Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 2:
Consumer Preferences and Utility

Outline

- Bundles
- Preferences for Bundles
- Utility Functions
- Marginal Utility
- Indifference Curves
- Marginal Rate of Substitution
- Special Types of Utility Functions
- A Look at Behavioral Economics–Social Preferences
- Appendix. Finding Marginal Rate of Substitution

Bundles

Bundles

- A bundle is a list of goods and services.
  - *Example*: If an individual consumes only 2 goods, x and y (apples and oranges), bundle $A = (40{,}30)$ indicates that she consumes $x = 40$ apples and $y = 30$ oranges.

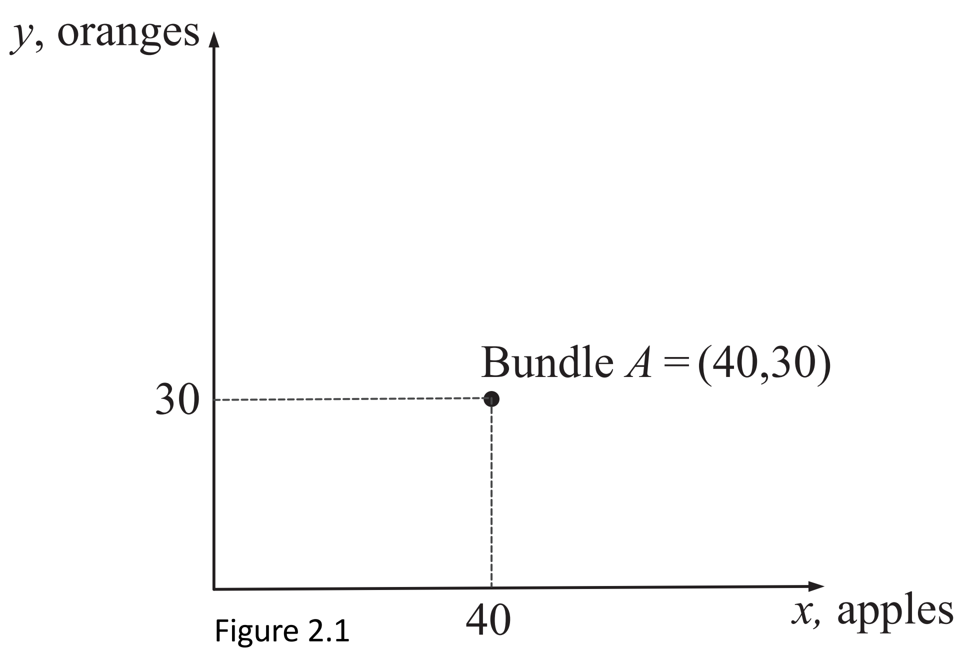

Figure 2.1

Preferences for Bundles

- Let's analyze consumer preferences over bundles or how a consumer ranks different bundles.
- Notation for comparing preferences for bundles:
  - Consider bundles $A = (x_A, y_A)$ and $B = (x_B, y_B)$.
  - $A \succ B$, the individual "strictly prefers" bundle A to B ("strictly" rules out the possibility she is indifferent between the two bundles).
  - $A \sim B$, she "indifferent" between bundles A and B.
  - $A \succsim B$, she "weakly prefers" bundle A to B (she can be indifferent between the two bundles or to strictly prefers A to B).

Preferences for Bundles

- Completeness:
  - A preference relation is *complete* if the consumer has the ability to compare every two bundles A and B:
    - $A \succ B$ (she strictly prefers bundle A),
    - $B \succ A$ (she strictly prefers bundle B), or
    - $A \sim B$ (she is indifferent between A and B).
  - Completeness implies that the consumer has time to be able to compare and rank two bundles.
  - We don't allow the consumer to respond "I don't know how to compare these two bundles!"

Preferences for Bundles

- Transitivity:
  - For every three bundles A, B, and C,
    - if the consumer prefers A to B ($A \succ B$),
    - and B to C ($B \succ C$),
    - she must also prefer A to C ($A \succ C$).
  - A consumer with intransitive preferences would have $A \succ B$ and $B \succ C$, but $C \succ A$. Her preferences would exhibit a cycle:

$$A \succ B \succ C \succ A.$$

  - An individual with intransitive preferences would be subject to exploitation.

Preferences for Bundles

- Exploitation of intransitive individuals:
  - Consider 3 goods, an orange, and apple, and a banana. And a consumer with the following preferences:

    $Orange \succ Banana$ and $Banana \succ Apple$

    but $Apple \succ Orange$ (which violates transitivity)

  - Assume she owns 1 orange, and she plays a game with a fruit seller. If the seller gives her preferred fruit, she pays $1.
    - The seller offers an apple, she pays $1 because $Apple \succ Orange$.
    - Next, the seller offers a banana, she pays $1 because $Banana \succ Apple$.
    - Next, the seller offers an orange, she pays $1 given $Orange \succ Banana$.
  - At the end, the consumer has 1 orange as at the beginning of the exchange, but she has lost $3 due to her intransitive preferences.

Preferences for Bundles

- Strict Monotonicity:
  - Consider an initial bundle A, and a new bundle B, where bundle B has
    - the same amount of good x as bundle A ($x_A = x_B$),
    - but more units of good y ($y_B > y_A$).
  - A consumer's preferences satisfy *strict monotonicity* if
    - she strictly prefers B to A ($B \succ A$).
  - Increasing the units of a single good, as y in bundle B, produces a new bundle that is strictly preferred to A → "more of *anything* is strictly preferred."

Preferences for Bundles

- Monotonicity:
  - Again, consider an initial bundle A, and a new bundle B, where bundle B has the same amount of good x as bundle A ($x_A = x_B$), but more units of good y ($y_B > y_A$),
  - whereas a new bundle C has more units of both goods than bundle A does ($x_c > x_A$ and $y_c > y_A$).
  - A consumer's preferences satisfy *monotonicity* if
    - she weakly prefers prefers B to A ($B \gtrsim A$),
    - but she strictly prefers C to A ($C \succ A$)
  - If the amounts of *all* goods are higher, as in bundle C, the consumer is better off → "more of *everything* is strictly preferred."

Preferences for Bundles

- *Example 2.1*: *Monotonic and strictly monotonic preferences.*
  - Consider bundles $A = (2,3)$ and $B = (2,4)$.
  - Eric strictly prefers bundle B to A ($B \succ A$).
    - If this ranking holds for any two bundles where only one of the good is increased, these preferences satisfy *strict monotonicity*.
  - Chelsea is indifferent between B and A ($B \sim A$).

    If we replace B with bundle $C = (3,4)$, she strictly prefers C to A ($C \succ A$).
    - If this ranking holds for any two bundles in which one has more units of all goods, her preferences satisfy *monotonicity.*

Preferences for Bundles

- Strict Monotonicity implies monotonicity:
  - If a consumer becomes strictly better off if we increase anyone of the goods, then she is not worse off, which is the minimal requirement to satisfy monotonicity.

$$Strict\ monotonicity \implies Monotonicity$$

  - Monotonicity and strict monotonicity require that the consumer regards all items in her bundle as goods rather than bads (e.g., pollution or garbage).
    - If some good were a bad, increasing the number of units in initial bundle A, would produce a new bundle B that would be *less* preferred than bundle A, violating monotonicity and strict monotonicity.

Preferences for Bundles

- Nonsatiation:
  - Preferences satisfy nonsatiation if, for every bundle A, there is another bundle B for which the consumer is better off,

$$B \succ A$$

    - Nonsatiation means there is no "bliss bundle" → the consumer cannot be made any happier by consuming an alternative bundle.
    - Nonsatiation allows the consumer to regard some goods as "bads."
    - Nonsatiation only requires the consumer to always find more preferred bundles.

$$Monotonicity \implies Nonsatiation$$

$$Monotonicity \not\Leftarrow Nonsatiation$$

Preferences for Bundles

- *Example 2.2*: ***Nonsatiated preferences***.
    - Consider bundles $A = (2,3)$ and $D = (2,1)$.

