

Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 10: Monopoly

Outline

- Barriers to entry
- Profit Maximization Problem (PMP)
- Common Misunderstandings of Monopoly
- The Lerner Index and Inverse Elasticity Pricing Rule
- Multiplant Monopolist
- Welfare Analysis under Monopoly
- Advertising in Monopoly
- Monopsony

Barriers to Entry

Barriers to entry

“Why do monopolies exist in the first place if they are bad for society?”

- **Structural barriers:** Incumbent firms may have advantages that are unattractive for potential entrants.
 - Cost advantage (e.g., superior technology)
 - Demand advantage (e.g., large group of loyal customers)
- **Legal barriers:** Incumbents firms may be legally protected.
 - *Example:* Patents.
- **Strategic barriers:** Incumbent firms can take actions to deter entry, by building a reputation of being a tough competitor.
 - *Example:* Price wars.

Profit Maximization Problem (PMP)

Profit Maximization Problem

- In a monopolized industry,
 - A single firm decides the output level, $q = Q$.
 - A change in q does not affect market prices, as measured by the inverse demand function $p(q)$, which decreases in q .
 - *Example* (linear inverse demand):

$$p(q) = a - bq, \text{ where } a, b > 0$$

- When the monopolist sells few units (low values of q), consumers are willing to pay a relatively high price for the scarce good.
- As the firm offers more units (larger values of q), consumers are willing to pay less for the relatively abundant good.

Profit Maximization Problem

- **PMP:** The monopolist chooses its output q to maximize its profits π

$$\max_q \pi = TR(q) - TC(q) = p(q)q - TC(q).$$

- Differentiating with respect to q ,

$$p(q) + \frac{\partial p(q)}{\partial q} q - \frac{\partial TC(q)}{\partial q} = 0.$$

- Rearranging,

$$\underbrace{p(q) + \frac{\partial p(q)}{\partial q} q}_{\text{Marginal revenue, } MR(q)} = \underbrace{\frac{\partial TC(q)}{\partial q}}_{\text{Marginal cost, } MC(q)}.$$

Profit Maximization Problem

- Therefore, to maximize profits, the monopolist increases its output q until

$$MR(q) = MC(q).$$

- If $MR(q) > MC(q)$, the monopolist would have incentives to increase output q because its revenues increases more than its cost.
- If $MR(q) < MC(q)$, the monopolist would have incentives to decrease its output q .

Profit Maximization Problem

- A closer look at marginal revenue,

$$MR(q) = \underbrace{p(q)}_{\text{Positive effect}} + \underbrace{\frac{\partial p(q)}{\partial q} q}_{\text{Negative effect}}.$$

- When monopolist increases output by 1 unit, this additional unit produces 2 effects on firm's revenue:
 - **Positive effect.** If the firm sells 1 more unit, it earns $p(q)$, and the firm's revenue increases.
 - **Negative effect.** When offering 1 more unit, the firm needs to decrease the price of previous units sold, $\frac{\partial p(q)}{\partial q} < 0$.
- In summary, the total effect of increasing output must exactly offset the additional costs of producing 1 more unit, $MR(q) = MC(q)$.

Profit Maximization Problem

- *Example 10.1: Positive and negative effects of selling more units.*

- Consider $p(q) = 10 - 3q$. If the firm were to marginally increase its output,

$$\begin{aligned}MR(q) &= p(q) + \frac{\partial p(q)}{\partial q}q \\ &= (10 - 3q) + (-3)q \\ &= 10 - 6q.\end{aligned}$$

- If the firm sells $q = 2$ units,

$$TR(1) = p(2)2 = (10 - 3 \times 2)2 = \$8.$$

Profit Maximization Problem

- *Example 10.1* (continued):

- Evaluating $MR(q)$ at $q = 2$ units yields

$$MR(2) = (10 - 3 \times 2) + (-3)2 = 4 - 6 = -\$2.$$

- The monopolist's revenue experiences:
 - A **positive effect** of \$4 because it now sells 1 more unit at price \$4.
 - A **negative effect** because selling 1 more unit entails applying a price discount of \$3 on all previous units.
 - Overall, these two effect generates a total (net) decrease in revenue of \$2.

Profit Maximization Problem

- *Example 10.2: Finding marginal revenue with linear demand.*

- Consider $p(q) = a - bq$. Marginal revenue is

$$MR(q) = p(q) + \frac{\partial p(q)}{\partial q}q = (a - bq) + (-b)q = a - 2bq.$$

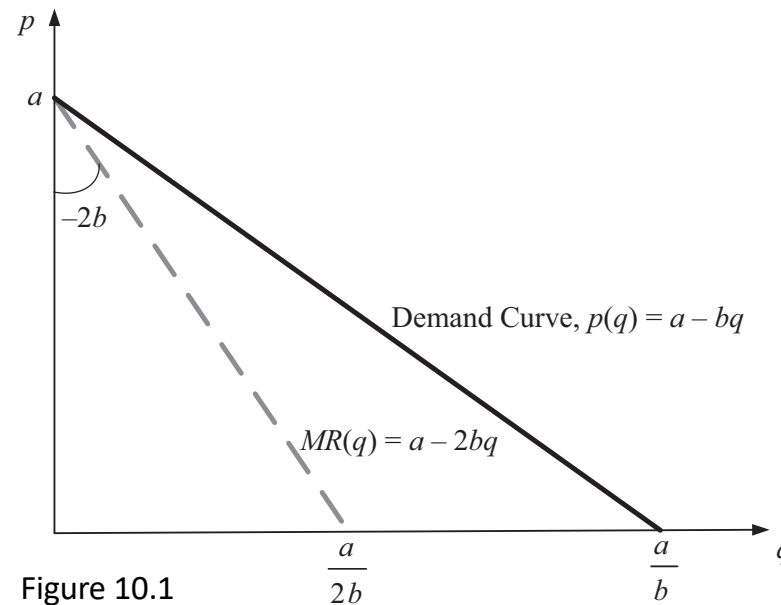


Figure 10.1

Profit Maximization Problem

- Two properties of marginal revenue curve:

1) $MR(q)$ lies below the demand curve.

