Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 5:
Measuring Welfare Changes

Outline

- Consumer Surplus
- Compensating Variation
- Equivalent Variation
- Measuring Welfare Changes with No Income Effects
- Appendix. An Alternative Representation of Compensating and Equivalent Variations.

Consumer Surplus

Consumer Surplus

- The demand curve identifies how many units of a good x that an individual is willing to purchase at price p_x and income $\$I$.
- In short, the demand function represents the maximum willingness-to-pay (WTP) for the good.
- Comparing this maximum WTP against the price the consumer pays, we find a measure of the utility gain that she makes buying the good.
- Consumer Surplus (CS). The area below the demand curve and above the price that consumers pay for the good.

Consumer Surplus

- *Example 5.1*: *Finding CS with linear demand.*
  - Consider demand curve $p(q) = 10 - 2q$ and market price of $p = \$4$.
  - The area given by the triangle A measures the CS.

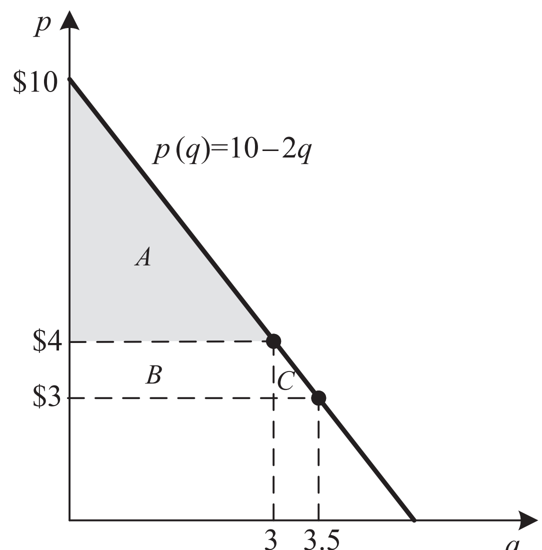

Figure 5.1

$$CS = \frac{1}{2}(10 - 4)3 = 9.$$

Consumer Surplus

- *Example 5.1* (continued):
  - If the price falls to $p = \$3$, output would increase to $3 = 10 - 2q \Longrightarrow q = 3.5$ units.
  - CS increases by the size of areas B and C.

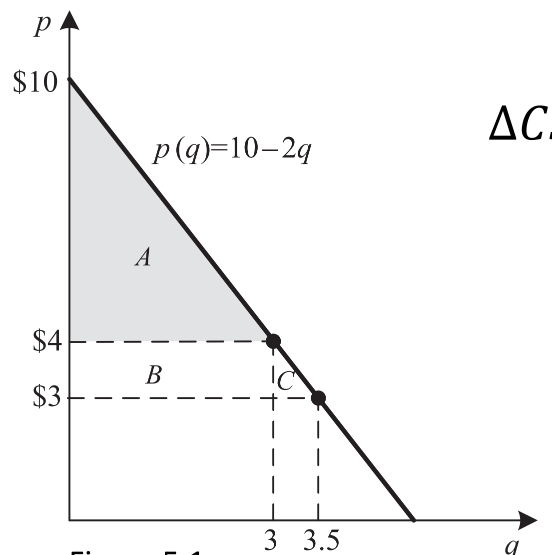

Figure 5.1

$$\Delta CS = B + C$$
$$= (4 - 3)3 + \frac{1}{2}(4 - 3)(3.5 - 3)$$
$$= 3 + 0.5 = 3.25.$$

  - The increase in CS produces a new CS of $9 + 3.25 = 12.25$.

Consumer Surplus

- *Example 5.2*: *Finding CS with nonlinear demand.*
  - Consider the nonlinear demand arising from a Cobb-Douglas utility function,
  $$x = \frac{I}{2p_x}.$$
  - If $p_x = \$4$ and $I = \$100$, she purchases
  $$x = \frac{100}{2 \times 4} = 12.5 \text{ units.}$$

Consumer Surplus

- *Example 5.2* (continued):
  - If the price decreases to $p_x = \$3$, she increases purchases to
$$x = \frac{100}{2 \times 3} = 16.6 \text{ units.}$$
  - In this case, to find the gain in CS we must use the integral of the demand function $x = \frac{I}{2p_x}$ between prices $p_x = \$4$ and , $p_x = \$3$ because the demand function is not linear.

