

# Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

## Chapter 17: Externalities and Public Goods

# Outline

- Externalities
- Social Optimum
- Restoring the Social Optimum
- Public Goods
- Common-Pool Resources

# Externalities

# Externalities

- **Externalities.** The effect that the action of an agent has on the welfare of another agent, beyond the effects transmitted by changes in prices.
  - *Examples of **negative** externalities:*
    - Production externality, a firm's pollution of a river being used for fishing downstream.
    - Consumption externality, a roommate streaming online which slows down the internet speed of other roommates.
    - The decrease in market prices after one firm brings more units for sale *cannot* be considered an externality.
      - This effect is transmitted via prices since a market for the good exists.

# Externalities

- *Examples of positive externalities:*
  - Consumption externality, an individual choosing to vaccinate helps other individuals around to be better protected.
  - Production externality, an unpatented R&D completed by a university which can be used for free.

# Unregulated Equilibrium

- In the case of a negative externality, like a factory polluting a river, the polluter ignores the effect that its actions have on others, such as poor air quality for citizen living in the area.
- If left unregulated, this polluting firm would produce a large amount of pollution, which is not necessarily optimal.

# Unregulated Equilibrium

- *Example 17.1: Unregulated equilibrium.*

- Consider a monopolist facing inverse demand function  $p(q) = 10 - q$ , and total cost  $TC(q) = 2q$ .

- The firm's PMP is:

$$\max_q (10 - q)q - 2q.$$

- Differentiating with respect to  $q$ ,

$$10 - 2q - 2 = 0,$$

$$8 = 2q.$$

- Solving for  $q$ , we obtain an unregulated equilibrium output of

$$q^U = 4 \text{ units.}$$

# Unregulated Equilibrium

- *Example 17.1* (continued):
  - Each unit of output generates  $\alpha \geq 0$  units of pollution, then the total amount of pollution the firm generates is  $4\alpha$ .
  - Figure 17.1 represents the firm's problem, which increases output  $q$  until marginal profits are zero.
    - Producing more than 4 units yields negative marginal profits.
    - Producing fewer than 4 units means the firm could still increase output and further increase its profits.

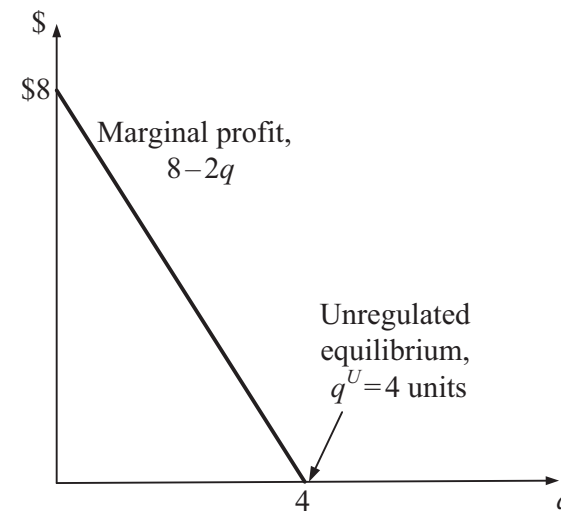


Figure 17.1



# Social Optimum

# Social Optimum

- *How can we evaluate whether the unregulated amount of pollution is socially excessive or not?*
- We first examine how much pollution would be generated by a social planner who considers both:
  - the firm's profits; and
  - the externality that pollution imposes on other individuals and firms.

# Social Optimum

- *Example 17.2: Finding the social optimum.*

- Consider the same scenario than in example 17.1.
- Every unit of emissions  $e \geq 0$  generates an external cost of  $EC = 3(e)^2$ , which is increasing and convex in  $e$ .
  - Emissions are damaging for individuals in the vicinity of the polluting factory, and at an increasing rate.
  - The first ton of  $CO_2$  might just create fog in the area, while the 10,000<sup>th</sup> ton creates serious health problems.
- Because emissions are defined as  $e = \alpha q$ ,

$$EC = 3(\alpha q)^2.$$

- If  $\alpha = \frac{1}{4}$ ,  $e = \frac{1}{4}q$ , then  $EC = 3\left(\frac{1}{4}q\right)^2 = \frac{3}{16}q^2$ .