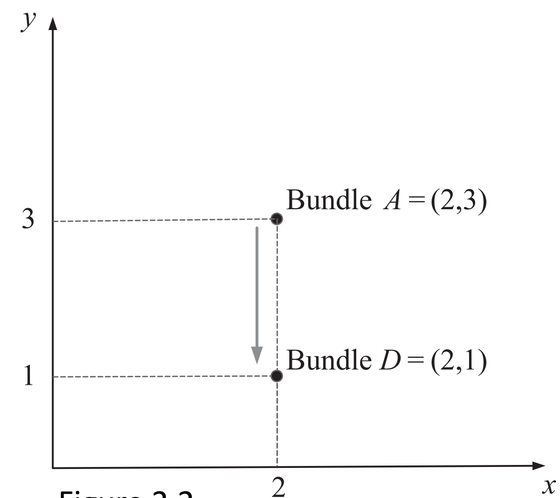

Figure 2.2

- Eric says he strictly prefers D to A, $D \succ A$, and that no other bundle makes him as happy as D does.

Do his preferences satisfy nonsatiation?

Preferences for Bundles

- *Example 2.2* (continued):
  - His preferences, $D \succ A$, can satisfy nonsatiation, but violate monotonicity.
    - Relative to bundle A, bundle D decreased the amount of good y, keeping x unaffected.
      - If $D \succ A$, it must be that y is a bad.
      - Bundle D is a "more preferred" bundle → nonsatiation is satisfied.
      - Monotonicity would require $\mathrm{A} \succ D$.

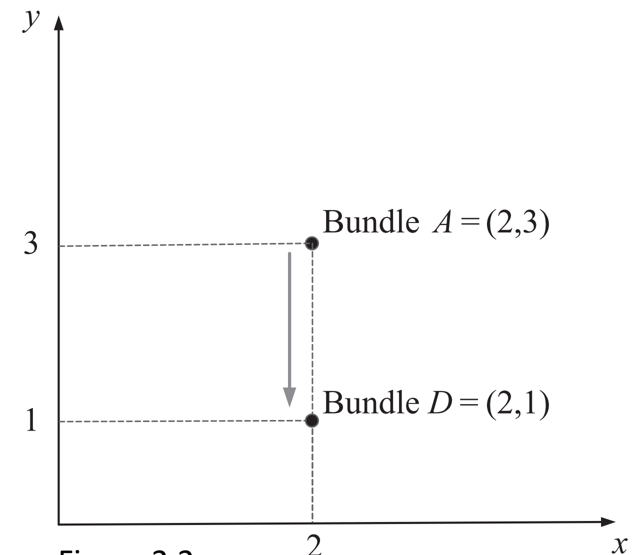

Figure 2.2

Utility Functions

Utility Functions

- A utility function mathematically represents the level of satisfaction that an individual enjoys from consuming a bundle of goods.
  - *Example*: If she consumes bundle $A = (40,30)$ and her utility function is $u(x, y) = 3x + 5y$, her level of utility at bundle A is

$$u(40,30) = (3 \times 40) + (5 \times 30) = 270$$

  - The utility level from bundle A is not as important as the *ranking* of utilities across bundles:
    - Only the utility ranking matters → "ordinality."
    - The specific utility level that the consumer reaches with each bundle does not matter → "cardinality."

Utility Functions

- *Example 2.3*: ***Utility ranking and increasing transformations of the utility function.***
  - Consider utility function $u(x, y) = xy$:
    - Bundle $A = (40{,}30)$ produces $u(40{,}30) = 1{,}200$.
    - Bundle $B = (20{,}30)$ generates $u(20{,}30) = 600$.
    - The consumer prefers bundle A to B ($A \gtrsim B$).
  - Consider now utility function $v(x, y) = 3xy + 8$, which is an increasing transformation of $u(x, y)$:
    - Bundle $A = (40{,}30)$ yields $v(40{,}30) = 3{,}608$.
    - Bundle $B = (20{,}30)$ still generates $v(20{,}30) = 1{,}808$.
    - The consumer stills prefers bundle A to B ($A \gtrsim B$).
  - The consumer's preference over bundle A and B is unaffected (i.e., her ranking does not change).

Utility Functions

- *Example 2.4*: *Testing properties of preference relations.*

  Consider utility function $u(x, y) = xy$. We check:

  a) Completeness:

  - For every two bundles, $A = (x_A, y_A)$ and $B = (x_B, y_B)$, completeness holds when
    - either $u(x_A, y_A) \geq u(x_B, y_B)$,
    - $u(x_B, y_B) \geq u(x_A, y_A)$, or
    - both, $u(x_A, y_A) = u(x_B, y_B)$.
  - If $u(x_A, y_A) = 1{,}200$ and $u(x_B, y_B) = 600$, we check that $u(x_A, y_A) \geq u(x_B, y_B)$ because $1{,}200 > 600$, and completeness is satisfied.

Utility Functions

- *Example 2.4* (continued):

  b) Transitivity:

  - For every three bundles, A, B, and C, where $(x_A, y_A) \geq (x_B, y_B)$ and $(x_B, y_B) \geq (x_C, y_C)$, transitivity holds when
    - $(x_A, y_A) \geq (x_C, y_C)$
  - If $u(x_A, y_B) = 1{,}200$, $u(x_B, y_B) = 600$ and $u(x_C, y_C) = 300$, we know
    - $1{,}200 > 600$,
    - $600 > 300$, and
    - $1.200 > 300$, implying that transitivity is satisfied.

Utility Functions

- *Example 2.4* (continued):
  - c) *Strict monotonicity:*
    - Consumers with strictly monotonic preferences prefer bundles with more units of *any* good.
    - For this property to hold, we need $u(x, y) = xy$ to be *strictly* increasing in both goods. We can check it by confirming

$$\frac{\partial x(x,y)}{\partial x} = y \geq 0 \text{ and } \frac{\partial x(x,y)}{\partial y} = x \geq 0$$

      - Increasing the units of x produces a strict increase in consumer's utility as far as $y > 0$.
      - If she does not consume good y at all, $y = 0$, increasing good x does not alter utility level.
    - Therefore, strict monotonicity does not hold because an increase in x does not necessarily increase consumer's utility.

Utility Functions

- *Example 2.4* (continued):
  - *d) Monotonicity:*
    - We need $u(x, y) = xy$ to be *weakly* increasing in x and y.
    - We know that an increase in x
      - produces a strict increase in consumer's utility (when $y > 0$),
      - or does not affect utility (when $y = 0$),
      - but it never reduces utility.

Utility Functions

- *Example 2.4* (continued):

  d) *Monotonicity* (cont.):

  - A similar argument applies to y. Then, an increase in both x and y produces a new bundle that generates a strictly greater utility.
  - Consider good x is increased by $a > 0$ and good y by $b > 0$. This yields a utility level of

  $$u(x + a, y + b) = (x + a)(y + b).$$

  - Monotonicity is satisfied because

  $$u(x + a, y + b) > u(x, y).$$

Utility Functions

- *Example 2.4* (continued):
  - *e) Nonsatiation:*
    - This property holds by monotonicity.
    - We found that increasing amounts of both goods produces a new bundle $(x + a, y + b)$, that is strictly preferred to the original bundle (x, y).
    - Starting from the original bundle we can always find another bundle for which the consumer is better off.
    - Thee consumer is never satiated.

Utility Functions

- *Utility functions and their properties.*

Table 2.1

Utility Function	Completeness	Transitivity	Strict Monotonicity	Monotonicity	Nonsatiation
$u(x,y) = by$	✓	✓	X	✓	✓
$u(x,y) = ax$	✓	✓	X	✓	✓
$u(x,y) = ax - by$	✓	✓	X	X	✓
$u(x,y) = ax + by$	✓	✓	✓	✓	✓
$u(x,y) = Amin\{ax, by)$	✓	✓	X	✓	✓
$u(x,y) = Ax^{\alpha}y^{\beta}$	✓	✓	X	✓	✓

Paramenters a, b, A, α, β are positive.

Marginal Utility

Marginal Utility

- Marginal utility of a good is the rate at which utility changes as the consumption of a good increases.
  - Intuitively, *how much better off do you become by consuming 1 more unit of good x?*
  - Mathematically, marginal utility of good x is

$$MU_x = \frac{\partial u(x,y)}{\partial x},$$

  and similarly for good y, $MU_y = \frac{\partial u(x,y)}{\partial y}$.