Consumer Surplus

- *Example 5.2* (continued):

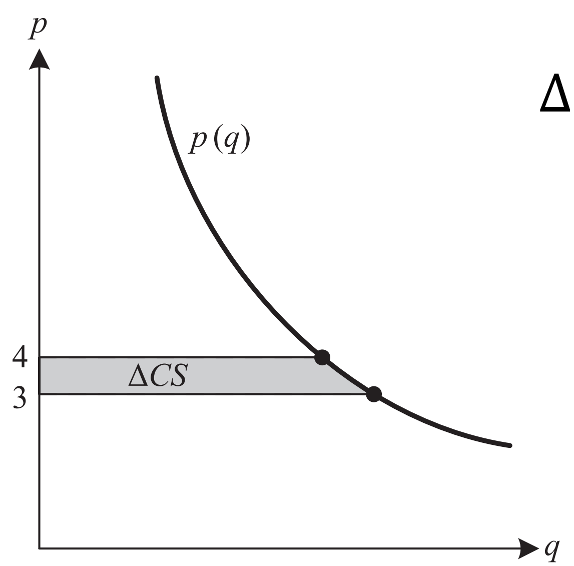

Figure 5.2

$$\Delta CS = \int_3^4 \frac{100}{2p_x} dp_x = 50 \int_3^4 \frac{1}{2p_x} dp_x$$

$$= 50[ln p_x]_3^4 = 50[ln4 - ln3] = 14.38.$$

Consumer Surplus

- *Example 5.2* (continued):
  - What if we tried to approximate the change in CS as if the demand curve was linear?

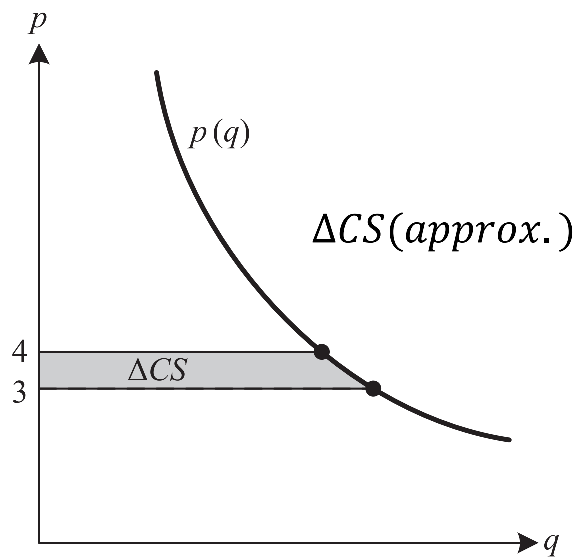

Figure 5.2

$$\Delta CS(approx.) = \underbrace{[1.5\times(4-3)]}_{\text{Area of rectangle } B} + \underbrace{\left[\frac{1}{2}(16.67-12.5)\times(4-3)\right]}_{\text{Area of triangle } C} = 14.59.$$

  - We would be overestimating the true change in CS, because $14.59 > 14.38$.

Compensating Variation

Compensating Variation

- An alternative measure of welfare change.
- Consider the price of good y is normalized to $\$1$:
  - We divide all prices by p_y.
  - For instance, if prices are $p_x = \$4$ and $p_y = \$3$, normalized prices are $p_x' = \$\frac{4}{3}$ and $p_y' = \$\frac{3}{3} = \$1$.
- The advantage is that the intercept of the budget line $y = \frac{I}{p_y}$, is now $y = \frac{I}{p_y'} = \frac{I}{I} = \$1$.
  - We can interpret the vertical intercept of the budget line as consumer's income, which facilitates income comparisons.

Compensating Variation

- Compensating Variation (CS). How much money one needs to take away from (give to) a consumer after a price decrease (increase) such that she is as well off as before the price change.
  - It measures the welfare change from a price change.
  - Intuitively, the consumer is better off (worse off) after a price decrease (increase), allowing her to achieve a higher (lower) utility.
- The CV focuses on the day *after* the price change and ask:
  - *How much do we need to reduce (increase) the consumer's income to make her as well off as she was before the price decrease (increase)?*

Compensating Variation

- CV with a price decrease:
  - The amount of money we need to subtract from the consumer's initial income to make her as well off as before the price change.
    - At initial prices, she faces BL_1, purchases A, and reaches u_1.
    - When the price of $x \downarrow$, the new budget line is BL_2, income I remains, and she chooses C.
    - CV ask: *How much money we need to take away from her income to make the new BL_2 tangent to u_1?*