We need,

$$MR(q) \leq p(q),$$

$$p(q) + \frac{\partial p(q)}{\partial q} q \leq p(q) \rightarrow \frac{\partial p(q)}{\partial q} q \leq 0.$$

2) $MR(q)$ and the demand curve originate at the same height.

At $q = 0$,

$$p(0) = a - b \times 0 = a,$$

$$MR(0) = p(0) + \frac{\partial p(q)}{\partial q} q = p(0) = a.$$

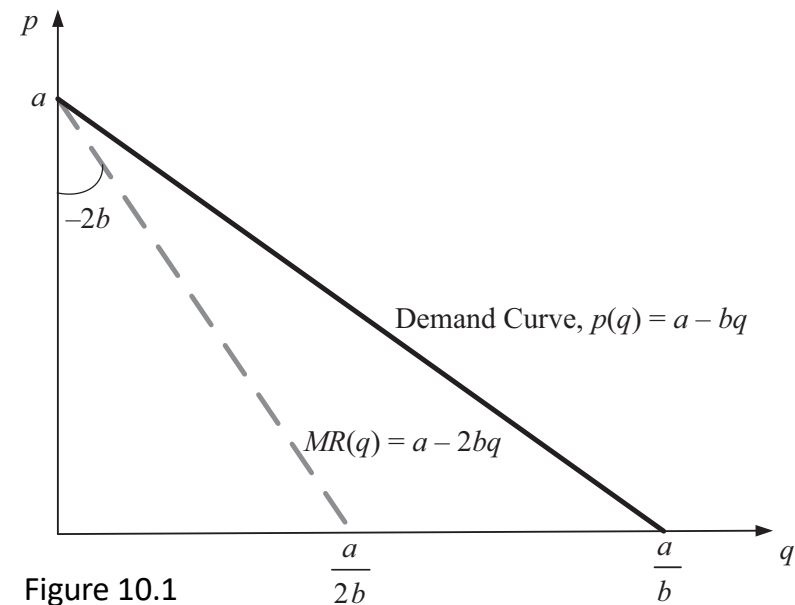


Figure 10.1

Profit Maximization Problem

- *Example 10.3: Finding monopoly output with linear demand.*
 - Consider $p(q) = a - bq$, and $TC(q) = cq$, where $c > 0$.
 - The monopolist maximizes its profits by solving

$$\max_q \pi = TR(q) - TC(q) = \underbrace{(a - bq)q}_{TR} - \underbrace{cq}_{TC}$$

- Differentiating with respect to q yields

$$a - 2bq - c = 0.$$

- Rearranging,

$$\underbrace{a - 2bq}_{MR(q)} = \underbrace{c}_{MC(q)}$$

Profit Maximization Problem

- *Example 10.3* (continued):

- Rearranging,

$$a - c = 2bq.$$

- Solving for output q ,

$$q^M = \frac{a - c}{2b}.$$

- We find the monopoly price by inserting this output into the inverse demand function

$$\begin{aligned} p(q^M) &= a - bq^M = a - b \overbrace{\left(\frac{a-c}{2b}\right)}^{q^M} \\ &= \frac{2ab - b(a - c)}{2b} = \frac{a + c}{2}. \end{aligned}$$

Profit Maximization Problem

- *Example 10.3* (continued):