# Social Optimum

- *Example 17.2* (continued):

- The social planner cares about society as a whole, by solving the following problem:

$$\max_q \underbrace{[(10 - q)q - 2q]}_{\text{Profits}} - \underbrace{3(\alpha q)^2}_{\text{External cost}}.$$

- Differentiating with respect to  $q$ ,

$$\begin{aligned}(10 - 2q - 2) - 6\alpha q &= 0, \\ 8 &= q(2 + 6\alpha).\end{aligned}$$

- And solving for  $q$ , the social optimum is

$$q^{so} = \frac{8}{2 + 6\alpha},$$

which is decreasing in the rate of emissions per unit of output,  $\alpha$ .

# Social Optimum

- *Example 17.2* (continued):
  - If every unit of output generates 1 unit of emissions,  $\alpha = 1$ , the social optimum is  $q^{SO} = \frac{8}{2+6} = 1$  unit.
  - When output does not generate any unit of emissions,  $\alpha = 0$ , the social optimum increases to  $q^{SO} = \frac{8}{2+0} = 4$  units, which coincides with the unregulated equilibrium.
    - External cost  $EC = 3(\alpha q)^2 = 0$ , when  $\alpha = 0$ , and the social planner's maximization problem coincides with that of the unregulated firm.

# Social Optimum

- *Example 17.2* (continued):
  - Figure 13.2 depicts the social planner's problem.
    - Leaving the firm unregulated yields  $q^U = 4$ .
    - The social optimum lies where marginal profit crosses marginal damage at  $q^{SO}$ .
      - $q > q^{SO}$  would generate more external costs than profits, being inefficient.
      - $q < q^{SO}$ , would generate more profits than externals, being not efficient either.

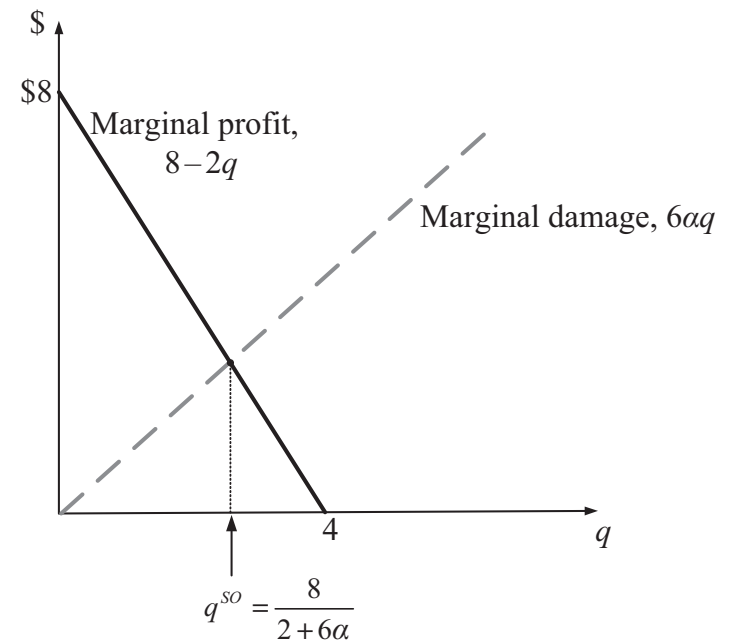


Figure 13.2

# Social Optimum

- *Example 17.2* (continued):
  - The regulator does not necessarily recommend the prohibition of the externality-generating activity.
  - The socially optimal output  $q^{SO} = \frac{8}{2+6\alpha}$ , decreases in  $\alpha$ , but it does not become zero for any value of  $\alpha$ .
    - Even in the case of  $\alpha = 100$ ,  $q^{SO} = \frac{8}{2+(6 \times 100)} = 0.013$  units.