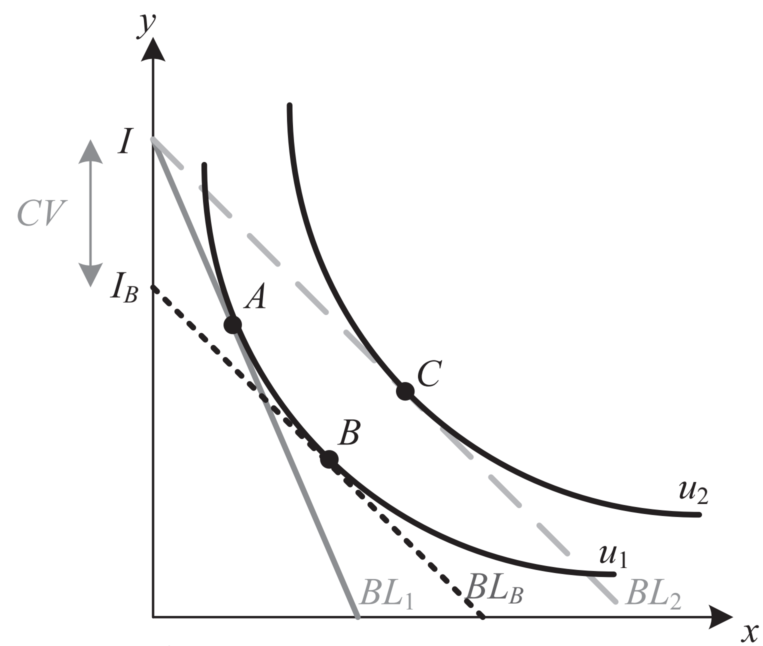

Figure 5.3

Compensating Variation

- CV with a price decrease (cont.):
  - We make a parallel shift of BL_2 downwards until we find BL_B tangent to u_1, where she purchases bundle B.
  - The vertical intercept I_B is the new income she needs to purchase B.
  - $CV = I - I_B$, measures the amount of money we need to subtract from the consumer's initial income to make her as well as before the price change.

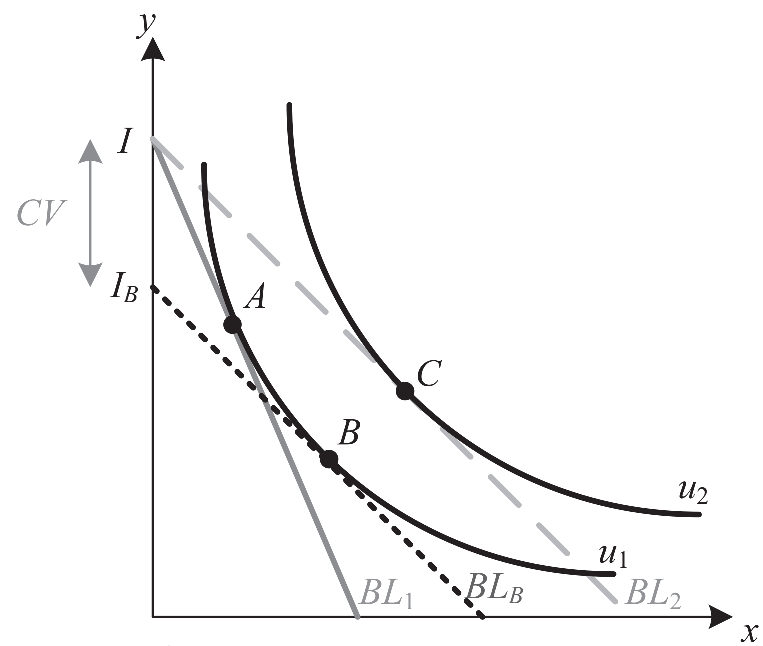

Figure 5.3

Compensating Variation

- *Example 5.3*: *Finding the CV of a price decrease.*
  - Consider a consumer with $u(x, y) = xy$ and, $I = \$100$, and a normalized $p_y = \$1$.
  - First, we need to find demand functions for goods x and y:
    - From the tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$,

$$\frac{y}{x} = \frac{p_x}{p_y} \Longrightarrow y = p_x x.$$

    - Inserting this result into the budget line, $p_x x + p_y y = I$,

$$p_x x + y = 100,$$
$$p_x x + p_x x = 100,$$
$$2p_x = 100.$$

Compensating Variation

- *Example 5.3* (continued):
  - Solving for x, we obtain the demand curve for good x

$$x = \frac{100}{2p_x} = \frac{50}{p_x}.$$

  - Inserting this result into the expression from the tangency condition, we find the demand for good y,

$$y = p_x \frac{50}{p_x} = 50 \text{ units.}$$

Compensating Variation

- *Example 5.3* (continued):
  - Consider the price of good x decreases from $p_x = \$3$ to $p'_x = \$2$, while the price of good y remains $p_y = \$1$.
    1. Finding the initial bundle A.
    2. Finding the final bundle C.
    3. Finding the decomposition bundle B.
    4. Evaluating the CV.