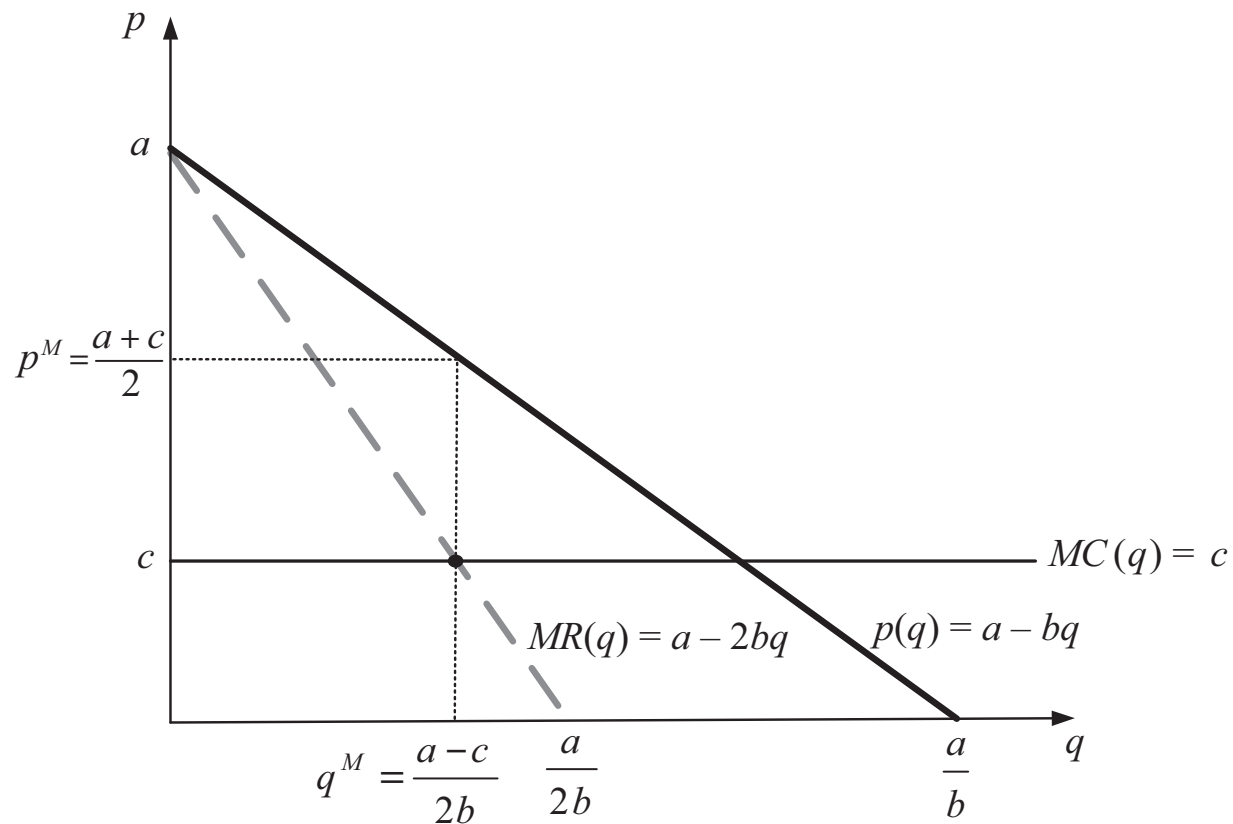


Figure 10.2

Profit Maximization Problem

- *Example 10.3* (continued):
 - Monopoly profits are

$$\begin{aligned}\pi^M &= p(q^M)q^M - cq^M \\ &= \frac{a+c}{2} \cdot \frac{a-c}{2b} - c \frac{a-c}{2b} \\ &= \left(\frac{a+c}{2} - c \right) \frac{a-c}{2b} \\ &= \frac{(a-c)^2}{4b}.\end{aligned}$$

Profit Maximization Problem

- *Example 10.3* (continued):
 - Consumer surplus under this monopoly is

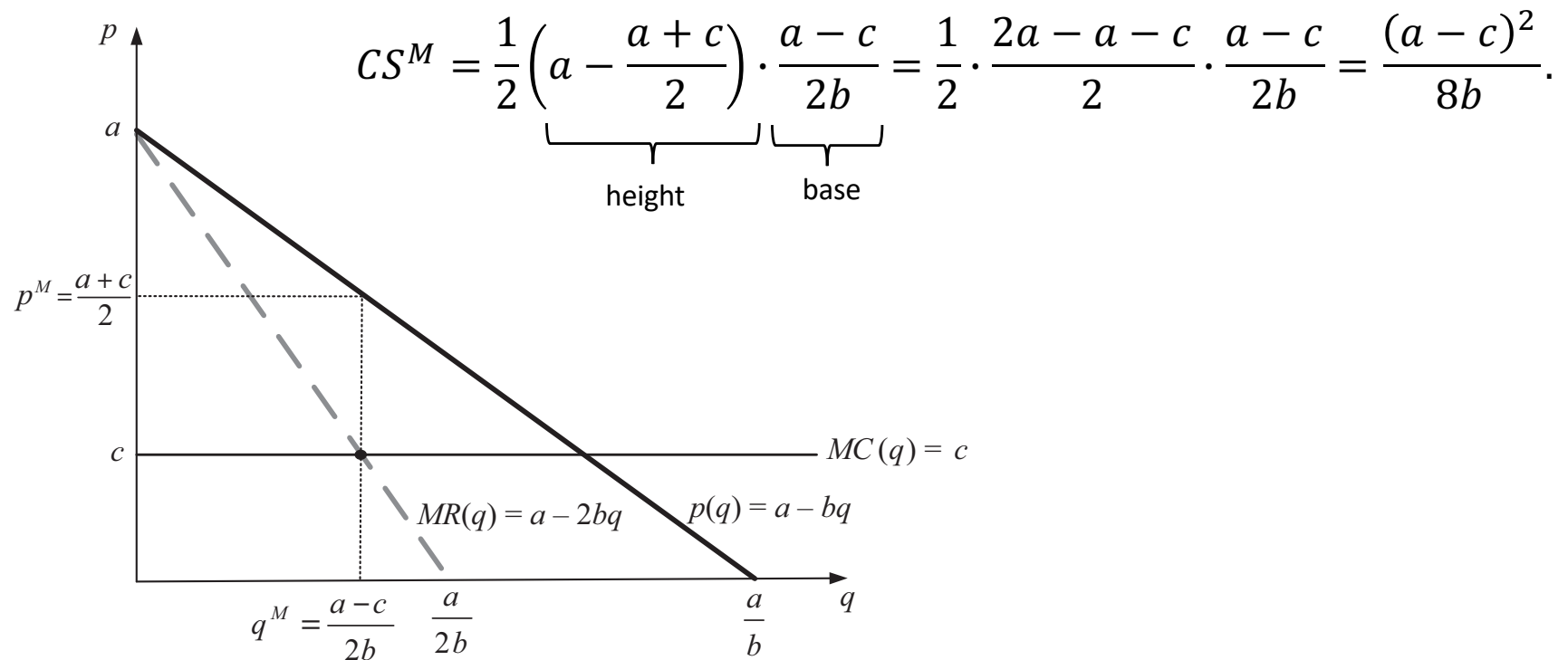


Figure 10.2

Profit Maximization Problem

- *Example 10.3* (continued):

- If inverse demand is $p(q) = 10 - q$ (i.e., $a = 10$ and $b = 1$), and $TC(q) = 4q$ (i.e., $c = 4$)

- $q^M = \frac{a-c}{2b} = \frac{10-4}{2} = 3$ units.

- $p^M = a - bq^M = \$7$.

- $\pi^M = \frac{(a-c)^2}{4b} = \frac{(10-4)^2}{4} = \frac{36}{4} = \9 .

- $CS^M = \frac{(a-c)^2}{8b} = \frac{(10-4)^2}{8} = \frac{36}{8} = \4.5 .

Common misunderstandings of Monopoly

Common Misunderstandings

1. *Monopolies do not set infinitely high prices.*

- While the monopolist is the only firm in its industry, it faces a demand curve $p(q)$, such as $p(q) = a - bq$.
- Setting higher prices might be attractive but could lead to fewer sales.
- This trade-off implies the monopolist does not set an infinitely high price because it would imply no sales at all.
 - In example 10.3, any price above $p = \$a$ (e.g., \$10 if $a = 10$) entails no sales.