  - Graphically, we measure the slope (rate of change) of the utility function as we increase the amount of good x, holding the amount of other goods constant.

Marginal Utility

- *Example 2.5*: *Finding marginal utility, MU*.
  - Consider utility function $u(x, y) = x^{1/2}y^{1/2}$.
  - Marginal utility of good x is

$$MU_x = \frac{1}{2}x^{\frac{1}{2}-1}y^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}}$$

  - Rearranging,

$$MU_x = \frac{1}{2}\frac{y^{1/2}}{x^{1/2}}.$$

  - $MU_x > 0$ when the individual consumes positive amounts of good x and y, indicating that 1 more unit of good x raises her utility.

Marginal Utility

- *Example 2.5* (continued):
  - Similarly, marginal utility of good y is

$$MU_y = \frac{1}{2}x^{\frac{1}{2}}y^{\frac{1}{2}-1} = \frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}$$

  - Rearranging,

$$MU_y = \frac{1}{2}\frac{x^{1/2}}{y^{1/2}}$$

  - When the individual consumes positive amounts of goods x and y, $MU_y > 0$.

Marginal Utility

- Diminishing Marginal Utility.
  - Marginal utilities of most utility functions are *decreasing* in the amount of the good that the individual consumes,

    MU_x decreases in x, or $\frac{\partial MU_x}{\partial x} \leq 0$ (similarly for y).
  - While more units of good x increase utility level, further increments in x produce smaller utility gains.
    - When the consumer has few units of good (e.g., food), giving her with 1 more unit increases her utility a great deal.
    - When she already has large amounts, giving her 1 more unit of food produces a small utility gain (or no gain at all!)

Marginal Utility

- *Example 2.6*: *Diminishing marginal utility*.
  - Consider $u(x, y) = x^{1/2}y^{1/2}$ in example 2.5.
  - Marginal utility of good x was $MU_x = \frac{1}{2}\frac{y^{1/2}}{x^{1/2}}$.
  - MU_x is decreasing in the amount the consumer enjoys of good x,

    $$\frac{\partial MU_x}{\partial x} = -\frac{y^{1/2}}{4x^{3/2}} < 0 \text{ for all values of } x \text{ of } y.$$

  - Similarly, $MU_y = \frac{1}{2}\frac{x^{1/2}}{y^{1/2}}$, is decreasing in good y because $\frac{\partial MU_x}{\partial x} = -\frac{x^{1/2}}{4y^{3/2}} < 0$ for all values of x of y.

Indifference Curves

Indifference Curves

- This figure depicts $u(x, y) = x^{1/2}y^{1/2}$ in example 2.4.
  - The height of the "mountain" is the utility that the individual achieves by consuming a specific amount of x and y.

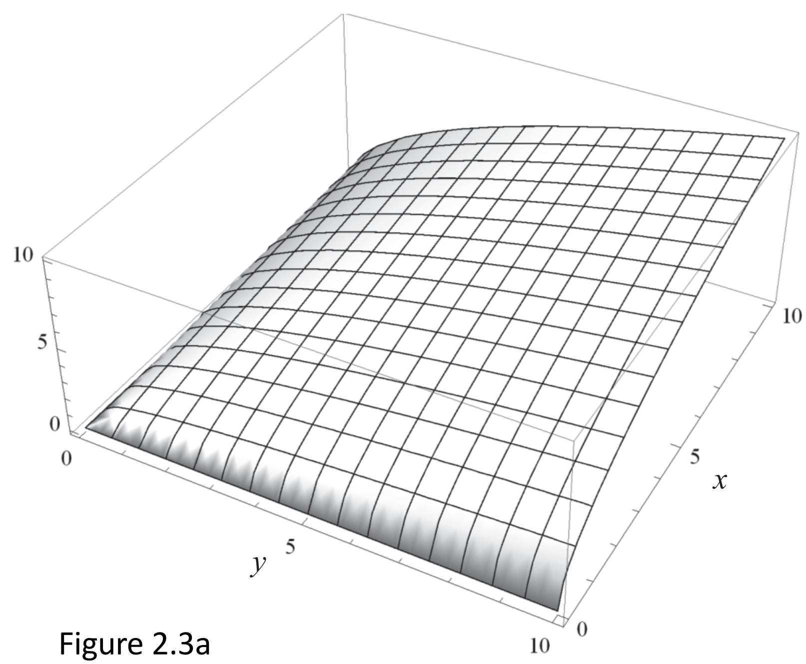

Figure 2.3a

- At bundle $(x, y) = (4,9)$, $u(4,9) = 4^{1/2}9^{1/2} = 6$.
- This utility level can also be obtained at bundles:
  - $(x, y) = (6,6)$, $u(6,6) = 6^{1/2}6^{1/2} = 6$.
  - $(x, y) = (9,4)$, $u(9,4) = 9^{1/2}4^{1/2} = 6$.

Indifference Curves

- The next figure depicts a “slice” of the utility mountain at a height of $u = 6$.

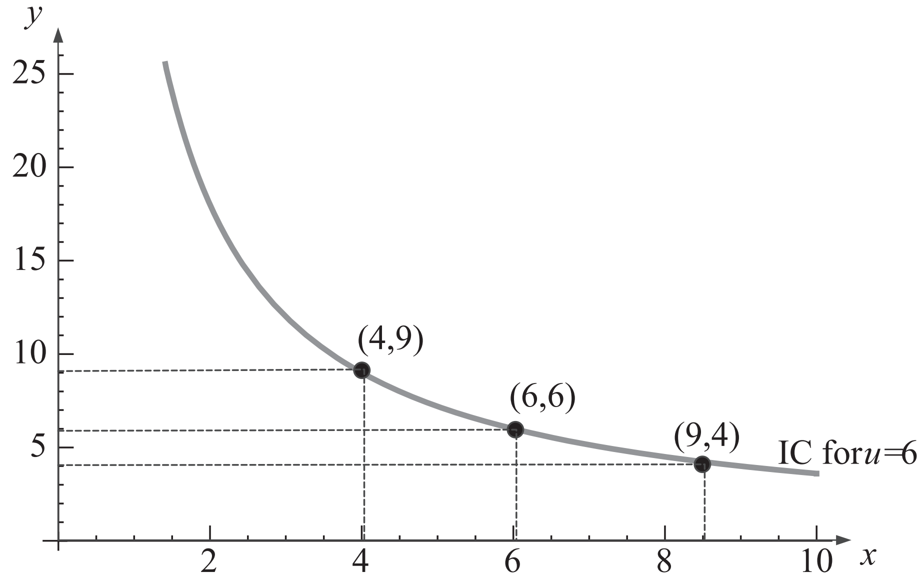

Figure 2.3b

- This curve connects bundles at which the consumer obtains the same utility $u = 6$. She is indifferent between consuming any of these bundles.

Indifference Curves

- Indifference curve (IC): A curve connecting consumption bundles that yield the same utility level.
  - IC for $u(x,y) = x^{1/2}y^{1/2}$ evaluated at $u = 6$. Solving for y, we find the expression of the indifference curve:

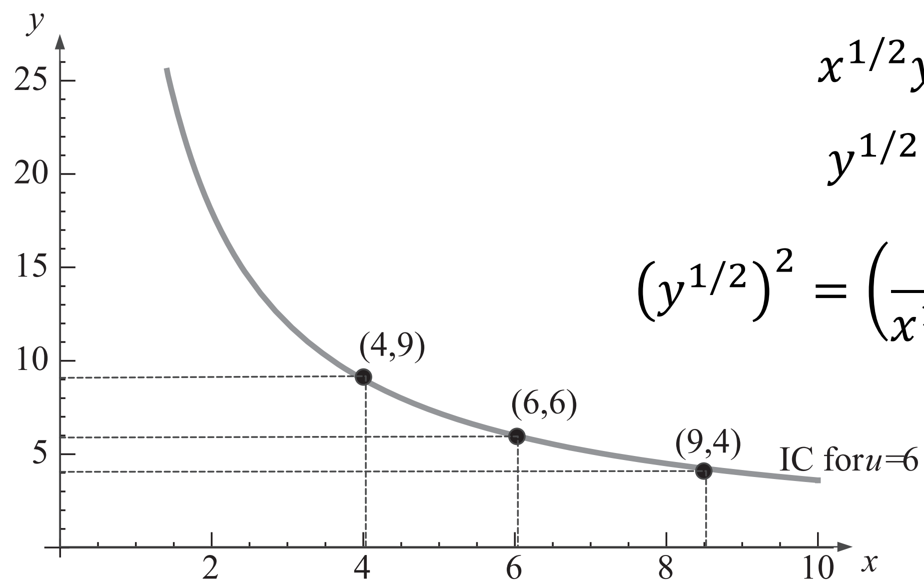

Figure 2.3b

$$x^{1/2}y^{1/2} = 6,$$

$$y^{1/2} = \frac{6}{x^{1/2}},$$

$$\left(y^{1/2}\right)^2 = \left(\frac{6}{x^{1/2}}\right)^2 \Longrightarrow y = \frac{36}{x}.$$

