Strategy and Game Theory: Practice Exercises with Answers,

by Felix Munoz-Garcia and Daniel Toro-Gonzalez, Springer-Verlag, 2019

Errata in Second Edition, Updated on June 15th, 2020

Chapter 1 – Dominance Solvable Games

- Page 14, The last paragraph of Exercise #7 should be deleted, from "Therefore, our most…" to "…equilibrium outcome.)"
- Page 30, the displayed equation at the top should read

$$S(h_i, h_j) = \alpha \sum_{i=1}^{I} h_i + \beta \prod_{i=1}^{I} h_i.$$

  This expression should also be the displayed equation at the beginning of the answer key, inside the large bracket.

Chapter 2 – Pure strategy Nash equilibrium and simultaneous-move games with complete information

- Page 51, last displayed equation at the bottom of the page should end with $-4r_i$, rather than with $+4r_i$.
- Page 62, last displayed equation at the bottom of the page should read (X, X), (X, Y), and (Y, X).
- Page 81.
  - Seventh displayed equation (which describes equilibrium prices) should read $\frac{1+3c}{4}$ at the end.
  - Last displayed equation of the page (describing equilibrium profits) should read
  $$\pi_c = \left(\frac{1+3c}{4} - c\right)\frac{1-c}{4} = \frac{(1-c)^2}{16}.$$
- Page 81.

Chapter 3 – Mixed strategies, strictly competitive games, and correlated equilibria

- Page 100, last displayed equation (immediately before Exercise #5) should read
$$msNE = \{pB, (1-p)NB; qS, (1-q)S\}$$
$$= \left\{\left(\frac{1}{2}\right)B, \left(\frac{1}{2}\right)NB; \left(\frac{1}{5}\right)S, \left(\frac{4}{5}\right)NS\right\}.$$
- Page 104, last displayed equation (immediately before Exercise #6) should read
$$msNE = \{pU, (1-p)D; qL, (1-q)R\}$$
$$= \left\{\frac{3}{5}U, \frac{2}{5}D; \frac{2}{5}L, \frac{3}{5}R\right\}.$$
- Page 113, the section describing "*Mixing between T and C*" at the top of the page should finish reporting the msNE of the game, as follows:

$$msNE = \left\{\left(\frac{1}{2}T, \frac{1}{2}C, 0B\right), \left(\frac{1}{3}L, \frac{2}{3}R\right)\right\}.$$

- Page 114. The paragraph after the first displayed equations should read: "Therefore, player 2 plays *R* in pure strategies. Player 1 is then indifferent between *C* and *B* since both yield a payoff of \$2. As a consequence, we found a third msNE which we report as follows:

  $$msNE = \left\{\left(0T, \frac{1}{2}C, \frac{1}{2}B\right), R\right\}$$

  Note that player 1 is not choosing *T* with probability $p_1 = \frac{1}{2}$ since this probability only holds when player 2 is indifferent between *L* and *R*. In this context, player 2 is choosing a pure strategy (*R*) while player 1 randomizes."
- Page 129. Figure 3.45 should have a label u_2 in the vertical axis rather than u_1.
- Page 134. Figure 3.52 should read "(1,5), psNE (*U*,*R*)" at the top left-hand label, and "(5,1), psNE (*D*,*L*)" at the bottom right-hand label.
- Page 136. Figure 3.55 at the top of the page. This matrix should have $\frac{1}{2}$ in the cell corresponding to (*U*,*R*) rather than in (*U*,*L*). In other words, the content of the cells at the top of the matrix should switched.
- Page 139. Last paragraph should delete the sentence in parenthesis starting at "(See the payoff matrix in…" until "...which only entails a payoff of zero.)"
- Page 143.
  - Figure 3.62 should have the payoffs at the bottom left cell and at the top right-hand cell switched, so it reads "-10, 0" at the bottom left cell and "0, -10" at the top right-hand cell.
  - The second paragraph of part (b) should read "When comparing (C, NC) and (NC, C), we find that player 1 prefers the former since
    $$u_1(C, NC) = 0 > -10 = u_1(NC, C).$$
    while player 2 prefers the latter because
    $$u_2(C, NC) = -10 < 0 = u_2(NC, C).$$
    Similarly, when comparing (C, C) and (C, NC), in the left-hand column of the matrix, we find that player 1 prefers the latter given that
    $$u_1(C, C) = -5 < 0 = u_1(C, NC).$$
- Page 148. The second displayed equation should read
  $$6 - 6q = 2q \Leftrightarrow q = \frac{3}{4}.$$