Common Misunderstandings

2. *The monopolist does not have a supply curve.*

- A common misunderstanding is to consider that q^M , where $MR(q) = MC(q)$, constitutes the monopolist's supply curve.
- In perfectly competitive markets, the firm observes the given market price offers the output that satisfies $p = MC(q)$, obtaining the supply function $q(p)$.
- In a monopoly, the monopolist determines output and price simultaneously.
 - In example 10.3, when the monopolist chooses $q^M = 3$ units, it simultaneously determines $p^M = 10 - 3 = \$7$, not allowing the firm to choose different output levels for a given market price of $p^M = \$7$.

Common Misunderstandings

3. *The monopolist produces in the elastic portion of the demand curve.*
- Goods with few (or no) close substitutes tend to have a relatively inelastic demand curve.
 - Monopolies often produce goods with no close substitutes. However, it does not mean that it produces in the inelastic portion of the demand curve.

Common Misunderstandings

3. *The monopolist produces in the elastic portion of the demand curve.*

- Consider the formula of price elasticity of demand

$$\varepsilon_{q,p} = \frac{\% \Delta q}{\% \Delta p},$$

- If the monopolist produces in the inelastic portion of the demand curve, $|\varepsilon_{q,p}| < 1$, an increase in price by 1% reduces sales by less than 1%. It would increase its price, as sales would not be greatly affected but it would not be profit maximizing.
- If it produces in the elastic segment, $|\varepsilon_{q,p}| > 1$, an increase in price by 1% reduces sales by more than 1%. The firm does not have incentives to adjust its price.

Common Misunderstandings

- *Example 10.5: Price elasticity of output q^M under linear demand.*
 - Consider the monopolist in example 10.3, facing $p(q) = 10 - q$.
 - We found $q^M = 3$ units, and $p^M = \$7$.
 - We find price elasticity as

$$\varepsilon_{q,p} = \frac{\% \Delta q}{\% \Delta p} = \frac{\Delta q}{\Delta p} \cdot \frac{p}{q}.$$

- If the change in price is small, $\varepsilon_{q,p} = \frac{\partial q(p)}{\partial p} \cdot \frac{p}{q}$.

Common Misunderstandings

- *Example 10.5* (continued):

- From the inverse demand function, we obtain the direct demand function, $q(p) = 10 - p$. Then,

$$\varepsilon_{q,p} = \frac{\partial q(p)}{\partial p} \cdot \frac{p^M}{q^M} = -1 \frac{7}{3} \cong -2.33.$$

- If the monopolist increases prices by 1%, its sales decrease by 2.33%.
- Therefore, $|\varepsilon_{q,p}| = 2.33 > 1 \rightarrow$ the monopolist sets a price p^M lying in the elastic portion of the demand curve.

The Lerner Index and Inverse Elasticity Pricing Rule

The Lerner Index

- We can rewrite the profit-maximizing condition for the monopolist, $MR(q) = MC(q)$, to show a relationship between margin, $p - MC(q)$, and price elasticity, $\varepsilon_{q,p}$,

$$p(q) + \frac{\partial p(q)}{\partial q} q = MC(q).$$

- Marginal revenue can be rearranged as

$$MR(q) = p \left(1 + \frac{\partial p(q)}{\partial q} \cdot \frac{q}{p} \right),$$

$$MR(q) = p \left(1 + \frac{1}{\frac{\partial q(p)}{\partial p} \cdot \frac{p}{q}} \right) = p \left(1 + \frac{1}{\varepsilon_{q,p}} \right).$$

The Lerner Index

- Substituting this expression of $MR(q)$ into $MR(q) = MC(q)$,

$$p \left(1 + \frac{1}{\varepsilon_{q,p}} \right) = MC(q).$$

- Rearranging, $p + p \frac{1}{\varepsilon_{q,p}} = MC(q)$, or $p - MC(q) = -p \frac{1}{\varepsilon_{q,p}}$.

- Dividing both sides by p yields,

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

- Which is known as the “Lerner Index”:
 - A monopolist’s ability to set a price above marginal cost is inversely related to the price elasticity of demand.

The Lerner Index

- The “Lerner Index” is also known as the “markup index” because it measures the price markup over marginal cost.
- As demand becomes relatively **elastic**, (i.e., a more negative number) the price markup decreases.

- *Example:* If $\varepsilon_{q,p} = -4$,

$$-\frac{1}{\varepsilon_{q,p}} = -\frac{1}{-4} = 0.25,$$

price markup over marginal cost decreases to 25%.

The Lerner Index

- As demand becomes relatively **inelastic**, the price markup increases.

- *Example:* If $\varepsilon_{q,p} = -0.5$,

$$-\frac{1}{\varepsilon_{q,p}} = -\frac{1}{-0.5} = 2,$$

price markup of 200%.

The Lerner Index

- *Example 10.6: Lerner index with a linear demand.*
 - Consider market inverse demand function is $p(q) = 10 - q$.
 - Solving for q , we obtain direct demand $q(p) = 10 - p$.
 - Which yields and elasticity of

$$\begin{aligned}\varepsilon_{q,p} &= \frac{\partial q(p)}{\partial p} \cdot \frac{p}{q} \\ &= -1 \frac{p}{10 - p}.\end{aligned}$$

The Lerner Index

- *Example 10.6* (continued):
 - Assuming $MC(q) = 4$, the Lerner Index becomes

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}},$$
$$\frac{p - 4}{p} = -\left(\frac{1}{-1 \frac{p}{10 - p}}\right).$$

The Lerner Index

- *Example 10.6* (continued):

- Rearranging terms,

$$\frac{p - 4}{p} = \frac{10 - p}{p}.$$

- Which simplifies to

$$p - 4 = 10 - p.$$

- And solving for price, $p = \$7$.

