower.

Regulation of Natural Monopolies

- Unregulated natural monopolist maximizes profits at the point where MR=MC, producing Q_1 units and selling them at a price p_1.
- Regulated natural monopolist will charge p_2 (where demand crosses MC) and produce Q_2 units.
- The production level Q_2 implies a loss of $p_2 - c_2$ per unit.



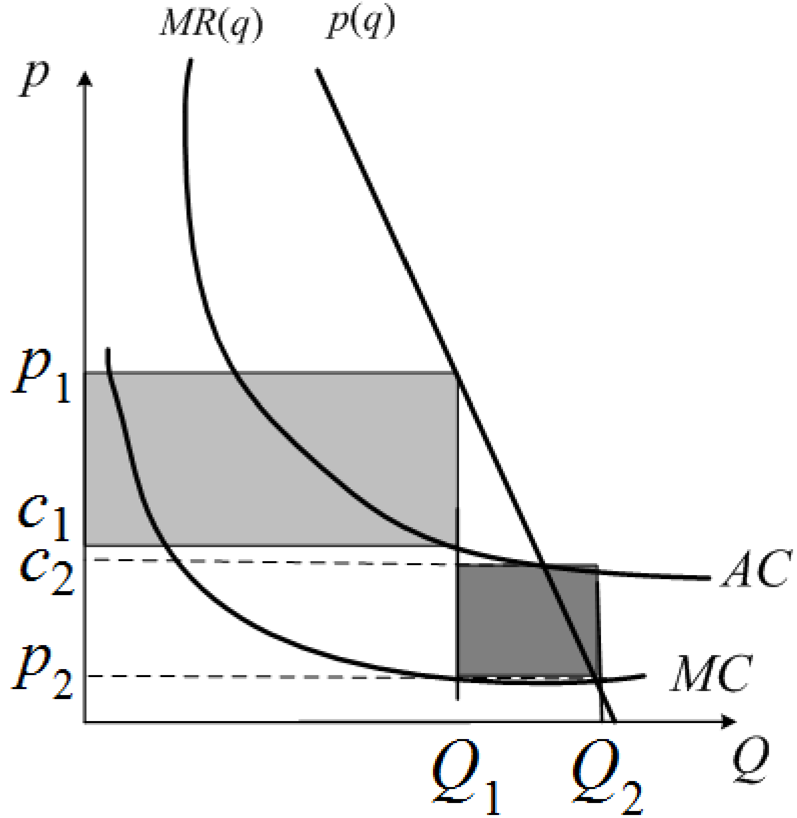
Regulation of Natural Monopolies

- Dilemma with natural monopolies:
  - abandon the policy of setting prices equal to marginal cost, OR
  - continue applying marginal cost pricing but subsidize the monopolist for his losses
- Solution to the dilemma:
  - A multi-price system that allows for price discrimination
  - Charging some users a high price while maintaining a low price to other users

Regulation of Natural Monopolies

- Multi-price system:
  - a high price p_1
  - a low price p_2
- *Benefit*: $(p_1 - c_1)$ per unit in the interval from 0 to Q_1
- *Loss*: $(c_2 - p_2)$ per unit in the interval $(Q_2 - Q_1)$
- The monopolist price discriminates iff

$$(p_1 - c_1)Q_1 > (c_2 - p_2)(Q_2 - Q_1)$$



Regulation of Natural Monopolies

- An alternative regulation:
  - allow the monopolist to charge a price above marginal cost that is sufficient to earn a “fair” rate of return on capital investments
- Two difficulties:
  - what is a “fair” rate of return
  - overcapitalization

Regulation of Natural Monopolies

- ***Overcapitalization of natural monopolies***:
  - Suppose a production function of the form $q = f(k, l)$. An unregulated monopoly with profit function $pf(k, l) - wl - rk$ has a rate of return on capital, r. Suppose furthermore that the rate of return on capital investments, r, is constrained by a regulatory agency to be equal to r_0.

Regulation of Natural Monopolies

- PMP:

$$L = pf(k,l) - wl - rk + \lambda[wl + r_0 k - pf(k,l)]$$

where $0 < \lambda < 1$.