Indifference Curves

- *Example 2.7*: *Finding ICs for two utility functions*.
  - Consider again utility function $u(x, y) = x^{1/2}y^{1/2}$.
  - We want to obtain the expression for the indifference cure when the consumer reaches utility level $u = 10$.
  - This indifference curve entails

$$x^{1/2}y^{1/2} = 10.$$

  - Solving for y,

$$y^{1/2} = \frac{10}{x^{1/2}}.$$

  - Squaring both sides we obtain the indifference curve:

$$\left(y^{1/2}\right)^2 = \left(\frac{10}{x^{1/2}}\right)^2 \quad \Rightarrow y = \frac{100}{x}.$$

Indifference Curves

- *Example 2.7* (continued):
  - Plugging in values for good x in indifference curve $y = \frac{100}{x}$,
    - $x = 4$, which produces $y = \frac{100}{4} = 25$;
    - $x = 8$, which yields $y = \frac{100}{8} = 12.5$;
    - $x = 10$, which entails $y = \frac{100}{10} = 10$;
  - We get bundles $(4{,}25)$, $(8{,}12.5)$, and $(10{,}10)$.
  - If we plot these bundles as points on the positive quadrant, and connect these points, we form the indifference curve for $u = 10$.

Indifference Curves

- *Example 2.7* (continued):
  - Consider now $u(x, y) = 5x + 3y$, and $u = 9$.
  - Solving for y in $5x + 3y = 9$,

$$y = 3 - \frac{5}{3}x.$$

  - This IC originates at $y = 3$, decreases at a rate of $5/3$ and cross the horizontal axis at $5/9$.
    - To find the horizontal intercept, set $3 - \frac{5}{3}x = 0$, rearrange $9 = 5x$, and and solve for x, $x = 9/5$.
  - We can evaluate the IC at several values of x (which need to be smaller than the horizontal intercept, $\frac{9}{5} \cong 1.8$).

Properties of Indifference Curves

- ICs are negatively sloped. It holds from monotonicity.
  - Consider bundle $A = (x_A, y_A)$ in a positively sloped IC.
    - The IC passing through bundle A cannot go through Regions I and II because the consumer strictly prefers
      - bundles in Region I than A,
      - bundle A than those in Region II.

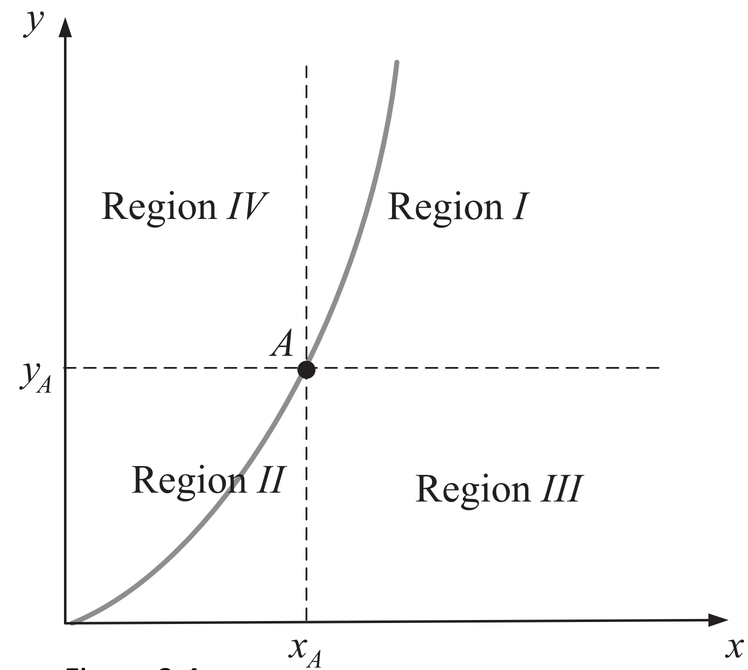

Figure 2.4

Properties of Indifference Curves

- ICs are negatively sloped.

  - The IC passing through bundle A can only go through Region III and IV.
  - IC must be negatively sloped.

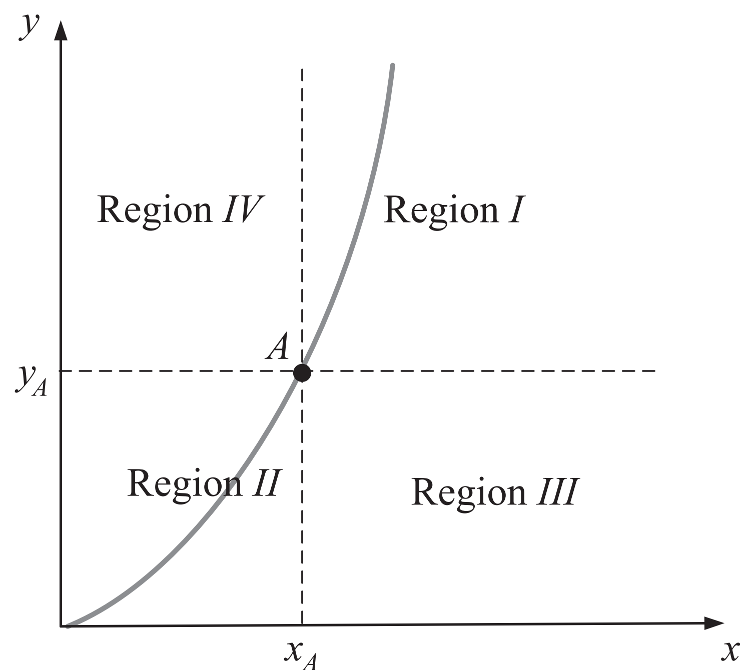

Figure 2.4

Properties of Indifference Curves

- ICs are negatively sloped.
  - Negatively sloped ICs are referred as "convex."

  For any two bundles, a straight line connecting them lies:

  (1) strictly above the curve, yielding a higher utility (when the IC curve is strictly decreasing); or

  (2) on the indifference curve, yielding the same utility level (when the IC is a straight line).

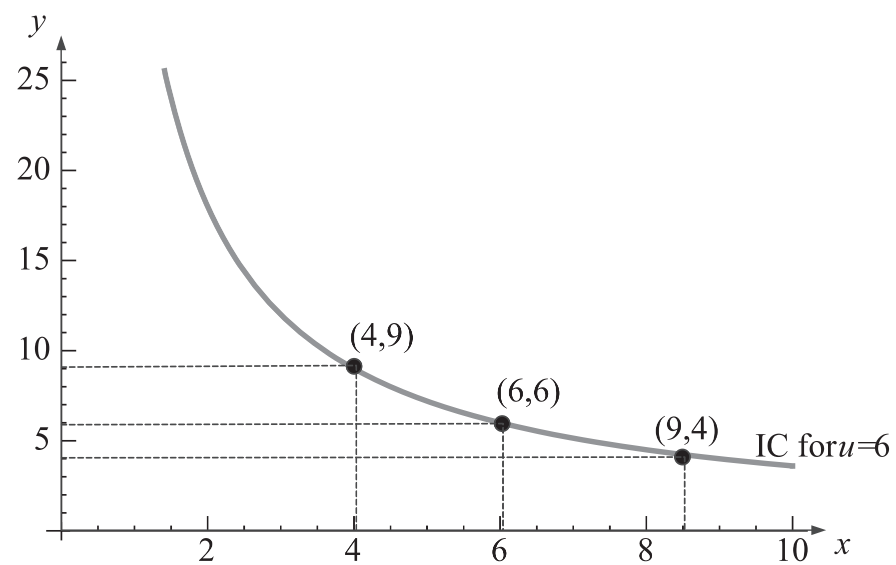

Figure 2.3b

Properties of Indifference Curves

- ICs cannot intersect. It holds from monotonicity.
  - ICs in the figure intersect at bundle A, violating monotonicity.