The Lerner Index

- *Example 10.7: Lerner index with constant elasticity demand.*

- Consider monopolist facing demand curve

$$q(p) = 5p^{-\varepsilon}.$$

- Assuming $MC(q) = \$4$, the Lerner Index becomes

$$\frac{p - 4}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

The Lerner Index

- *Example 10.7* (continued):

- If demand curve is $q(p) = 5p^{-2}$ (i.e., $\varepsilon = -2$),

$$\frac{p - 4}{p} = -\frac{1}{-2},$$

which simplifies to $2p - 8 = p$, or $p = \$8$.

- If demand function changes to $q(p) = 5p^{-5}$,

$$p = \frac{20}{4} = \$5.$$

As demand becomes more elastic, price decreases.

Inverse elasticity pricing rule (IEPR)

- Using the Lerner index, and solving for price

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}},$$

$$p = \frac{MC(q)}{1 + \frac{1}{\varepsilon_{q,p}}}.$$

which is known as the “inverse elasticity price rule” (IEPR)

- Example:* If $MC(q) = \$4$ and $\varepsilon_{q,p} = -2$,

$$p = \frac{4}{1 + \frac{1}{-2}} = \frac{4}{\frac{1}{2}} = \$8.$$

Multipoint Monopoly

Multipiant Monopoly

- Consider a monopoly producing in two plants (factories),
 - q_1 is the output produced in plant 1,
 - q_2 is the output produced in plant 2,
 - $Q = q_1 + q_2$ represents total output across plants.
- The monopolist maximizes the joint profits from both plants

$$\begin{aligned}\max_{q_1, q_2} \pi &= \pi_1 + \pi_2 = \underbrace{TR_1(q_1, q_2) - TC_1(q_1)}_{\pi_1} + \underbrace{TR_2(q_1, q_2) - TC_2(q_2)}_{\pi_2} \\ &= [p(q_1, q_2) \times q_1 - TC_1(q_1)] + [p(q_1, q_2) \times q_2 - TC_2(q_2)] \\ &= p(q_1, q_2) \times (q_1 + q_2) - TC_1(q_1) - TC_2(q_2)\end{aligned}$$

Multipiant Monopoly

- Differentiating with respect to q_1 , yields

$$\underbrace{p(q_1, q_2) + \frac{\partial p(q_1, q_2)}{\partial q_1}}_{MR_1} = \underbrace{\frac{\partial TC_1(q_1)}{\partial q_1}}_{MC_1},$$

- And differentiating with respect to q_2 ,

$$\underbrace{p(q_1, q_2) + \frac{\partial p(q_1, q_2)}{\partial q_2}}_{MR_2} = \underbrace{\frac{\partial TC_2(q_1)}{\partial q_2}}_{MC_2},$$

- In the special case $\frac{\partial p(q_1, q_2)}{\partial q_1} = \frac{\partial p(q_1, q_2)}{\partial q_2}$, $MR_1 = MR_2 = MR$.
- The multipiant monopoly maximizes its joint profits at

$$MR = MC_1 = MC_2.$$

Multiplant Monopoly

- When $\frac{\partial p(q_1, q_2)}{\partial q_1} = \frac{\partial p(q_1, q_2)}{\partial q_2}$,

$$MR_1 = MR_2 = MR.$$

- This occurs when prices are affected to the same extent when either plant increases its productions, if $p(q_1, q_2) = 300 - q_1 - q_2$.
- The multiplant monopoly only needs to equate marginal costs across plants.

Multipoint Monopoly

- When $\frac{\partial p(q_1, q_2)}{\partial q_1} \neq \frac{\partial p(q_1, q_2)}{\partial q_2}$,

$$MR_1 \neq MR_2.$$

- This may occur if $p(q_1, q_2) = 300 - q_1 - 0.5q_2$.
- The multipoint monopoly maximizes joint profits when $MR_1 = MC_1$ and $MR_2 = MC_2$.

Multipiant Monopoly

- *Example 10.8: Multipiant Monopoly.*

- Consider $p(Q) = 100 - Q = 100 - q_1 - q_2$
- Assume the e monopolist operates 2 plants
 - Plant 1 (US) with $TC_1(q_1) = 5 + 12q_1 + 6(q_1)^2$
 - Plant 2 (Chile) with $TC_2(q_2) = 5 + 18q_2 + 3(q_2)^2$
- The monopolist chooses q_1 and q_2 to maximize joint profits from both plants

$$\max_{q_1 \geq 0, q_2 \geq 0} \pi = \pi_1 + \pi_2 = \underbrace{(100 - q_1 - q_2)q_1 - TC_1(q_1)}_{\pi_1} + \underbrace{(100 - q_1 - q_2)q_2 - TC_2(q_2)}_{\pi_2}$$

Multiplant Monopoly

- *Example 10.8* (continued):

- Differentiating with respect to q_1 ,

$$100 - 2q_1 - q_2 - 12 - 12q_1 - q_2 = 0,$$

$$88 - 14q_1 - 2q_2,$$

$$q_1 = \frac{44 - q_2}{7}.$$

- Similarly, differentiating total profits with respect to q_2 ,

$$100 - q_1 - 2q_2 - 18 - 6q_2 - q_1 = 0,$$

$$82 - 2q_1 - 8q_2,$$

$$q_2 = \frac{41 - q_1}{4}.$$

Multipoint Monopoly

- *Example 10.8* (continued):

- Inserting the result for q_2 into q_1 , we obtain

$$q_1 = \frac{44 - q_2}{7} = \frac{44 - \left(\frac{41 - q_1}{4}\right)}{7},$$

which simplifies to $7q_1 = \frac{135 - q_1}{4}$, yielding an optimal production in the US plant of $q_1 = 5$ units.

- The optimal production in the Chilean plant is $q_2 = \frac{41 - 5}{4} = 9$ units.
- Aggregate output is $Q = q_1 + q_2 = 5 + 9 = 14$ units.
- In summary, the monopoly produces a share of $\frac{q_1}{Q} = \frac{5}{14} \cong 0.36$ in the US plant, and $\frac{q_2}{Q} = \frac{9}{14} \cong 0.64$ in the Chilean plant.