Compensating Variation

- *Example 5.3* (continued):
  1. *Finding the initial bundle A:*
     - At $p_x = \$3$, the demand for good x is

$$x_A = \frac{50}{3} \cong 16.67 \text{ units.}$$

  2. *Finding the final bundle C:*
     - At $p'_x = \$2$, the demand for good x is

$$x_C = \frac{50}{2} \cong 25 \text{ units.}$$

Compensating Variation

- *Example 5.3* (continued):
  3. *Finding the decomposition bundle B:*
     a. The consumer must reach the same utility as with A.

$$u_A = \left(\frac{50}{3}\right)(50) = \frac{2500}{3} \approx 833.33.$$

The amount of x and y consumed at B must yield this utility level,

$$u_B = (x_B)(y_B) = 833.33.$$

Compensating Variation

- *Example 5.3* (continued):
  3. *Finding the decomposition bundle* B (cont.):
     b. The consumer IC must be tangent to the BL,

$$y = p_x x.$$

Because $p_x = 2$, the tangency condition can be written as

$$y = 2x.$$

Compensating Variation

- *Example 5.3* (continued):
    3. *Finding the decomposition bundle B* (cont.):

Substituting $y = 2x$ in $(x_B)(y_B) = 833.33$,

$$u_B = (x_B)(2x_B) = 833.33 \text{ } (y_B = 2x_B),$$

$$2(x_B)^2 = 833.33 \Rightarrow (x_B)^2 = 416.67,$$

$$\sqrt{(x_B)^2} = \sqrt{416.67,}$$

$$x_B \cong 20.41.$$

Inserting this result into the tangency condition,

$$y_B = 2\times 20.41 = 40.82 \text{ units.}$$

Compensating Variation

- *Example 5.3* (continued):
  4. *Evaluating the CV*:
     - The CV is given by $CV = I - I_B$,
       - $I = \$100$;
       - I_B is the income the consumer needs to purchase $B = (20.41, 40.82)$. Specifically,

         $I_B = (\$2 \times 20.41) + (\$1 \times 40.82) = \$81.64.$

Compensating Variation

- *Example 5.3* (continued):
  4. *Evaluating the CV* (cont.):
     - The CV is

$$CV = I - I_B = \$100 - \$81.64 = \$18.36.$$

In words, if after experiencing the price decrease, we reduce the consumer's income by $\$18.36$, her utility coincides with that before the price change.

Equivalent Variation

Equivalent Variation

- Equivalent Variation (EV). How much money one needs to give to (take from) a consumer before a price decrease (increase) such that she is as well off as after the price change.
- The The CV focuses on the day *before* the price change and ask:
  - *How much do we need to increase (decrease) the consumer's income today to make her as well off as she will be after the price decrease (increase)?*

Equivalent Variation

- EV with a price decrease:
  - How much money we need to give to the consumer (on top of her initial income I) to make her as well off as she will be once she enjoys the price decrease.
    - First, she faces BL_1, purchases A, and reaches u_1.
    - When the price of x ↓, the new budget line is BL_2 which leads her to choose C.
    - EV ask: *How much money we need to give to the consumer to make her as well off as she will be once the price decreases?*

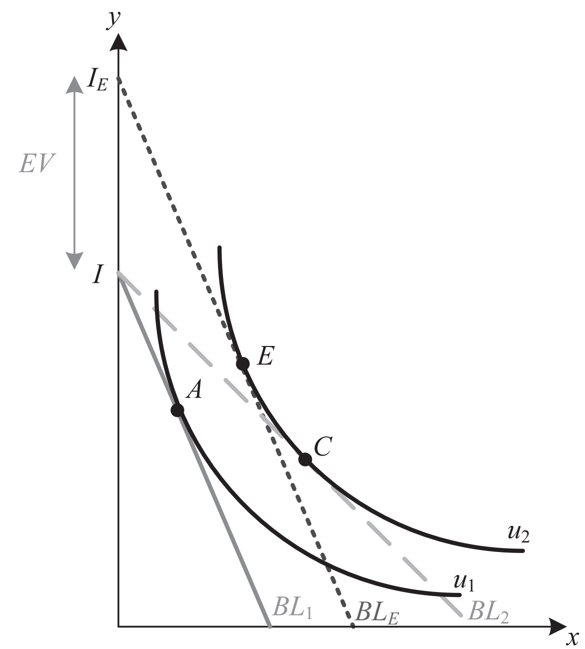

Figure 5.4

Equivalent Variation

- EV with a price decrease (cont.):

  - We make a parallel shift of BL_1 upward until we find BL_E tangent to u_2, where she purchases bundle B.
  - The vertical intercept I_E indicates the income she needs to reach u_2 at initial prices.
  - $EV = I_E - I$, measures the additional income we need to give to the consumer to make her as well as after the price change.