# Social Optimum

- *Example 17.3: Prohibiting pollution.*

- Consider example 17.2, but assume  $EC = 3(e)^2 + 7e$ , which is increasing and convex in  $e$ , but yields a higher marginal damage than the  $EC$  in example 17.2.

- The social planner's problem is:

$$\max_q \underbrace{[(10 - q)q - 2q]}_{\text{Profits}} - \underbrace{[3(\alpha q)^2 + 7\alpha q]}_{\text{External cost}}.$$

- Differentiating with respect to  $q$ ,

$$(10 - 2q - 2) - (6\alpha q + 7\alpha) = 0,$$
$$8 - 7\alpha = q(2 + 6\alpha).$$

- Solving for  $q$ , the social optimum is

$$q^{so} = \frac{8 - 7\alpha}{2 + 6\alpha}.$$



# Social Optimum

- *Example 17.3* (continued):

- Social optimum  $q^{SO} = \frac{8-7\alpha}{2+6\alpha}$  is decreasing in the rate at which output transforms into emissions,  $\alpha$ , because its derivative is

$$\begin{aligned}\frac{\partial q^{SO}}{\partial \alpha} &= \frac{-7(2+6\alpha) - 6(8-7\alpha)}{(2+6\alpha)^2} = -\frac{62}{4(1+3\alpha)^2} \\ &= -\frac{31}{4(1+3\alpha)^2},\end{aligned}$$

which is negative for all values of  $\alpha$ .

# Social Optimum

- *Example 17.3* (continued):

- Social optimum  $q^{SO} = \frac{8-7\alpha}{2+6\alpha} \leq 0$  so long as  $8 - 7\alpha \leq 0$  or  $\alpha \geq \frac{8}{7}$ .
  - If every unit of output generates slightly more than 1 unit of emissions, the socially optimal output should be reduced to zero, banning the pollution-generating activity.

# Restoring the Social Optimum

# Restoring the Social Optimum

- *How to induce agents to internalize externalities, rather than ignoring them completely in the unregulated equilibrium?*
  1. Bargaining between the affected parties.
  2. Market intervention through government policy.

# Bargaining between the Affected Parties

- **Coase theorem** (Coase, 1960). The agents producing the externality and those affected can negotiate, generating a socially optimal amount of externality, if:
  - (1) All parties are perfectly informed about each other's benefits and costs.
  - (2) The negotiation and transaction costs are zero.
  - (3) The amount of the externality is observable by a third part.
  - (4) Their agreement is enforceable.

This result holds both when the property rights for the resource are assigned to the agent generating the externality, and when they are assigned to the agent affected by the externality.

# Bargaining between the Affected Parties

- *Example:*
  - An upstream firm pollutes a river, affecting a fishing farm located downstream.
  - As water becomes more polluted, the fishing farm needs to spend more resources in filtering water.
  - A production externality arises because pollution affects the costs of the fishing farm.
  - We analyze two cases:
    - Property rights over the river are assigned to the fishing farm.
    - The polluting firm owns the river.

# Bargaining between the Affected Parties

- *Example* (continued):

*Fishing farm* owns the river.

- The river would be initially completely clean. The externality-generating activity would be  $q = 0$ .
- *Is this outcome efficient?* No!
- The polluting firm could negotiate with the fishing farm and pay for an increase in the externality-generating activity from  $q = 0$  to exactly  $q^{SO}$  units.

# Bargaining between the Affected Parties

- *Example* (continued):

*Fishing farm* owns the river (cont.).

- Output levels between  $q = 0$  and  $q^{SO}$ 
  - generate more profits for the polluting firm than the external cost imposed on the fishing farm.
- Beyond  $q^{SO}$ ,
  - the polluting firm would obtain additional profits but they are small than the additional compensation the fishing farm needs to accept the increase in pollution.