Multipiant Monopoly

- The analysis about how the multipiant monopolist determines Q , and how it distributes such production among its plants, q_1 and q_2 , is analogous to a “cartel” problem.
- A **cartel** is a group of firms (equivalent to a monopolist with different plants) coordinating their production decisions to increase their joint profits.
 - *Example: Organization of the Petroleum-Exporting Countries (OPEC).*
 - Some countries have a lower MC (i.e., lower cost of extracting an additional barrel of oil), such as Saudi Arabia.
 - Other countries have higher MC , such as Angola o Venezuela.
 - They coordinate their total production and distribute it among the cartel participants.

Welfare Analysis under Monopoly

Welfare Analysis

- Output is lower under monopoly than under perfectly competitive industries, entailing a higher price.
- Consumer surplus is much smaller than under perfect competition because customers pay more per unit and buy fewer units.
- In contrast, profits are larger.
- However, the firm's profit gain does not compensate for the loss in consumer surplus, yielding a net loss in social welfare.

Welfare Analysis

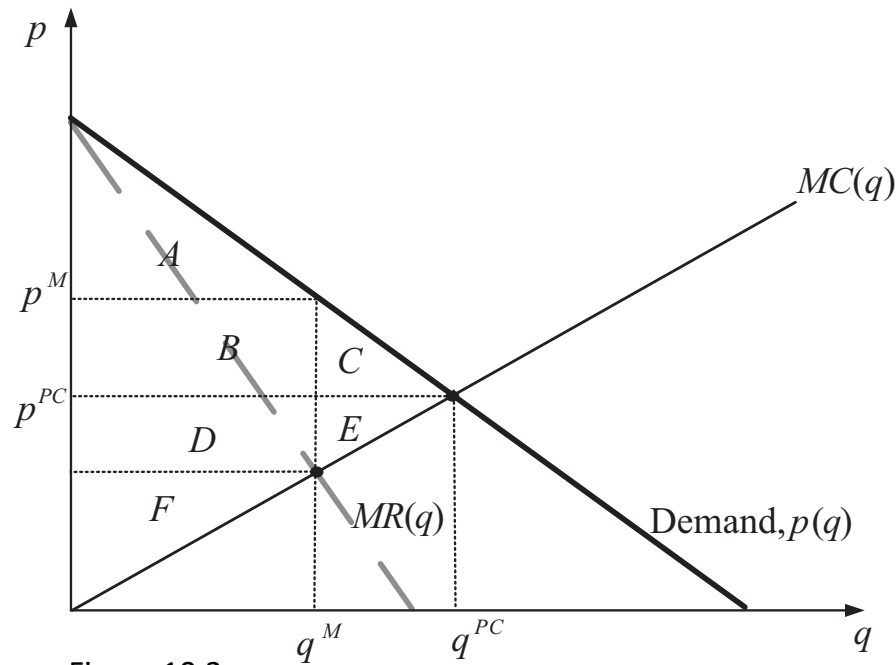


Figure 10.3

Table 10.1

	Perfect Competition	Monopoly	Difference
Consumer Surplus	$A + B + C$	A	$-B - C$
Profits	$D + E + F$	$D + F + B$	$B - E$
Welfare	$A + B + C + D + E + F$	$A + D + F + B$	$-C - E$ } “Deadweight loss”

Welfare Analysis

- *Example 10.9: Finding the deadweight loss of a monopoly.*
 - Consider $p(q) = 10 - q$ and $MC(q) = 4$.

Monopoly	Perfect Competition
$q^M = 3$ units	$q^{PC} = 6$ units
$p^M = \$7$	$p^{PC} = \$4$
$CS^M = \frac{1}{2}(10 - 7)3 = \4.50	$CS^{PC} = \frac{1}{2}(10 - 4)6 = \18
$\pi^M = (7 \times 3) - (4 \times 3) = \9	$\pi^{PC} = (4 \times 6) - (4 \times 6) = \0
$W^M = CS^M + \pi^M$ $= 4.5 + 9 = \$13.50$	$W^{PC} = CS^{PC} + \pi^{PC}$ $= 18 + 0 = \$18$