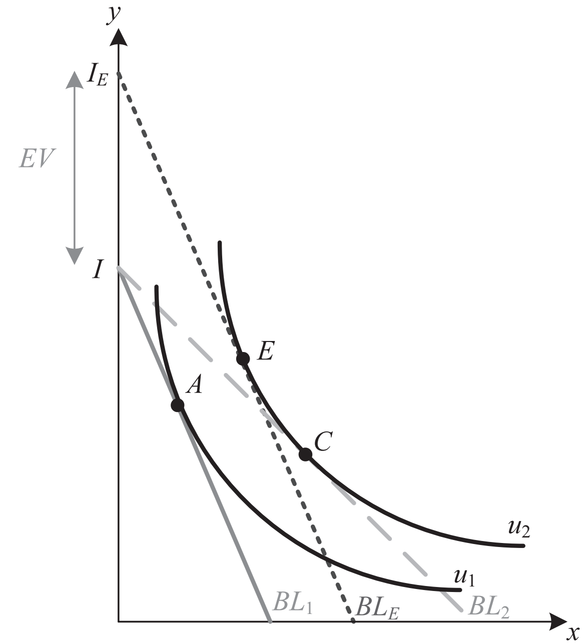

Figure 5.4

Equivalent Variation

- *Example 5.4*: ***Finding the EV of a price decrease.***
  - Consider a consumer with $u(x, y) = xy$, $I = \$100$, and a $p_y = \$1$, as in example 4.8.
  - The price of good x decreases from $p_x = \$3$ to $p_x' = \$2$.
  - From example 4.8., we know:
    - Initial bundle is $A = \left(\frac{50}{3}, 50\right)$.
    - Final bundle is $C = (25,50)$.
    - Decomposition bundle is $B = (20.4,40.8)$.
  - The EV of this price decrease is given by $EV = I_E - I$.
  - We need to find bundle E.

Equivalent Variation

- *Example 5.4* (continued):
  - Finding bundle E.

1. Bundle E must reach the same utility level as the final bundle $C = (25{,}50)$,

$$u_c = 25 \times 50 = 1{,}250.$$

Bundle $E = (x_E, y_E)$ must also yields this utility level,

$$u_c = x_E y_E = 1{,}250.$$

2. Bundle E must be a tangency point; the tangency condition must hold,

$$\frac{MU_x}{MU_x} = \frac{p_x}{p_y},$$

$$\frac{y}{x} = \frac{3}{1} \Longrightarrow y = 3x.$$

Equivalent Variation

- *Example 5.4* (continued):
  - Finding bundle E.

Plugging $y = 3x$ into $x_E y_E = 1{,}250$

$$x_E(3x_E) = 1{,}250,$$

$$3(x_E)^2 = 1{,}250 \Longrightarrow (x_E)^2 = \frac{1{,}250}{3} \cong 416.67,$$

$$\sqrt{(x_E)^2} = \sqrt{416.67,}$$

$$x_E = 20.41 \text{ units},$$

Inserting this result into the tangency condition $y = 3x$, we find the amount of good y at bundle E,

$$y_E = 3 \times 20.41 = 61.2 \text{ units.}$$

Equivalent Variation

- *Example 5.4* (continued):
  - Therefore, the income that the consumer spends to purchase bundle $E = (20.41, 61.2)$ at the initial prices ($p_x = \$3$ and $p_y = \$1$) is

$$I_E = (\$3 \times 20.41) + (\$1 \times 61.2) = \$122.43,$$

  - The EV is

$$EV = I_E - I = \$122.43 - \$100 = \$22.43.$$

  In words, if, before enjoying the price decrease, we increase the consumer's income by $\$22.43$, we help her reach the same utility level that she will enjoy after the price decrease.

Measuring Welfare Changes with No Income Effects

Welfare Changes with No IE

- We have discussed three approaches to measure the welfare change that consumers experience after a price change:
  i. The change in consumer surplus (CS).
  ii. The Compensating Variation (CV).
  iii. The Equivalent Variation (EV).
- While these welfare measures generally differ, they produce the same number if income effects (IE) are absent.
  - $IE = 0$ when the consumer has quasilinear preferences.
  - This type of preferences produce $CS = CV = EV$.