    - Bundle B lies northeast of C. With monotonicity, $u_B > u_C$.
    - Bundle D lies northeast of E. With monotonicity $u_D > u_E$.
    - Bundles C and D lie on the same IC, $u_C = u_D$. Similarly, $u_B = u_E$.

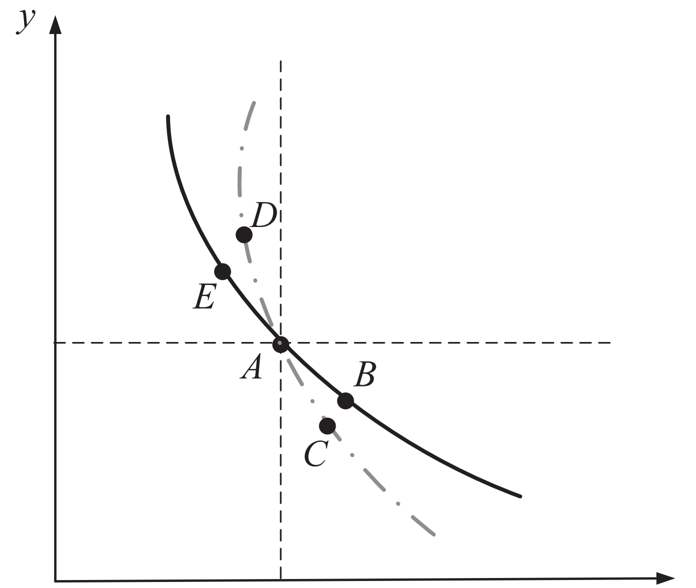

Figure 2.5

Properties of Indifference Curves

- ICs cannot intersect.

  - Combining with $u_B > u_C$,

    $$u_E = u_B > u_C = u_D,$$
    $$u_E > u_D,$$

    which contradicts the result about bundles E and D ($u_D > u_E$).

  - Monotonicity → ICs cannot intersect.

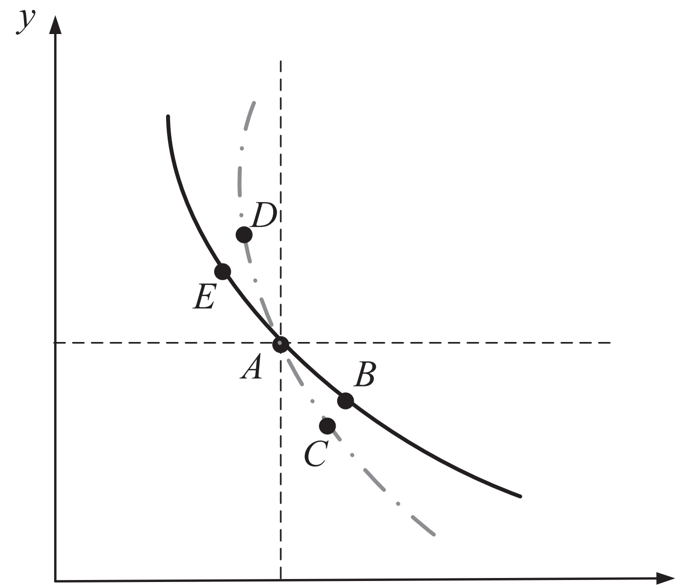

Figure 2.5

Properties of Indifference Curves

- ICs are not thick. It holds from monotonicity.
  - The thick IC depicted in the figure violates monotonicity.
    - Bundles A and B lie in the same thick IC.
    - But, bundle B contains larger amounts of goods x and y than A. Then,
      - The consumer is not indifferent between A and B.
      - By monotonicity, $u_B > u_A$.
    - Monotonicity → ICs cannot be thick.

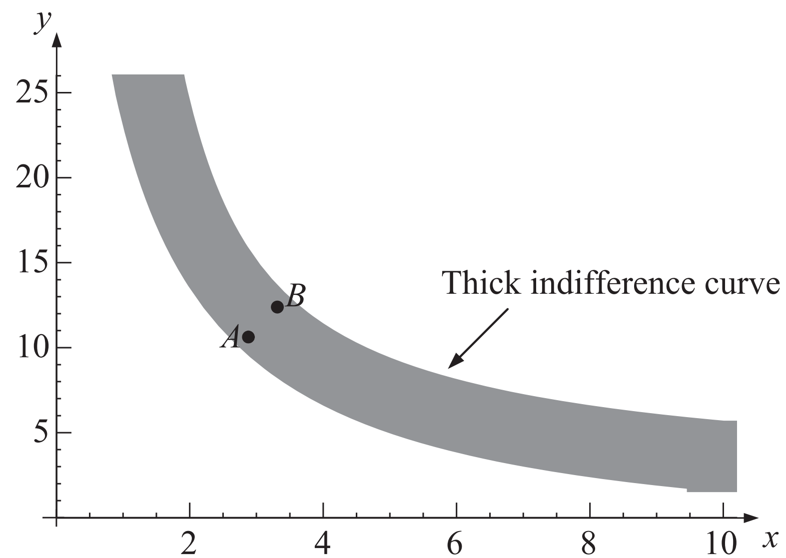

Figure 2.6

Marginal Rate of Substitution

Marginal Rate of Substitution

- Marginal rate of substitution (MRS) is the rate at which a consumer is willing to give up units of good y as she receives an additional unit of good x, in order to keep where utility level constant.

  Formally,

$$MRS_{x,y} = \frac{MU_x}{MU_y}.$$

  - When $MU_x > 0$ and $MU_y < 0$,

$$MRS_{x,y} = \frac{(+)}{(-)} = (-).$$

  - MRS represents the slope of the IC.

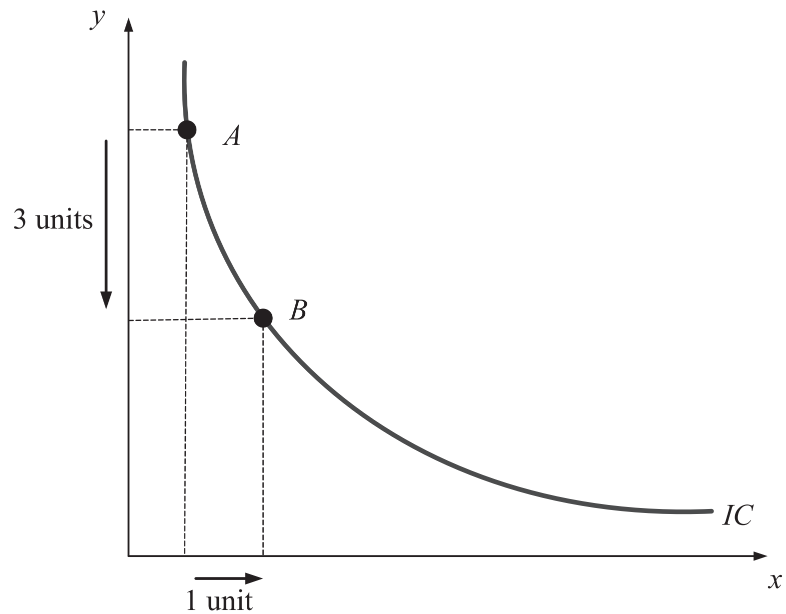

Figure 2.7

Marginal Rate of Substitution

- Diminishing MRS. The IC is relatively steep for small amounts of good x, but becomes flatter as we move rightward toward greater amounts of good x.
  1. *Preference for variety*. ICs are bowed in toward the origin.
     - The consumer is indifferent between extreme bundles, such as A and C, which yield utility level of u_1.
     - She prefers more balanced bundles, such C, yielding a higher utility of u_2.