$$W^{PC} - W^M = 18 - 13.50 = \$4.50.$$

Welfare Analysis

- *Example 10.9* (continued):
 - Deadweight loss under this monopoly is

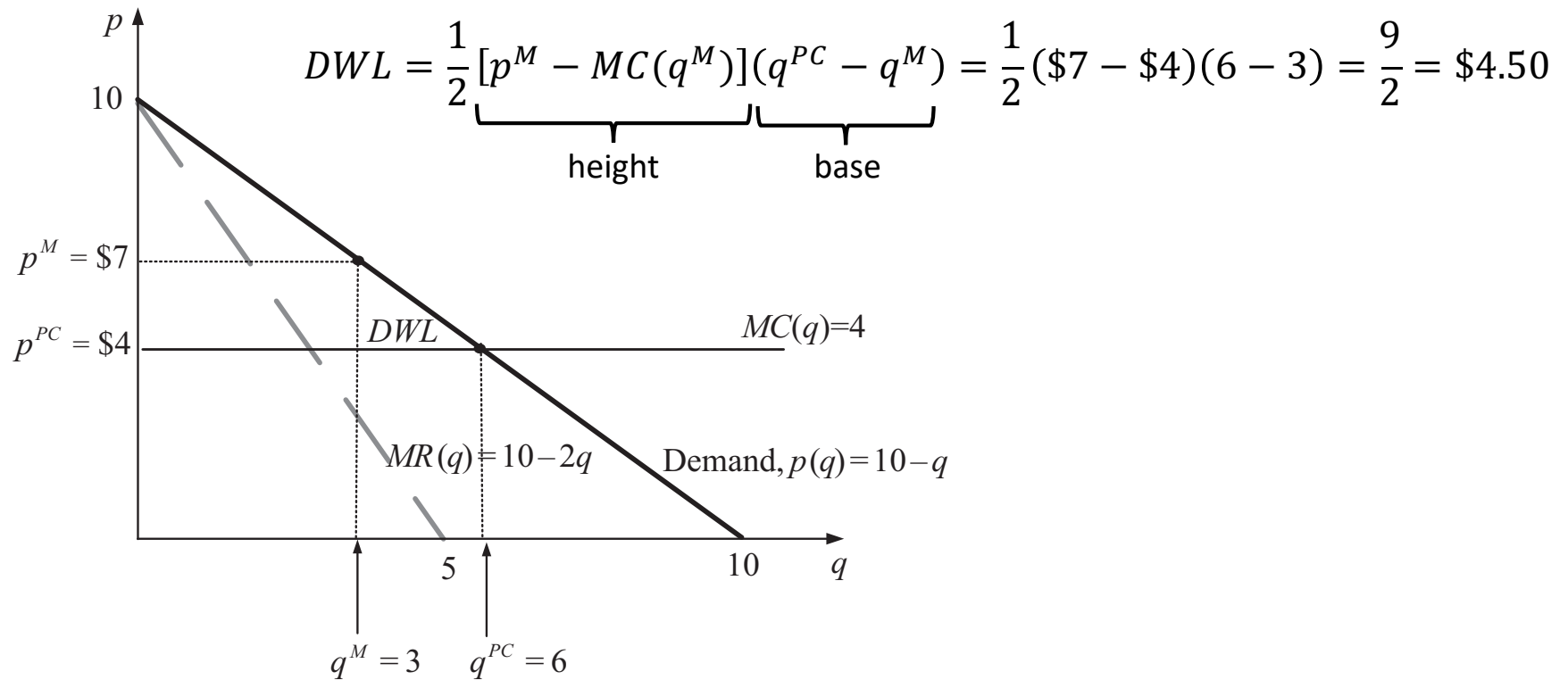


Figure 10.4

Advertising in Monopoly

Advertising in Monopoly

- When investing in advertising, the monopolist faces a trade-off: advertising increases demand but it is costly.
- To find the profit-maximizing amount of advertising, A ,

$$\max_A \pi = TR - TC - A.$$

- We can rewrite this problem as

$$\begin{aligned} \max_A \pi &= (p \times q) - TC(q) - A \\ &= [p \times q(p, A)] - TC[q(p, A)] - A. \end{aligned}$$

- where $q = q(p, A)$ represents the demand function (sales) which is decreasing p , and increasing in A .

Advertising in Monopoly

- Differentiating with respect to the amount of advertising A ,

$$p \frac{\partial q(p, A)}{\partial A} - \frac{\partial TC}{\partial q} \cdot \frac{\partial q(p, A)}{\partial A} - 1 = 0.$$

- Rearranging,

$$(p - MC) \cdot \frac{\partial q(p, A)}{\partial A} = 1.$$

Advertising in Monopoly

- Let us define the advertising elasticity of demand, $\varepsilon_{q,A}$, as

$$\varepsilon_{q,A} = \frac{\% \text{ increase in } q}{\% \text{ increase in } A} = \frac{\frac{\Delta q}{q}}{\frac{\Delta A}{A}} = \frac{\Delta q}{\Delta A} \cdot \frac{A}{q}.$$

- In the case of a small change in A , the elasticity $\varepsilon_{q,A}$ can be written as $\varepsilon_{q,A} = \frac{\partial q(p,A)}{\partial A} \cdot \frac{A}{q}$.
- Rearranging, we find $\varepsilon_{q,A} \cdot \frac{q}{A} = \frac{\partial q(p,A)}{\partial A}$.

Advertising in Monopoly

- Therefore, we can rewrite the profit-maximizing condition as

$$(p - MC) \underbrace{\varepsilon_{q,A}}_{\frac{\partial q(p,A)}{\partial A}} \cdot \frac{q}{A} = 1.$$

- Dividing both sides by $\varepsilon_{q,A}$ and rearranging,

$$p - MC = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{q}.$$

- Dividing both sides by p , we find

$$\frac{p - MC}{p} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{pq}.$$

Advertising in Monopoly

- From the IERP, we know

$$\frac{p-MC}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

- Hence,

$$-\frac{1}{\varepsilon_{q,p}} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{pq}.$$
$$-\frac{\varepsilon_{q,A}}{\varepsilon_{q,p}} = \frac{A}{pq}.$$

- The right side represents the **advertising-to-sales ratio**.
- For two markets with the same $\varepsilon_{q,p}$, the advertising-to-sales ratio must be larger in the market where demand is more sensitive to advertising (higher $\varepsilon_{q,A}$).

Advertising in Monopoly

- *Example 10.11: Monopolist's optimal advertising ratio.*
 - Consider a monopolist with price elasticity of demand of $\varepsilon_{q,p} = -1.5$ and advertising elasticity $\varepsilon_{q,A} = 0.1$.
 - The advertising-to-sales ratio should be

$$\begin{aligned}\frac{A}{pq} &= -\frac{\varepsilon_{q,A}}{\varepsilon_{q,p}} \\ &= -\frac{0.1}{-1.5} = 0.067.\end{aligned}$$

- Advertising should account for 6.7% of this monopolist's total revenue.

Monopsony

Monopsony

- **Monopsony**: only one buyer in the market and several sellers.
 - *Examples*: small labor markets, such as a mine or Walmart superstore in a small town.
- The buyer (employer) will be able to pay less for each hour of labor (lower wages) than if it had to compete against other employers, as in a perfectly competitive market.

Monopsony

- Consider a firm (e.g., a coal mine) with production function $q = f(L)$, which:
 - increases with the number of workers hired, $f'(L) > 0$,
 - but at a decreasing rate, $f''(L) < 0$.