- FOCs:

$$\frac{\partial L}{\partial l} = pf_l - w + \lambda(w - pf_l) = 0$$

$$\frac{\partial L}{\partial k} = pf_k - r + \lambda(r_0 - pf_k) = 0$$

$$\frac{\partial L}{\partial \lambda} = wl + r_0 k - pf(k,l) = 0$$

Regulation of Natural Monopolies

- From the first FOC:

$$pf_l = w$$

- From the second FOC:

$$(1-\lambda)pf_k = r - \lambda r_0$$

and rearranging

$$pf_k = \frac{r-\lambda r_0}{1-\lambda} = r - \frac{\lambda(r_0-r)}{1-\lambda}$$

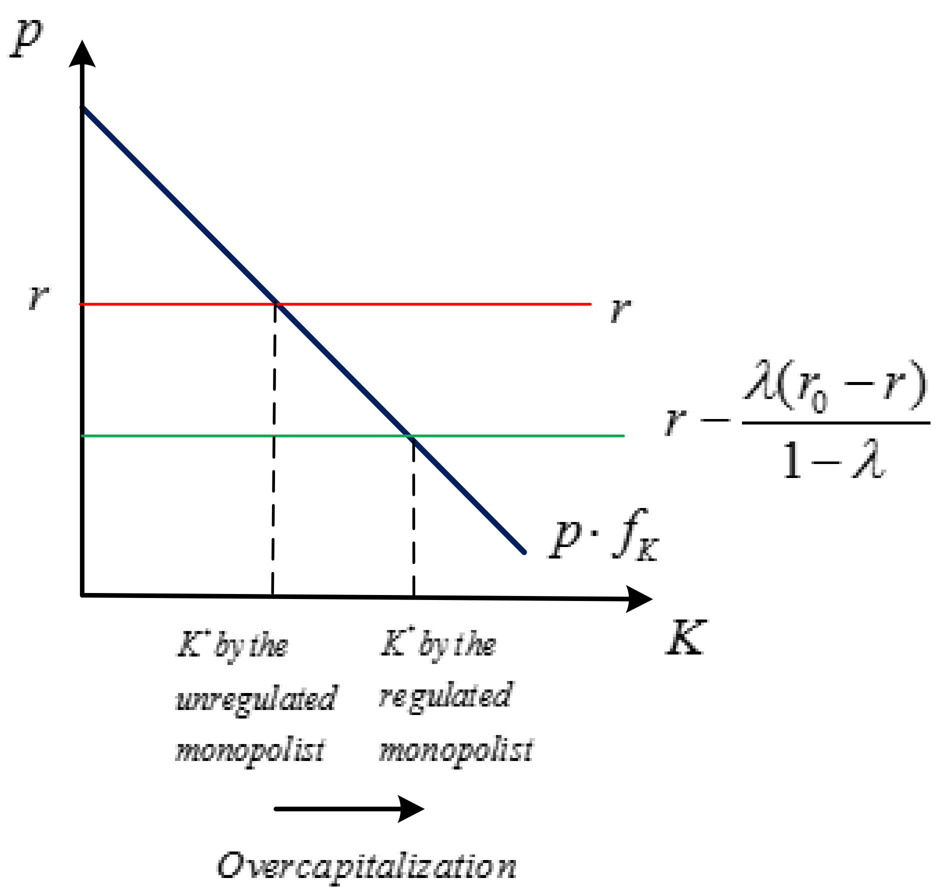
- Since $r_0 > r$ and $0 < \lambda < 1$, then $pf_k < r$.
- Hence, the firm would hire *more capital* than under unregulated condition, where $pf_k = r$.

Regulation of Natural Monopolies

- pf_k is the value of the marginal product of capital
  - It is decreasing in k (due to diminishing marginal return, i.e., $f_{kk} < 0$)
- r and $r - \frac{\lambda(r_0 - r)}{1-\lambda}$ are the marginal cost of additional units of capital in the unregulated and regulated monopoly, respectively