Welfare Changes with No IE

- *Example 5.5: CS, CV, and EV with a quasilinear utility function*.
  - Consider a consumer with $u(x, y) = 2\sqrt{x} + y$, $I = \$100$, and $p_y = \$1$.
  - We first find the demand function for goods x and y,
    - Using the tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_u}$, we find the demand for x,

$$\frac{\frac{1}{\sqrt{x}}}{1} = \frac{p_x}{1} \quad \Longrightarrow x = \frac{1}{p_x^2}.$$

    - Inserting the demand for x into the budget line, $p_x x + p_y y = I$, we find the demand for y,

$$p_x \frac{1}{p_x^2} + 1 \times y = 100 \quad \Longrightarrow \frac{1}{p_x} + y = 100 \quad \Longrightarrow y = 100 - \frac{1}{p_x}.$$

Welfare Changes with No IE

- *Example 5.5* (continued):
  - Consider the price of good x decreases from $\$4$ to $\$3$.
  - We find the increase in consumer welfare measured through the CS, CV, and EV.
  - **Finding the CS**. We integrate the demand curve of good x between $4 and $3 to obtain the welfare change,

$$\Delta CS = \int_3^4 \frac{1}{p_x^2} dp_x = \left[-\frac{1}{p_x}\right]_3^4 = \left(-\frac{1}{4}\right) - \left(-\frac{1}{3}\right) = \frac{1}{12} = 0.08.$$

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the CV**. $CV = I - I_B$. We need to find the income that the consumer needs to purchase bundle E, I_B.
    1. *Finding the initial bundle A*. At $p_x = \$4$, the demands for goods x and y simply to

$$x_A = \frac{1}{(p_x^2)} = \frac{1}{4^2} = \frac{1}{16},$$

$$y_A = 100 - \frac{1}{p_x} = 100 - \frac{1}{4} = \frac{399}{4}.$$

Bundle A is $A = \left(\frac{1}{16}, \frac{399}{4}\right)$.

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the CV** (cont.).

    2. *Finding the final bundle C*. At $p_x' = \$3$, the demands for x and y change to

$$x_A = \frac{1}{3^2} = \frac{1}{9},$$

$$y_A = 100 - \frac{1}{3} = \frac{299}{3}.$$

Bundle C is $C = \left(\frac{1}{9}, \frac{299}{3}\right)$.

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the CV** (cont.).
    3. *Finding the decomposition bundle B.*
       a. The consumer must reach the same utility level as with $A = \left(\frac{1}{16}, \frac{399}{4}\right)$,

       $$u_A = 2\sqrt{\frac{1}{16}} + \frac{399}{4} = 100.25.$$

       Bundle $B = (x_B, y_B)$ must yield the same utility,

       $$u_B = 2\sqrt{x_B} + y_B = 100.25. \quad (5.1)$$

       b. The IC must be tangent to the budget line at final prices,

       $$\frac{MU_x}{MU_x} = \frac{p_x'}{p_y},$$

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the CV** *(cont.)*.

$$\frac{MU_x}{MU_x} = \frac{p_x'}{p_y},$$

$$\frac{\frac{1}{\sqrt{x_B}}}{1} = \frac{3}{1} \Rightarrow \frac{1}{\sqrt{x_B}} = 3,$$

$$\left(\frac{1}{\sqrt{x_B}}\right)^2 = 3^2 \Rightarrow \frac{1}{x_B} = 9 \Rightarrow x_B = \frac{1}{9} \cong 0.11.$$

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the CV** (cont.).

    Substituting $x_B = 0.11$ into the expression for utility level at B, equation (5.1), $u_B = 2\sqrt{x_B} + y_B = 100.25$,

$$2\sqrt{\frac{1}{9}} + y_B = 100.25 \Rightarrow \frac{2}{3} + y_B = 100.25,$$

$$y_B \cong 99.58.$$

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the CV** (cont.).

    Therefore, the income that the consumer needs to purchase the decomposition bundle $B = (0.11, 99.58)$ is

    $$I_B = 3(0.11) + 1(99.58) = \$99.91.$$

4. *Evaluating the CV*. The CV is given by

$$CV = I - I_B = 100 - 99.9133 \approx 0.08,$$

which coincides with the CS we found previously because the consumer exhibits a quasilinear utility function.