- The profits of the coal mine is given by

$$\pi = TR - TC = pq - w(L)L.$$

- The firm extracts q units of coal, each sold at price p , yielding $TR = pq$.
- The firm hires L workers, paying each of them a wage of $w(L)$.
 - $w'(L) > 0$, as the firm hires more workers, labor becomes scarce, and a more generous wage must be offered to attract new workers.

Monopsony

- The monopsonist's PMP is

$$\max_{L \geq 0} \pi = pq - w(L)L = pf(L) - w(L)L.$$

- Intuitively, this problem says “choose the number of workers you plan to hire, L , so as to maximize your profits.”
- Differentiating with respect to L ,

$$pf'(L) - [w(L) + w'(L)L] = 0.$$

- Rearranging,

$$\underbrace{pf'(L)}_{MRP_L} = \underbrace{w(L) + w'(L)L}_{ME_L}.$$

Monopsony

$$\underbrace{pf'(L)}_{MRP_L} = \underbrace{w(L) + w'(L)L}_{ME_L}$$

- MRP_L (“marginal revenue product” of labor):
 - After hiring 1 more worker (increase in L), the firm produces $f'(L)$ more units of output (e.g., coal), sold at a price p .
- ME_L (“marginal expenditure” on labor). After hiring 1 more worker, the firm experiences an increase in cost:
 - This extra worker must be paid $w(L)$.
 - The additional worker is only attracted to the job if the firm offers her a higher salary because labor becomes scarcer. Such a wage increase, $w'(L)$, must be passed on to all existing worker, entailing a cost increase of $w'(L)L$.

Monopsony

- *Example 10.12: Finding optimal L in monopsony.*
 - Consider a coal company in a small town with production function $q = 100 \times \ln(L)$.
 - It faces an international perfectly competitive price of coal, $p = \$8$.
 - Assume the supply curve for labor is $w(L) = 3 + \frac{1}{2}L$. Then,

$$MRP_L = pf'(L) = 8 \times 100 \frac{1}{L} = \frac{800}{L}.$$

$$ME_L = w(L) + w'(L)L = \left(3 + \frac{1}{2}L\right) + \frac{1}{2}L = 3 + L.$$

Monopsony

- *Example 10.12* (continued):

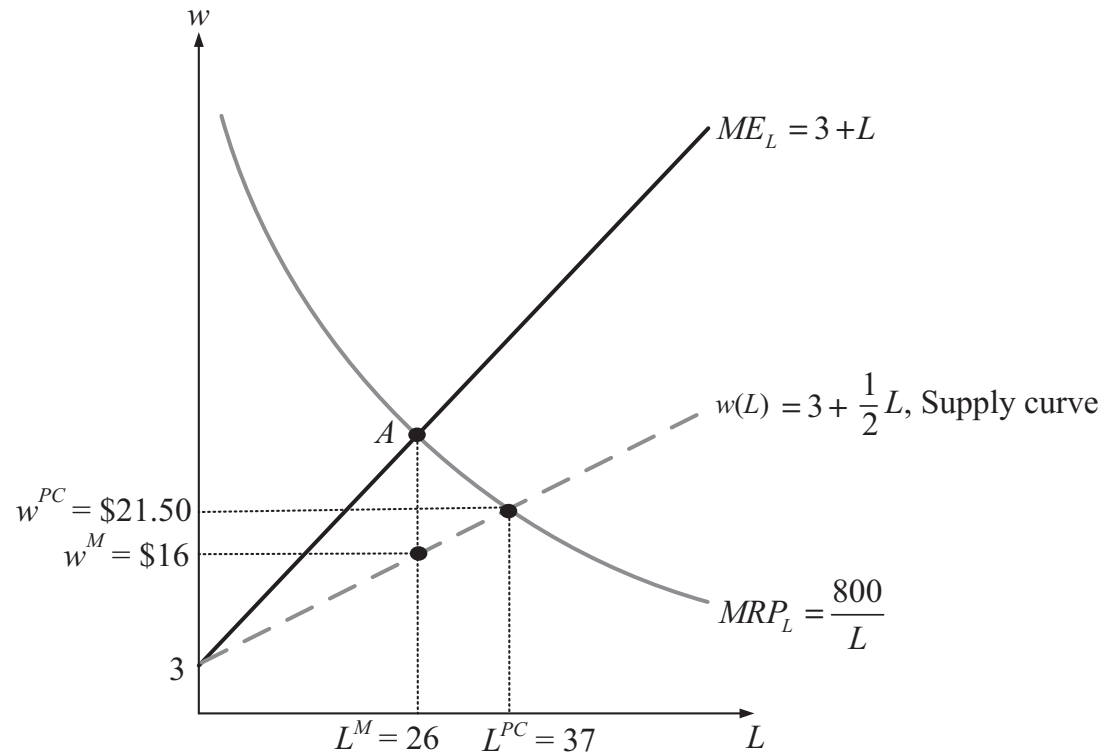


Figure 10.5

Monopsony

- *Example 10.12* (continued):

- Setting $MRP_L = ME_L$,

$$\frac{800}{L} = 3 + L,$$

which expanding yields $800 = 3L + L^2$ or

$$L^2 + 3L - 800 = 0.$$

- Solving for L , we find $L = -29.82$ and $L = 26.82$. Because the firm must hire a positive number of workers (or zero), we find that $L^M = 26$ workers is optimal.
- At $L^M = 26$, wages become $w(26) = 3 + 26 \times \frac{1}{2} = \16 .

Monopsony

- *Example 10.12* (continued):

- Under a perfectly competitive labor market, we have $MRP_L = w(L)$, that is,

$$\frac{800}{L} = 3 + \frac{1}{2}L,$$

which expanding yields $800 = 3L + \frac{L^2}{2}$.

- Solving for L , we obtain $L = -43.11$ and $L = 37.11$. Then $L^{PC} = 37$ is the optimal number of workers.
- At $L^{PC} = 37$, wages become $w(37) = 3 + \frac{1}{2}37 = \21.5 .