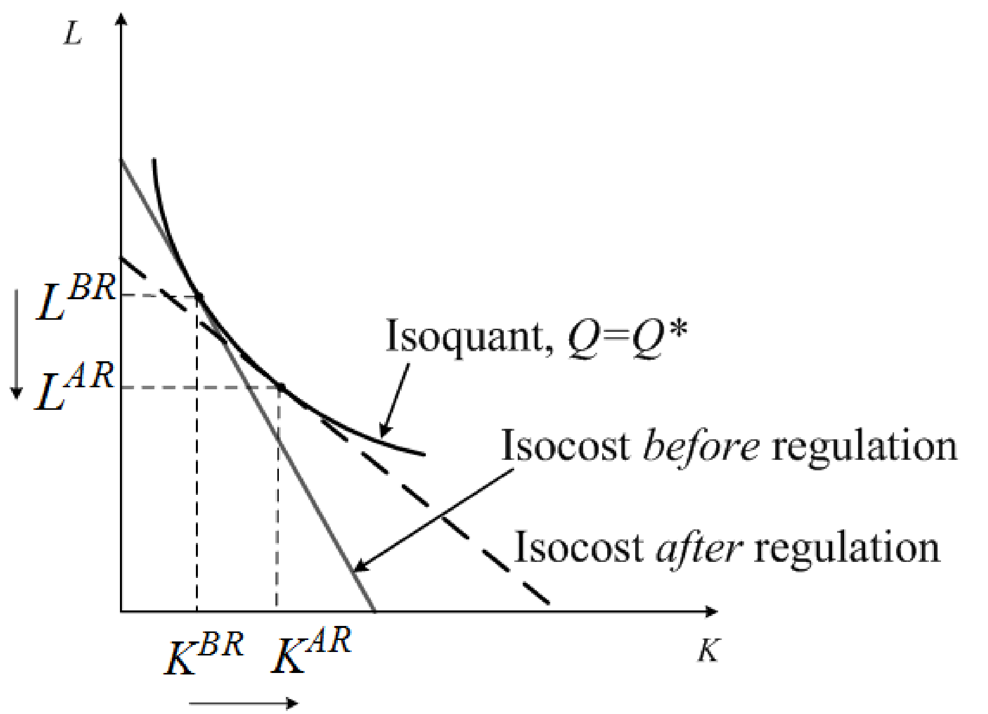
$$r > r - \frac{\lambda(r_0 - r)}{1-\lambda}$$

- *Example*: electricity and water suppliers



Regulation of Natural Monopolies

- An alternative illustration of the overcapitalization (***Averch-Johnson effect***)
- Before regulation, the firm selects (L^{BR}, K^{BR})
- After regulation, the firm selects (L^{AR}, K^{AR}), where $K^{AR} > K^{BR}$ but $L^{AR} < L^{BR}$
- The overcapitalization result only captures the substitution effect of a cheaper input.
  - Output effect?



Monopsony

Monopsony

- ***Monopsony***: A single buyer of goods and services exercises "buying power" by paying prices below those that would prevail in a perfectly competitive context.
- Monopsony (single buyer) is analogous to that of a monopoly (single seller).
- *Examples*: a coal mine, Walmart Superstore in a small town, etc.

Monopsony

- Consider that the monopsony faces competition in the product market, where prices are given at $p > 0$, but is a monopsony in the input market (e.g., labor services).
- Assume an increasing and concave production function, i.e., $f'(x) > 0$ and $f''(x) \leq 0$.
  - This yields a total revenue of $pf(x)$.
- Consider a cost function $w(x) \cdot x$, where $w(x)$ denotes the inverse supply function of labor x.
  - Assume that $w'(x) > 0$ for all x.
  - This indicates that, as the firm hires more workers, labor becomes scarce, thus increasing the wages of additional workers.