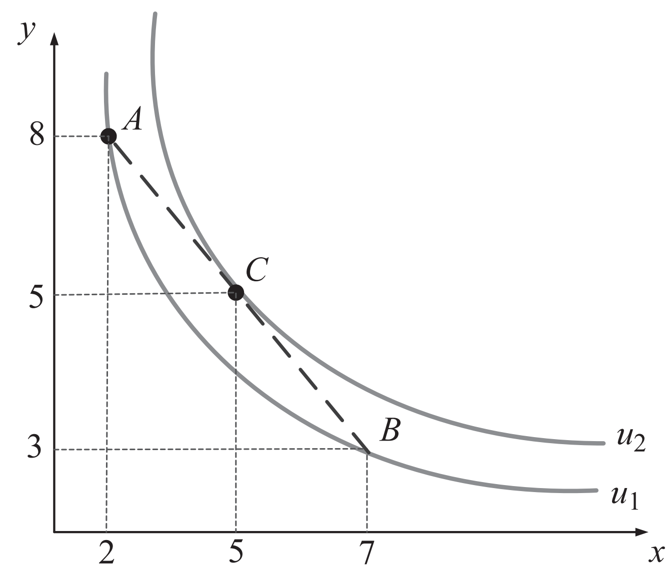

Figure 2.8

Marginal Rate of Substitution

- Diminishing MRS.

  2. *Decreasing willingness to substitute*.

     - At A, MU_x is high while MU_y is low.
       - The consumer is willing to give up several units of y to obtain more units of x.
     - At C, MU_x is low and MU_y becomes high.
       - Willingness to give up units of y decreases once she has more units of x.

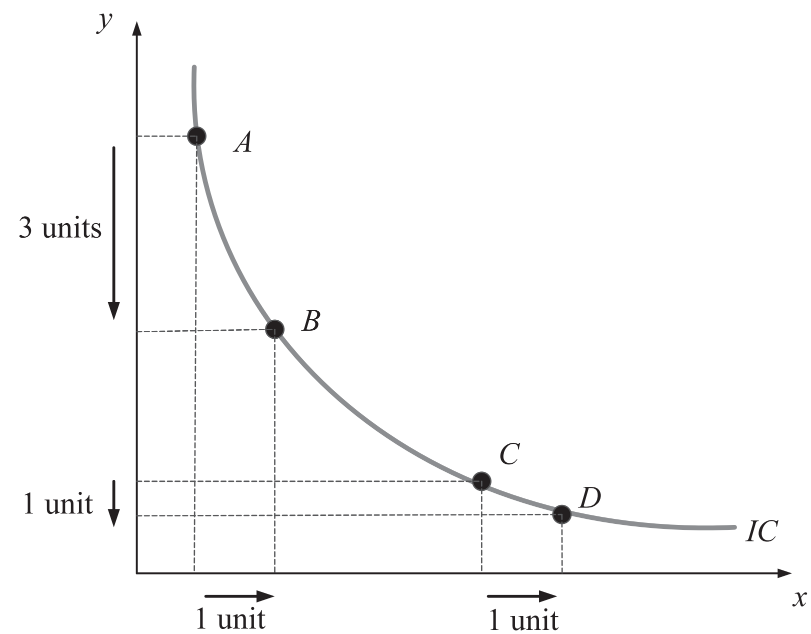

Figure 2.9

Marginal Rate of Substitution

- *Example 2.8*: *Finding MRS.*
  1. Consider utility function $u(x, y) = x^{1/2}y^{1/2}$ from example 2.5,

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}}}{\frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}} = \frac{y^{\frac{1}{2}-\left(-\frac{1}{2}\right)}}{x^{\frac{1}{2}-\left(-\frac{1}{2}\right)}} = \frac{y}{x},$$

where we cancel $1/2$ on numerator and denominator; and we use the property $\frac{x^a}{x^b} = x^{a-b}$ for exponents a and b.

$MRS_{x,y}$ is decreasing in x, yielding ICs that are bowed in toward the origin.

Marginal Rate of Substitution

- *Example 2.8* (continued):
  2. Consider the linear utility function $u(x, y) = ax + by$ where $a, b > 0$,

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{a}{b}.$$

$MRS_{x,y}$ is <u>constant</u> in x.

For instance, if $a = 10$ and $b = 4$, $MRS_{x,y} = 2.5$, indicating that the slope of the IC is -2.5 along all its points (i.e., a straight line).

Marginal Rate of Substitution

- *Example 2.8* (continued):
  3. Consider utility function $u(x,y) = ax^2 + by^3$.

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{2ax}{3by^2}.$$

$MRS_{x,y}$ is increasing in x, yielding ICs bowed away from the origin. The IC is relatively flat for low values of x, but becomes steeper as we move rightward along the x-axis.

Special Types of Utility Functions

Special Types of Utility Functions

- Perfect Substitutes:
  - Consider goods x and y. The consumer can use either good without significantly affecting her utility.
    - *Examples*: Two brands of mineral water, butter and margarine.
  - The consumer's utility function takes the form

$$u(x, y) = ax + by, \text{ where } a, b > 0.$$

  - This utility is linear in both goods because marginal utilities are constant, $MU_x = a$ and $MU_y = b$.
  - MRS is also constant,

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{a}{b}.$$

Special Types of Utility Functions

- Perfect Substitutes (cont.):
  - Solving for y in $u(x,y) = ax + by$,

$$y = \frac{u}{b} - \frac{a}{b}x.$$

  - ICs are straight lines:
    - Originating at $\frac{u}{b}$.
    - Decreasing at rate $\frac{a}{b}$.
    - Crossing the x-axis at $\frac{u}{a}$.
  - Figure 2.10 illustrates ICs evaluated at $u = 1$, and at $u = 2$.

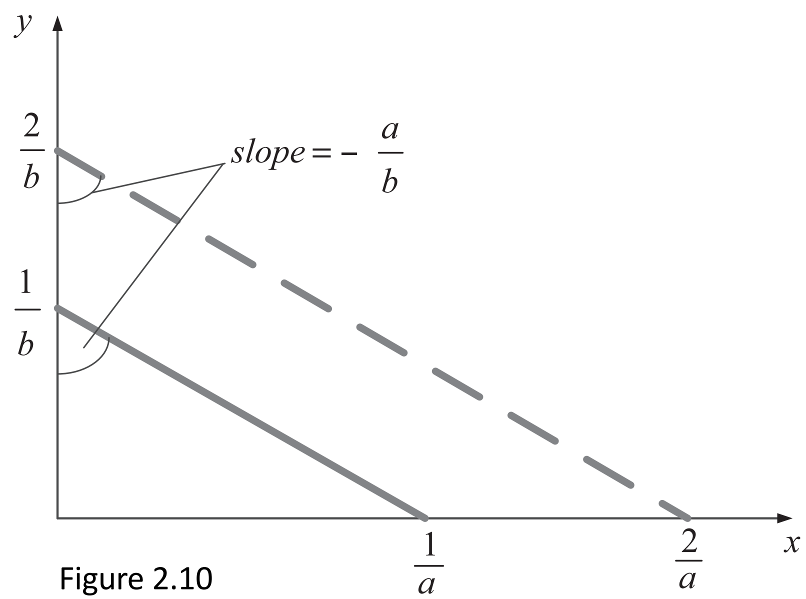

Figure 2.10

Special Types of Utility Functions

- Perfect Substitutes (cont.):
  - Recall that MRS measures the consumer's willingness to give up units of good y to obtain 1 more unit of x, keeping her utility level unaffected.
    - A constant MRS (i.e., a number) → the consumer's willingness to substitute y for additional units of x is "always the same."
    - A decreasing MRS → the consumer is willing to give up more units of good y when x becomes relatively scarce.

Special Types of Utility Functions

- Perfect Complements:
  - The consumer must consume goods in fixed proportions.
    - *Examples*: cars and gasoline, left and right shoes.
  - The utility function (referred as "Leontief") takes the form
    $$u(x, y) = A \min\{ax, by\}, \text{ where } \mathrm{A}, a, b > 0.$$
    - If $A = 1$ and $a = b = 2$, the utility function reduces to
    $$u(x, y) = \min\{2x, 2y\} = 2 \min\{x, y\}.$$

Special Types of Utility Functions

- Perfect Complements (cont.):
  - If the consumer increases the amount of x by 1 unit without increasing the amount of y, her utility does not necessarily increase.
    - If $x \geq y$, an increase in x does not increase her utility.
    - If $y > x$, an increase in x does increase her utility.