# Bargaining between the Affected Parties

- *Example* (continued):

*Polluting firm* owns the river.

- The river would initially be completely dirty, as the firm would choose  $q^U$ .
- *Is this outcome efficient?* No!
- The fishing farm could negotiate with the polluting firm and for a decrease in the externality-generating activity, from  $q^U$  to exactly  $q^{SO}$ .

# Bargaining between the Affected Parties

- *Example* (continued):

*Polluting firm* owns the river (cont.).

- Output levels between  $q^{SO}$  and  $q^U$ 
  - generate a larger external cost for the fishing farm than additional profits for the polluting firm.
- When reducing pollution below  $q^{SO}$ ,
  - the fishing farm would obtain additional reduction in external costs, but smaller than the additional compensation the polluting firm needs to further decrease pollution.

# Bargaining between the Affected Parties

- The Coase theorem holds when its main assumptions are satisfied:
  - *Zero negotiation costs.*
    - Negotiation costs increase as more agents generate the externality and more agents are affected.
  - *Well-defined property rights.*
    - Allowing both parties to know who should be compensated for an increase or decrease of the externality.
  - *Perfect information.*
    - Agents must be well informed about the benefits and costs that the other party experiences from the externality.

# Bargaining between the Affected Parties

- *Observable pollution and enforceable contracts.*
  - The amount of pollution must be observable by a third party, and the contract must be enforceable in case one of the parties breaks it.
- When any of these conditions does not hold, the negotiation does not generate an efficient amount of the externality.

# Government Intervention

- Public policy seeking to correct externalities can take the form of:
  - **Quota**, which sets an upper limit on the amount of the externality that agents can generate. *Examples:*
    - Maximum tons of  $CO_2$  that firms can emit per year.
    - Maximum amount of fish that a fishing company can appropriate.
  - **Emission fee**, which increases the cost that the firm faces per unit of the output generating externality. *Example:*
    - The firm pays \$7 per ton of cement being produced, as this production generates emissions.

# Emission Quotas

- If the regulator seeks to induce a socially optimal output  $q^{SO}$ , she can set an emission quota of exactly  $q^{SO}$ :
  - When the firm emits *less* than  $q^{SO}$ , no fines are imposed.
  - When the firm emits *more* than  $q^{SO}$ , a hefty fine is levied.
- In the case of example 17.2:
  - The regulator can set the emission quota at  $q^{SO} = \frac{8}{2+6\alpha'}$ , where  $\alpha$  denotes the rate at which every unit of output transforms into emissions.
    - If  $\alpha = 1/3$ , the emission quota would be  $q^{SO} = \frac{8}{2+6\frac{1}{3}} = 2$  tons of  $CO_2$ .

# Emission Fees

- If the regulator seeks to induce a socially optimal output  $q^{SO}$ , she only needs to set an emission fee  $t$  that induces the firm to produce exactly  $q^{SO}$ .
  - By anticipating the firm's production behavior, the regulator knows how the firm reacts to the emission fee (which increases its unit cost by  $t$ ).

# Emission Fees

- *Example 17.4: Finding optimal emission fees.*
  - From example 17.2, a polluting monopolist faces a linear demand  $p(q) = 10 - q$  and  $MC(q) = 2$ .
  - The regulator seeks to induce  $q^{SO} = 2$  tons of  $CO_2$ .
  - She faces a two-period game:
    - *First stage*, the regulator sets emission fee  $t$ .
    - *Second stage*, observing this fee, the polluting firm responds choosing its output  $q$ .
  - We solve this sequential-move game by applying backward induction.



# Emission Fees

- *Example 17.4* (continued):

- *Second stage.* If the regulator sets a fee  $t$  on every unit of output, the monopolist's PMP becomes

$$\max_q (10 - q)q - (2 + t)q,$$

where the firm's unit cost increases from  $2q$  under to regulation to  $(2 + t)q$  under regulation.