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the EV**. $EV = I_B - I$. We start finding I_B.
    1. Bundle E must reach the same utility level as the final bundle C. We already found that $C = \left(\frac{1}{9}, \frac{299}{3}\right)$, which yields utility level of

$$u_C = 2\sqrt{\frac{1}{9}} + \frac{299}{3} = 100.3.3$$

Bundle $E = (x_E, y_E)$ must also yield this utility level

$$u_E = 2\sqrt{x_E} + y_E = 100.33. \qquad (5.2)$$

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the EV** (cont.).
    2. The consumer's IC must be tangent to the budget line at the initial prices, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$,

$$\frac{\frac{1}{\sqrt{x_E}}}{1} = \frac{4}{1} \Rightarrow \sqrt{\frac{1}{x_E}} = 4 \Rightarrow \left(\sqrt{\frac{1}{x_E}}\right)^2 = 4^2,$$

$$x_E = \frac{1}{16} \cong 0.0625.$$

Substituting this result in equation (5.2), $2\sqrt{x_E} + y_E = 100.33$,

$$2\sqrt{\frac{1}{16}} + y_E = 100.33 \Rightarrow \frac{2}{4} + y_E = 100.33 \Rightarrow y_E = 99.83.$$

The income the consumer needs to purchase $E = (0.0625, 99.83)$ is

$$I_E = 4(0.0625) + 1(99.83) = 100.08.$$

Welfare Changes with No IE

- *Example 5.5* (continued):
  - **Finding the EV** (cont.).

    3. *Evaluating the EV*. The EV is given by

$$EV = I_E - I = 100.08 - 100 = 0.08.$$

This result coincides with those we found for the CS and CV because the consumer exhibits a quasilinear utility function.

Appendix A.
An Alternative Representation of CV and EV

Alternative Representation of CV

- We can measure the CV using the consumer's expenditure minimization problem (EMP).
- Consider an individual facing p_x and p_y, and seeking to reach a utility target of u in her EMP.
- From Appendix B in chapter 3, we know she would set the tangency condition,

$$MRS_{x,y} = \frac{p_x}{p_y},$$

and then insert this result into her constraint of reaching utility target u, $u(x,y) = u$.

Alternative Representation of CV

- She obtains a demand for good x of

$$x^E(p_x, p_y, u).$$

  And a demand for good y of

$$y^E(p_x, p_y, u).$$

- These demands help her minimize her expenditure while reaching utility target u.

Alternative Representation of CV

- We can find the cost of buying these demands as

$$e(p_x, p_y, u) = p_x x^E(p_x, p_y, u) + p_y y^E(p_x, p_y, u).$$

This is the "expenditure function", which represents the minimal expenditure that the individual needs to incur to reach u at current prices.

Alternative Representation of CV

- When price of good x decrease from p_x to p'_x , she can reach a higher utility u', where $u' > u$.

  The demands are

$$x^E(p'_x, p_y, u') \text{ and } y^E(p'_x, p_y, u').$$

  And the expenditure function becomes

$$e(p'_x, p_y, u') = p'_x x^E(p'_x, p_y, u') + p_y y^E(p'_x, p_y, u'). \quad (5.3)$$

Alternative Representation of CV

- We repeat the analysis p_x', but requiring she reaches the same utility as before the price change, u.

  The demands are

$$x^E(p_x', p_y, u) \text{ and } y^E\left(p_x', p_y, u\right).$$

  And the expenditure function becomes

$$e\left(p_x', p_y, u\right) = p_x' x^E\left(p_x', p_y, u\right) + p_y y^E\left(p_x', p_y, u\right).$$

Alternative Representation of CV

- Using the monetary amounts found in the expenditure functions before and after the price decrease to find the CV,

$$CV = e(p_x', p_y, u') - e(p_x', p_y, u).$$

- The CV measures the amount of money the consumer is willing to give up *after* the price decrease (after improving utility level from u to u') to be just as well as *before* the price decrease (where she reached u).

Alternative Representation of CV

- Yet another alternative representation of CV, using $x^E(p_x', p_y, u)$ alone.
- Note that the individual's expenditure function must satisfy

$$e(p_x, p_y, u) = I.$$

when u coincides with the maximal utility when solving UMP.

Alternative Representation of CV

- Yet another alternative representation of CV (cont.).
- Similarly, $e(p_x', p_y, u) = I$, which allows to rewrite CV as

$$CV = I - e(p_x', p_y, u),$$

or using $e(p_x, p_y, u) = I$,

$$CV = e(p_x, p_y, u) - e(p_x', p_y, u).$$

- CV represents the consumer's minimal expenditure of reaching u when prices decreases from p_x to p_x'.

Alternative Representation of CV

- Yet another alternative representation of CV (cont.).
- The CV can be written as

$$CV = \int_{p_x'}^{p_x} \frac{\partial e(p_x, p_y, u)}{\partial p_x} dp_x \, .$$

- To simplify this expression, recall that from equation (5.3), we know $\frac{\partial e(p_x,p_y,u)}{\partial p_x} = x^E\left(p_x, p_y, u\right)$.