Special Types of Utility Functions

- Perfect Complements (cont.):
  - Consider the consumer has 10 units of each good, yielding

    $$u(10,10) = \min\{2\times10, 2\times10\} = \min\{20,20\} = 20$$

  - If good x is increased from 10 to 11 units, but good y is unaffected, her utility remains the same

    $$u(11,10) = \min\{2\times11, 2\times10\} = \min\{22,20\} = 20$$

  - Increasing the amount of one of the goods alone does not yield utility gains, as the consumer needs to enjoy both goods in fixed proportions.
  - Formally, preferences for complementary goods violate the monotonicity property.

Special Types of Utility Functions

- Perfect Complements (cont.):
  - ICs have an L-shape:
    - The king occurs at points where $ax = by$.
    - The slope is zero in the flat segment.
    - The slope is $-\infty$ in the vertical segment.

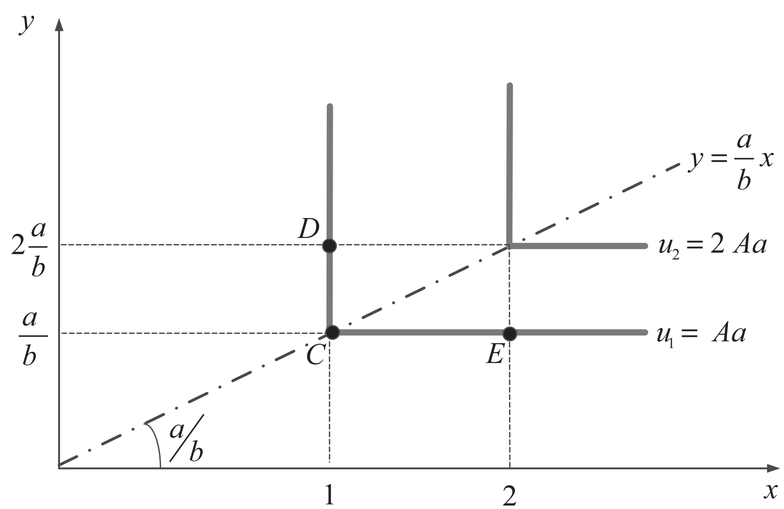

Figure 2.11

Special Types of Utility Functions

- Cobb-Douglas:
  - The consumer regards goods x and y as neither perfectly substitutable nor complementary.
  - The utility function takes the form

$$u(x, y) = Ax^{\alpha}y^{\beta}, \text{ where } \mathrm{A}, \alpha, \beta > 0.$$

  - Marginal utilities are

$$MU_x = A\alpha x^{\alpha-1}y^{\beta} \text{ and } MU_y = A\beta x^{\alpha}y^{\beta-1}.$$

which yield

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{A\alpha x^{\alpha-1}y^{\beta}}{A\beta x^{\alpha}y^{\beta-1}} = \frac{\alpha y^{\beta-(\beta-1)}}{\beta x^{\alpha-(\alpha-1)}} = \frac{\alpha y}{\beta x}.$$

Special Types of Utility Functions

- Cobb-Douglas (cont.):
  - $MRS_{x,y} = \frac{\alpha y}{\beta x}$ is decreasing in x.
  - ICs are bowed in the origin, they become flatter as x increases.

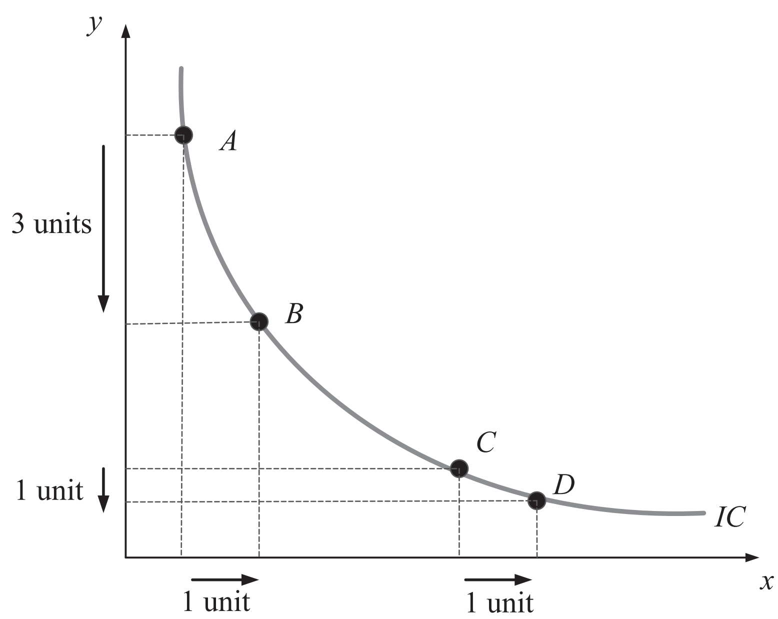

Figure 2.9

Special Types of Utility Functions

- Cobb-Douglas (cont.):
  - Special cases:

1. $A = \alpha = \beta = 1,$

$$u(x,y) = xy \quad \Rightarrow MRS_{x,y} = \frac{y}{x}.$$

2. $A = 1, \alpha = \beta,$

$$u(x,y) = x^{\alpha}y^{\alpha} = (xy)^{\alpha} \quad \Rightarrow MRS_{x,y} = \frac{y}{x}.$$

3. $A = 1, \beta = 1 - \alpha,$

$$u(x,y) = x^{\alpha}y^{1-\alpha} \quad \Rightarrow MRS_{x,y} = \frac{\alpha}{1-\alpha}\frac{y}{x}.$$

Utility Elasticity of good

- Exponents in the Cobb-Douglas utility function can be interpreted as elasticities.

- "Utility elasticity" of good x, $\varepsilon_{u,x}$, is the % increase in utility (if $\varepsilon_{u,x} > 0$) or % decrease in utility (if $\varepsilon_{u,x} < 0$) that the consumer experiences after increasing the amount of good x by 1%. Formally,

$$\varepsilon_{u,x} = \frac{\%\Delta u(x,y)}{\%\Delta x}.$$

Rearranging,

$$\varepsilon_{u,x} = \frac{\%\Delta u(x,y)}{\%\Delta x} = \frac{\frac{\Delta u(x,y)}{u(x,y)}}{\frac{\Delta x}{x}} = \frac{\Delta u(x,y)}{\Delta x}\frac{x}{u(x,y)}.$$

Utility Elasticity of a good

- When the increase in the amount of good x is marginally small,

$$\varepsilon_{u,x} = \underbrace{\frac{\partial u(x,y)}{\partial x}}_{MU_x} \frac{\overbrace{x}^{\text{Amount of } x \text{ consumed}}}{\underbrace{u(x,y)}_{\text{Utility function}}}.$$

- Applying the definition of $\varepsilon_{u,x}$ to the Cobb-Douglas utility function,

$$\varepsilon_{u,x} = \frac{\partial u(x,y)}{\partial x} \frac{x}{u(x,y)} = \underbrace{A\alpha x^{\alpha-1} y^{\beta}}_{\frac{\partial u(x,y)}{\partial x}} \frac{x}{\underbrace{A x^{\alpha} y^{\beta}}_{u(x,y)}}.$$

Utility Elasticity of a good

Simplifying,

$$\varepsilon_{u,x} = \frac{A\alpha x^{\alpha-1+1} y^{\beta}}{A x^{\alpha} y^{\beta}} = \frac{A\alpha x^{\alpha} y^{\beta}}{A x^{\alpha} y^{\beta}} = \alpha.$$

- Hence, when facing a utility function like $u(x,y) = Ax^{\alpha}y^{\beta}$, we can claim the exponent in good x, α, represents the utility elasticity of a marginal increase in x.
  - A 1% increase in the amount of good x increases utility by $\alpha\%$.
- And β is the utility elasticity of good y.