- Differentiating and solving with respect to  $q$ ,

$$10 - 2q - (2 + t) = 0,$$

$$q(t) = \frac{8 - t}{2}.$$

- When  $t = 0$ ,  $q(0) = 4$  units (unregulated scenario).
- When  $t > 0$ , output decreases in the severity of the fee.

# Emission Fees

- *Example 17.4* (continued):

- *First stage.* Anticipating the output that maximizes the firm's profits  $q(t) = \frac{8-t}{2}$ , she sets it equal to the socially optimal output  $q^{SO} = 2$  tons of  $CO_2$  that she seeks to induce

$$\frac{8-t}{2} = 2,$$

$$8-t = 4,$$

$$t = \$4.$$

- By setting a fee of \$4 per unit of output, the regulator increases the monopolist's costs, which induces the firm to voluntarily produce  $q^{SO}$ .

# Public Goods

# Public Goods

- “Public goods” are goods and services which are:
  - *Nonrival*, its consumption by one individual does not reduce the amount of the good available to others.
  - *Nonexcludable*, preventing an individual from enjoying the good is extremely expensive or impossible.
- *Examples:*
  - National defense.
  - Clean air.
- Goods that not satisfy either property are “private goods,” such as an apple.

# Public Goods

- Taxonomy of goods when combining rivalry and excludability properties.

Table 17.1

	Excludable	Nonexcludable
Rival	Private goods (Apples)	Common-Pool Resources (Fishing grounds)
Nonrival	Club goods (Gyms)	Public Goods (National defense)

- Non-excludable goods result in **free-riding behavior**, in which consumers do not pay for the goods because they expect that others will pay.

# Public Goods

- *Example 17.5: Free-riding of public goods.*

- Consider 2 roommates cleaning their apartments on Saturday.
- Every roommate  $i$  simultaneously and independently chooses the number of hours she spends cleaning,  $h_i \in [0, 24]$ .
- Her utility from cleaning is

$$u_i(h_i, h_j) = \underbrace{(24 - h_i)}_{\text{Leisure}} + \underbrace{\beta h_i (h_i + h_j)}_{\text{Cleaner apartment}}.$$

- $(24 - h_i)$  indicates the number of hours she enjoys in leisure.
- $\beta h_i (h_i + h_j)$  reflects the benefit she obtains from living in a cleaner apartment, which is increasing  $h_i, h_j$ , and in parameter  $\beta > 0$ .

# Public Goods

- *Example 17.5* (continued):

- Roommate  $i$  chooses the hours she spends cleaning,  $h_i$ , to maximize her utility  $u_i(h_i, h_j)$ . Differentiating with respect to  $h_i$ ,

$$\begin{aligned} -1 + 2\beta h_i + \beta h_j &= 0, \\ 2\beta h_i &= 1 - \beta h_j. \end{aligned}$$

- Solving for  $h_i$ ,

$$h_i(h_j) = \frac{1 - \beta h_j}{2\beta} = \frac{1}{2\beta} - \frac{1}{2} h_j.$$

- This expression determines the optimal cleaning time for individual  $i$ , as a function of individual  $j$ 's cleaning time. It can be understood as a “best response function.”

# Public Goods

- *Example 17.5* (continued):

- Roommate  $i$ 's best response function,  $h_i(h_j) = \frac{1}{2\beta} - \frac{1}{2}h_j$ .

- When  $h_j = 0$ ,  $h_i = \frac{1}{2\beta}$ .
- When roommate  $j$  increases her cleaning time, roommate  $i$  decreases her own because  $i$  can free-ride off  $j$ 's cleaning time.
- When  $h_j \geq \frac{1}{\beta}$ , roommate  $i$  does not spend time cleaning.