Alternative Representation of CV

- Yet another alternative representation of CV (cont.).
- The CV is reduced to

$$CV = \int_{p_x'}^{p_x} x^E(p_x, p_y, u)dp_x\,.$$

- Graphically, the CV becomes the area below the demand curve for good x, we found from the EMP, $x^E(p_x, p_y, u)$ between prices p_x and p_x'.

Alternative Representation of CV

- *Example 5.6: An alternative representation of CV.*
  - Consider a consumer with Cobb-Douglas $u(x, y) = xy$.
  - If we solve the EMP we find the demand for good x is

$$x^E(p_x, p_y, u) = \sqrt{u\frac{p_y}{p_x}}.$$

  - Consider the price of x decreases from $p_x = \$3$ to $p_x' = \$2$, the price of y is constant, $p_y' = \$1$.

Alternative Representation of CV

- *Example 5.6* (continued):
  - The consumer seeks to reach a utility target of

$$u = xy = \frac{50}{3} \times 50 = 833.33,$$

    which is the utility the consumer reaches at bundle $A = \left(\frac{50}{3}, 50\right)$ (from example 5.4).

  - In this case, the demand function from the EM simplifies to

$$x^E(p_x, p_y, u) = \sqrt{u\,\frac{p_y}{p_x}} = \sqrt{833.33\,\frac{1}{p_x}} = 28.87\sqrt{\frac{1}{p_x}}.$$

Alternative Representation of CV

- *Example 5.6* (continued):
  - Therefore, the CV becomes

$$CV = \int_{p_x'}^{p_x} x^E(p_x, p_y, u)dp_x = \int_2^3 28.87\sqrt{\frac{1}{p_x}}dp_x = 28.87\int_2^3 \sqrt{\frac{1}{p_x}}dp_x$$

$$= 28.87[2\sqrt{p_x}]_2^3 = 28.87\left[2\left(\sqrt{3} - \sqrt{2}\right)\right] \cong \$18.35,$$

which coincides with the answer obtained in example 5.3 using an alternative approach (with a small difference of $\$0.01$ due to approximations while solving).

Alternative Representation of EV

- We follow a similar approach. Recall the EV takes the "before-the-price change" perspective.
- The EV can be expressed as

$$EV = e(p_x, p_y, u') - e(p_x, p_y, u).$$

- The EV measures the amount of money the consumer needs to receive *before* the price decrease (when her utility level is still u) to be just as well as *after* the price decrease (where she reaches a higher utility u').

Alternative Representation of EV

- Yet we can obtain one more expression for the EV. Consumer's minimal expenditure satisfies $e(p_x, p_y, u) = I$, then

$$EV = e(p_x, p_y, u') - I.$$

- Because $e(p'_x, p_y, u') = I$ also holds,

$$EV = e(p_x, p_y, u') - e(p'_x, p_y, u').$$

Alternative Representation of EV

- Therefore, the EV can be written as

$$EV = \int_{p_x'}^{p_x} \frac{\partial e(p_x, p_y, u')}{\partial p_x} dp_x \, .$$

- To simplify this expression, recall that

$$\frac{\partial e(p_x, p_y, u')}{\partial p_x} = x^E(p_x, p_y, u').$$

Alternative Representation of EV

- The EV is reduced to

$$EV = \int_{p_x'}^{p_x} x^E(p_x, p_y, u')dp_x \,.$$

- Graphically, the EV is the area below the demand curve for good x, we found from the EMP, $x^E(p_x, p_y, u')$ between prices p_x and p_x'.

Alternative Representation of EV

- *Example 5.7: An alternative representation of EV*.
  - From example 5.6 consider an individual with the demand for good x as
$$x^E(p_x, p_y, u) = \sqrt{u\frac{p_y}{p_x}}.$$
  - Consider the price of x decreases from $p_x = \$3$ to $p_x' = \$2$, the price of y is constant, $p_y' = \$1$.
  - The utility level at final bundle C (after the price change) is $u' = 1{,}250$ (from example 5.4). Inserting it into the demand for good x,
$$x^E(p_x, p_y, u') = \sqrt{u'\frac{p_y}{p_x}} = \sqrt{1{,}250\frac{1}{p_x}} = 25\sqrt{\frac{2}{p_x}}.$$

Alternative Representation of EV

- *Example 5.7* (continued):
  - Therefore, the EV becomes

$$EV = \int_{p_x'}^{p_x} x^E(p_x, p_y, u)dp_x = \int_2^3 25\sqrt{\frac{2}{p_x}}dp_x$$

$$= 25\sqrt{2}[2\sqrt{p_x}]_2^3 = 502\sqrt{2}(\sqrt{3} - \sqrt{2}) \cong \$22.47,$$

which coincides with the answer obtained in example 5.4 using an alternative approach.