Monopsony

- The monopsony PMP is

$$\max_{x} \quad pf(x) - w(x)x$$

- FOC wrt the amount of labor services (x) yields

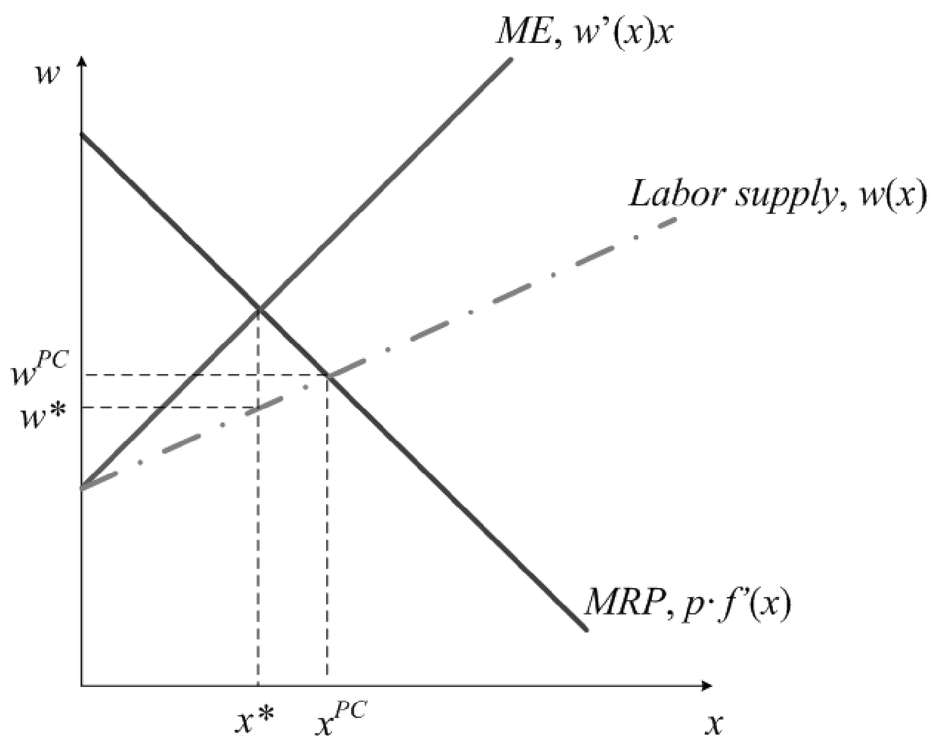
$$pf'(x^*) - w(x^*) - w'(x^*)x^* = 0$$

$$\Longrightarrow \underbrace{pf'(x^*)}_{A} = \underbrace{w(x^*) + w'(x^*)x^*}_{B}$$

  – A: "marginal revenue product" of labor.
  – B: "marginal expenditure" (ME) on labor.
    - The additional worker entails a monetary outlay of $w(x^*)$.
    - Hiring more workers make labor become more scarce, ultimately forcing the monopsony to raise the prevailing wage on all inframarginal workers, as captured by $w'(x^*)x^*$.

Monopsony

- Monopsonist hiring and salary decisions.
  - The marginal revenue product of labor, $pf'(x)$, is decreasing in x given that $f''(x) \leq 0$.
  - The labor supply, $w(x)$, is increasing in x since $w'(x) > 0$.
  - The marginal expenditure (ME) on labor lies above the supply function $w(x)$ since $w'(x) > 0$.
  - The monopsony hires x^* workers at a salary of $w(x^*)$.



Monopsony

- A deadweight loss from monopsony is

$$DWL = \int_{x^*}^{x^{PC}} [pf'(x) - w(x)]dx$$

- That is, the area below the marginal revenue product and above the supply curve, between x^* and x^{PC} workers.

Monopsony

- We can write the monopsony profit-maximizing condition, i.e., $pf'(x^*) = w(x^*) + w'(x^*)x^*$, in terms of labor supply elasticity, using the following steps:

$$pf'(x^*) = w(x^*) + \frac{\partial w(x^*)}{\partial x^*}x^*$$
$$= w(x^*)\left(1 + \frac{\partial w(x^*)}{\partial x^*}\frac{x^*}{w(x^*)}\right)$$

- And rearranging,

$$pf'(x^*) = w(x^*)\left(1 + \frac{1}{\frac{\partial x^*}{\partial w}\frac{w(x^*)}{x^*}}\right)$$

Monopsony

- Since $\frac{\partial x^*}{\partial w}\frac{w(x^*)}{x^*}$ represents the elasticity of labor supply ε, then

$$pf'(x^*) = w(x^*)\left(1 + \frac{1}{\varepsilon}\right)$$

- Intuitively, as $\varepsilon \to \infty$ (labor supply becoming perfectly elastic), the behavior of the monopsonist approaches that of a pure competitor.

Monopsony

- The equilibrium condition above is also sufficient as long as
$$pf''(x^*) - 2w'(x^*) - w''(x^*)x^* < 0$$
- Since $f''(x^*) < 0$, $w'(x^*) > 0$ (by assumption), we only need that either:
  a) the supply function is convex, i.e., $w''(x^*) > 0$; or
  b) if it is concave, i.e., $w''(x^*) < 0$, its concavity is not very strong, that is
$$pf''(x^*) - 2w'(x^*) < w''(x^*)x^*$$

Monopsony

- ***Example***:
  - Consider a monopsonist with production function $f(x) = ax$, where $a > 0$, and facing a given market price $p > 0$ per unit of output.
  - Labor supply is $w(x) = bx$, where $b > 0$.
  - The marginal revenue product of hiring an additional worker is
$$pf'(x) = pa$$
  - The marginal expenditure on labor is
$$w(x) + w'(x)x = bx + bx = 2bx$$

Monopsony

- ***Example*** (continued):
  - Setting them equal to each other, $ap = 2bx^*$, yields a profit-maximizing amount of labor:
  $$x^* = \frac{ap}{2b}$$
  - x^* increases in the price of output, p, and in the marginal productivity of labor, a; but decreases in the slope of labor supply, b.
  - Sufficiency holds since
  $$pf''(x^*) - 2w'(x^*) = p0 - 2b < 0 = w''(x^*)x^*$$