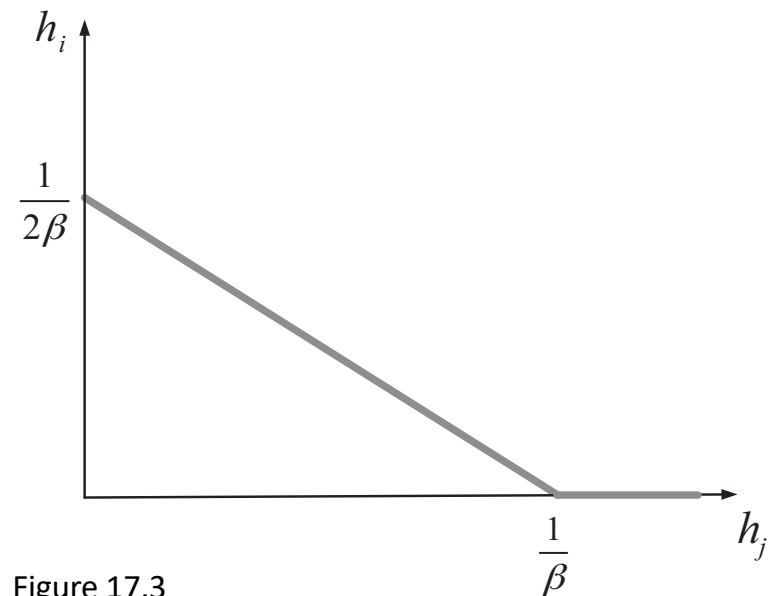


Figure 17.3



# Public Goods

- *Example 17.5* (continued):

- A symmetric best response function applies to roommate  $j$ ,  $h_j(h_i)$ .
- Invoking symmetry,  $h_i^* = h_j^* = h^*$ .
- Inserting  $h^*$  into  $h_i(h_j)$ , and solving for  $h^*$ ,

$$h^* = \frac{1 - \beta h^*}{2\beta},$$

$$2\beta h^* = 1 - \beta h^*,$$

$$h^* = \frac{1}{3\beta}.$$

- If  $\beta = 1/10$ , every roommate would spend  $\frac{1}{3(1/10)} \cong 3.3$  hours cleaning on Saturday.

# Public Goods

- *How can free-riding be prevented?*
  - Require all individuals (users and nonusers) to pay for the provision of the good via taxes rather than voluntary contributions.
  - Require users to pay a certain amount every time they use the good.
  - *Example: Highways*
    - Drivers pay tolls every time they access a road, transforming the nature of the good from nonexcludable (freeway) to excludable (controlled-access highway).
    - Tolls help to alleviate traffic congestion, as they can vary significantly depending on the time of the day.

# Common-Pool Resources

# Common-Pool Resources

- Assume  $N$  individuals have access to a resource (e.g., fishing ground).
- Every unit of appropriation is sold in the international market, which is perfectly competitive.
  - Every fisherman's appropriation (e.g., 20 tons of cod) represents a small share of the industry catches, which does not affect market prices for this variety of fish.
- Market price  $p = \$1$  is given.
- Every firm faces the cost function

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}, \quad (17.1)$$

where  $Q_{-i} = \sum_{j \neq i} q_j$  represents the sum of all appropriations by individuals different than  $i$ .

# Common-Pool Resources

- When only two fishermen exploit the resource, the cost function in equation (17.1) simplifies to

$$C(q_1, q_2) = \frac{q_1(q_1 + q_2)}{S}$$

for fisherman 1 (so that  $Q_{-i} = q_2$ ).

And similarly,

$$C(q_2, q_1) = \frac{q_2(q_2 + q_1)}{S}$$

for fisherman 2 (so that  $Q_{-i} = q_1$ ).

- $S > 0$  denotes the stock of the resource.
  - A more abundant resource (higher  $S$ ) decrease fisherman  $i$ 's cost because fish is easier to catch.