Special Types of Utility Functions

- Quasilinear:
  - Consumers who use all their additional income on one good alone, y (e.g., video games).
  - Additional income is never spent on good x (e.g., toothpaste).
  - This utility function takes the form

$$u(xy) = v(x) + by.$$

where $b > 0$, and $v(x)$ is a nonlinear function in x.

Special Types of Utility Functions

- Quasilinear (cont.):
  - *Examples*:
    - $v(x) = x^{1/2}$
    - $v(x) = lnx.$
    - Any $v(x)$ which $v'(x)$ is not a constant, but instead depends on the units of good x, good y, or both.
      - E.g., $v(x) = axy$, which $v'(x) = ay$ (not constant).

Special Types of Utility Functions

- Quasilinear (cont.):
  - For $u(x, y) = v(x) + by$, the marginal utilities are $MU_x = v'(x)$ and $MU_y = b$, which yield

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{v'(x)}{b}.$$

  - For a given value of x, the MRS is constant because it does not depend on the amount of good y.
  - *Example*: $u(x, y) = x^{1/2} + 3y$, where $v(x) = x^{1/3}$, $b = 3$.

$$MRS_{x,y} = \frac{\frac{1}{2}x^{-1/2}}{3} = \frac{1}{6\sqrt{x}}.$$

For $x = 16$, $MRS_{x,y} = \frac{1}{6\sqrt{16}} = \frac{1}{24}$, which is constant in y.

Special Types of Utility Functions

- Quasilinear (cont.):
  - ICs are parallel shifts of each other.
  - If we fix constant the value of good x (e.g., $x = 16$), the slope of the IC ($MRS_{x,y}$) is unaffected by the amount of good y.

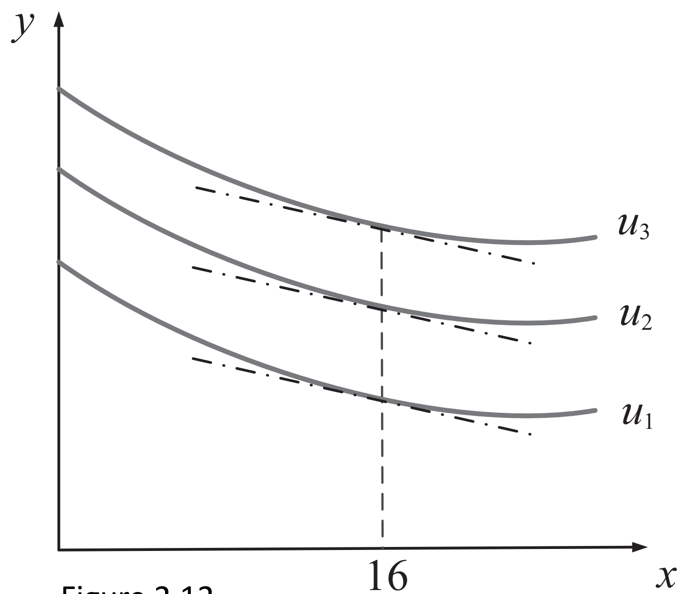

Figure 2.12

Special Types of Utility Functions

- Stone-Geary:
  - It takes a Cobb-Douglas shape, but requires the individual have a minimum amount of each good (e.g., half a gallon of water), represented as $\bar{x}$ and $\bar{y}$.
  - This utility function takes the form

    $$u(x, y) = A(x - \bar{x})^{\alpha}(y - \bar{y})^{\beta}, \text{ where } A, \alpha, \beta > 0.$$

    - The consumer obtains a positive utility from good x only after exceeding her minimal consumption $\bar{x}$, when $x > \bar{x}$. And similarly, for good y, $y > \bar{y}$.
    - When $\bar{x} = \bar{y} = 0$, the utility reduces to $u(x, y) = Ax^{\alpha}y^{\beta}$, which coincides with Cobb-Douglas utility function.

Special Types of Utility Functions

- Stone-Geary (cont.):
  - For $u(x,y) = A(x-\bar{x})^{\alpha}(y-\bar{y})^{\beta}$, marginal utilities are

$$MU_x = A\alpha(x-\bar{x})^{\alpha-1}(y-\bar{y})^{\beta},$$

$$MU_y = A\beta(x-\bar{x})^{\alpha}(y-\bar{y})^{\beta-1}.$$

  which imply

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{A\alpha(x-\bar{x})^{\alpha-1}(y-\bar{y})^{\beta}}{A\beta(x-\bar{x})^{\alpha}(y-\bar{y})^{\beta-1}} = \frac{\alpha(y-\bar{y})^{\beta-(\beta-1)}}{\beta(x-\bar{x})^{\alpha-(\alpha-1)}} = \frac{\alpha(y-\bar{y})}{\beta(x-\bar{x})}.$$

  - When $\bar{x} = \bar{y} = 0$, MRS collapses to MRS with Cob-Douglas function,

$$MRS_{x,y} = \frac{\alpha(y-\bar{0})}{\beta(x-\bar{0})} = \frac{\alpha y}{\beta x}.$$

A Look at Behavioral Economics– Social Preferences

Social Preferences

- Previous utility functions assume the consumer cares about the bundle she receives but ignore the bundle (or money) that other individuals enjoy.
- However, there are scenarios where we care about the well-being of family members or friends.
- We next explore utility functions where individuals exhibit social, rather than selfish, preferences.
    - Fehr-Schmidt Social Preferences (1999).
    - Bolton and Ockenfels Social Preferences (2000).

Social Preferences

- Fehr-Schmidt Social Preferences:
  - Consider individuals 1 and 2, and let x_1 and x_2 represent their incomes.
  - When $x_2 > x_1$, individual 2 is richer than 1. The utility of individual 1 is

$$x_1 - \underbrace{\alpha(x_2 - x_1)}_{\text{Disutility from envy}}, \text{ where } \alpha \geq 0.$$

  - When $x_2 < x_1$, individual 2 is poorer than 1. The utility of individual 1 is

$$x_1 - \underbrace{\beta(x_1 - x_2)}_{\text{Disutility from guilt}}, \text{ where } \beta \geq 0.$$

- When $\alpha = \beta = 0$, this utility function reduces to x_1 both when $x_2 > x_1$ and otherwise, which reflects selfish preferences as the individual does not suffer from envy or guilt.

Social Preferences

- Bolton and Ockenfels Social Preferences:
  - For individuals 1 and 2, the utility function of individual 1 is

$$u_1\left(x_1, \frac{x_1}{x_1 + x_2}\right).$$

    - The first term in the parentheses, x_1, represents the selfish component because individual 1 considers only her own wealth x_1.
    - The second argument, $\frac{x_1}{x_1+x_2}$, measures the share that individual 1's wealth represents of the total wealth in the group.

Social Preferences

- Bolton and Ockenfels Social Preferences (cont.):
  - *Example*:

$$u_1\left(x_1, \frac{x_1}{x_1 + x_2}\right) = x_1 + \alpha\left(\frac{x_1}{x_1 + x_2}\right)^{1/2}.$$

- If $\alpha \geq 0$, individual 1 enjoys a utility from owning a larger share of total wealth.
- If $\alpha < 0$, individual 1 suffers from owning a larger share of wealth.

Appendix. Finding the Marginal Rate of Substitution

Finding MRS

- We increase good x by 1 unit and seek to measure how many units of good y the consumer must give up to preserve her utility level.
- Because we simultaneously alter the amounts of x and y, we totally differentiate $u(x, y)$,

$$du = \frac{\partial u(x, y)}{\partial x} dx + \frac{\partial u(x, y)}{\partial y} dy.$$

- Because the consumer is moving along an IC, her utility does not vary, implying $du = 0$.
- Plugging this result and using $MU_x = \frac{\partial u(x,y)}{\partial x}$ and $MU_y = \frac{\partial u(x,y)}{\partial y}$,

$$\underbrace{0}_{du=0} = MU_x dx + MU_y dy.$$

Finding MRS

- After rearranging,

$$-MU_y dy = MU_x dx.$$

- Because we are interested in the rate at which y changes for a 1-unit increase in x,

$$-\frac{dy}{dx} = \frac{MU_x}{MU_y}.$$

- Therefore, the slope of the indifference curve, coincides with the ratio of marginal utilities.
- This ratio is referred to as the marginal rate of substitution between goods x and y, or $MRS_{x,y}$.