# Common-Pool Resources

- The cost function is increasing in fisherman  $i$ 's own appropriation,  $q_i$ , and his rivals' appropriation,  $Q_{-i}$ .
- Every fisherman chooses its appropriation level  $q_i$  to maximize its profits as follows:

$$\max_{q_i} \pi_i = q_i - \frac{q_i(q_i + Q_{-i})}{S}.$$

## Finding Equilibrium Appropriation.

- Differentiating with respect to  $q_i$ ,

$$\underbrace{1}_{MR} - \underbrace{\frac{2q_i + Q_{-i}}{S}}_{MC} = 0$$

The fisherman increases his appropriation until the marginal revenue and the cost exactly offset each other.

# Common-Pool Resources

## Finding Equilibrium Appropriation (cont.).

- Rearranging the previous expression and solving for  $q_i$ ,

$$S = 2q_i + Q_{-i},$$
$$q_i(Q_{-i}) = \frac{S}{2} - \frac{1}{2}Q_{-i}. \quad (BRF_i)$$

- $q_i(Q_{-i})$  describes how many units to appropriate,  $q_i$ , as a response to how many units his rivals' appropriate,  $Q_{-i}$ .
- He appropriates  $\frac{S}{2}$  when  $Q_{-i} = 0$ .
- His appropriation decreases as  $Q_{-i} > 0$ .

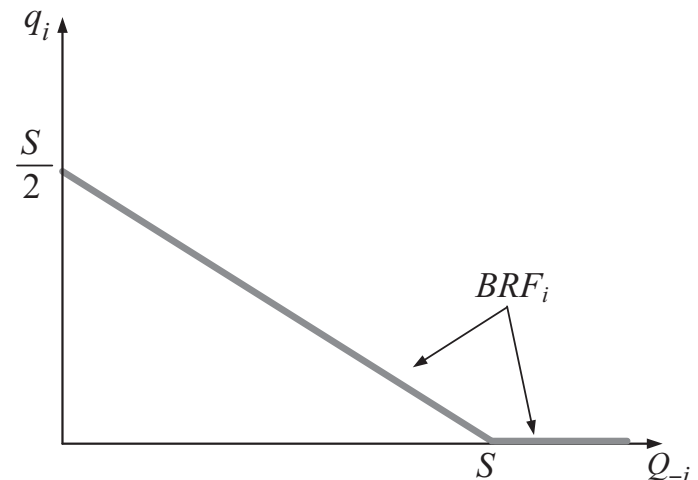


Figure 17.4

# Common-Pool Resources

## Finding Equilibrium Appropriation (cont.).

- Firms are symmetric in this scenario because they face the same price for each unit of fish (\$1), and the same cost function.
- Therefore, the best response function of any other firm  $j$  ( $j \neq i$ ) is symmetric to  $BRF_i$ ,  $q_j(Q_{-j}) = \frac{S}{2} - \frac{1}{2}Q_{-j}$ .
- In a symmetric equilibrium,  $q_1^* = q_2^* = \dots = q_N^* = q^*$ . Therefore  $Q_{-i}^* = \sum_{j \neq i} q^* = (N - 1)q^*$ .
- Inserting this result in the best response function,

$$q^* = \frac{S}{2} - \frac{1}{2}(N - 1)q^*. \quad (17.2)$$



# Common-Pool Resources

## Finding Equilibrium Appropriation (cont.).

- Rearranging (17.2) and solving for  $q^*$ ,

$$\frac{2q^* + (N - 1)q^*}{2} = \frac{S}{2},$$

$$(N + 1)q^* = S,$$

$$q^* = \frac{S}{N + 1}.$$

- If  $S = 100$  tons of fish and  $N = 9$  fishermen,  $q^* = \frac{100}{9+1} = 10$  tons.
- Generally, the equilibrium appropriation  $q^*$  increases in the stock of the resource,  $S$ , but decreases in the number of firms competing for the resource,  $N$ .

# Common-Pool Resources— Joint Profit Maximization

# Common-Pool Resources— Joint Profit Maximization

- Can fishermen increase their profits if they coordinate their catches? YES!
- Consider the case of 2 fishermen,  $N = 2$ .
- When fishermen 1 and 2 coordinate their catches, they maximize joint profits as follows:

$$\max_{q_1, q_2} \pi_1 + \pi_2 = \left( q_1 - \frac{q_1(q_1 + q_2)}{S} \right) + \left( q_2 - \frac{q_2(q_2 + q_1)}{S} \right),$$

$$\max_{q_1, q_2} (q_1 + q_2) - \frac{(q_1 + q_2)^2}{S}.$$

- Differentiating with respect to  $q_1$ ,

$$\underbrace{1}_{MR} - \frac{2(q_1 + q_2)}{\underbrace{S}_{MC}} = 0. \quad (17.3)$$

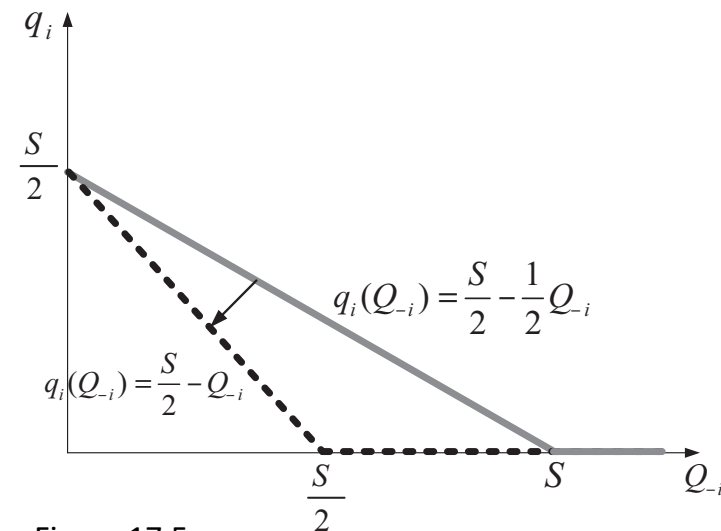
# Common-Pool Resources— Joint Profit Maximization

- Every fisherman internalizes the cost externality that his appropriation generates on other fishermen, as larger  $q_i$  increases the cost of fisherman  $j$ .
- Rearranging equation (17.3) and solving for  $q_1$ ,

$$S = 2(q_1 + q_2)$$
$$q_1(q_2) = \frac{S}{2} - q_2. \quad (17.4)$$

# Common-Pool Resources— Joint Profit Maximization

- $q_1(q_2) = \frac{S}{2} - q_2$  originates at the same height as fisherman  $i$ 's best response function, but decreases in his rival's appropriation faster.
- For a given  $q_2$ , firm 1 chooses to appropriate fewer units when firms coordinate their exploitation of the resource than when every firm independently selects its own appropriation.



# Common-Pool Resources— Joint Profit Maximization

- To confirm this finding, simultaneously solve for  $q_1$  and  $q_2$  in equation (17.4),

$$q_1(q_2) = \frac{S}{2} - q_2 \text{ for fisherman 1,}$$

$$q_2(q_1) = \frac{S}{2} - q_1 \text{ for fisherman 2.}$$

- These equations overlap each other, indicating that a continuum of optimal pairs  $(q_1, q_2)$  solves the joint maximization problem.

# Common-Pool Resources— Joint Profit Maximization

- Because firms are symmetric, a natural equilibrium is

$$q_1^{JP} = q_2^{JP} = q^{JP}.$$

- Inserting it in equation  $q^{JP} = \frac{S}{2} - q^{JP}$  and solving for  $q^{JP}$ ,

$$q^{JP} = \frac{S}{4}.$$

# Common-Pool Resources— Joint Profit Maximization

- Comparing  $q^{JP} = \frac{S}{4}$  with  $q^* = \frac{S}{N+1} = \frac{S}{2+1}$  (evaluated for the case of  $N = 2$ ),

$$q^* > q^{JP}$$

$$\frac{S}{3} > \frac{S}{4}.$$

- Agents exploit the resource less intensively when they coordinate their appropriation decisions than when they do not coordinate their exploitation.