Chapter 4 – Sequential-move games with complete information

- Page 163, Exercise 4.
  - Part (a) should read "Find the husband's best responses, for each of his wife's actions in the first stage."
  - Part (b) should read "Find the wife's equilibrium action and…"
- Page 165. Part c, line 4, should read "we found that only the former can be sustained in equilibrium."
- Page 174. Exercise 8, line 2, should read "The game is similar to that in Exercise 7, but with…"
- Page 184. Fourth displayed equation should read
  $$P(Q^{Cournot}) = 1 - \frac{2(1-c)}{3} = \frac{1+2c}{3}.$$

- Page 189. Exercise 13, part c, should read “Repeat part (b), but assuming that…”
- Page 197. Last displayed equation at the bottom of the page should read

$$\left(q_1^S, q_2(q_1), q_3(q_1, q_2)\right) = \left(8, 8 - \frac{1}{2}q_1, \frac{16 - q_1 - q_2}{2}\right).$$

Chapter 5 – Applications to industrial organization

- Page 250, figure 5.14 should read “Quantities” rather than “Quatities” in both the bottom row and right-hand column.

Chapter 6 – Repeated Games and Correlated Equilibria

- Page 275. Paragraph after the second displayed equation should read “…detects that player *i* is defecting (i.e., playing C), he finds that…”
- Page 285, the last displayed equation should read $\delta \geq 0.53$.
- Page 286, top paragraph should read “…discount factor δ has to be at least 0.53. In other words, firms must put…”
- Page 287, the last sentence of exercise 3 should read Typo: “Therefore, when deviations are punished for three periods, cooperation can be sustained…”
- Page 311. The case of “*Cooperative outcome (Monopoly)*” should read as follows. “The merged firm selects the individual output of firm *i* and *j*, q_i and q_j, that maximizes the joint profits, as follows

$$\max_{q_i, q_j} \quad \pi = \pi_i + \pi_j = \left(a - bq_i - dq_j\right)q_i + (a - bq_j - dq_i)q_j.$$

Differentiating with respect to q_i, we obtain

$$\frac{\partial \pi}{\partial q_i} = a - 2\left(bq_i + dq_j\right) = 0$$

and differentiating with respect to q_j, we find a symmetric expression, as follows

$$\frac{\partial \pi}{\partial q_j} = a - 2\left(bq_i + dq_j\right) = 0$$

In a symmetric output profile, we have that $q_i = q_j$. Inserting this property in any of the above first-order conditions yields

$$a - 2(b + d)q_i = 0$$

Solving for q_i, we find

$$q_i = q_j = \frac{a}{2(b + d)}.$$

Thus, the joint profits that firms earn if the collusive agreement is respected are

$$\pi^m = \left(a - b\frac{a}{2(b+d)} - d\frac{a}{2(b+d)}\right)\frac{a}{2(b+d)} + \left(a - b\frac{a}{2(b+d)} - d\frac{a}{2(b+d)}\right)\frac{a}{2(b+d)}$$

which simplifies to

$$\pi^m = \frac{a^2}{2(b+d)}$$

implying that every firm *i* earns half of these profits, that is, $\pi_i^m = \frac{1}{2}\frac{a^2}{2(b+d)} = \frac{a^2}{4(b+d)}$.

<u>*Best deviation for firm i*</u>. Let us assume that firm $j \neq i$ produces the cooperative output level $q_i^m = \frac{a}{2(b+d)}$. In order to determine the optimal deviation for firm *i*, we just need to plug firm *j*'s output q_j^m into firm *i*'s best response function, as follows

$$q_i(q_j^m) = \frac{a - d\left(\frac{a}{2(b+d)}\right)}{2b} = \frac{a(2b+d)}{4b(b+d)}.$$

Therefore, the profits that firm i earns from deviating are

$$\pi_i^D = \left(a - b\frac{a(2b+d)}{4b(b+d)} - d\frac{a}{2(b+d)}\right)\frac{a(2b+d)}{4b(b+d)} = \frac{a^2(2b+d)^2}{16b(b+d)^2}.$$

- Page 312. Third displayed equation should read:

$$\frac{1}{1-\delta}\frac{a^2}{4(b+d)} \geq \frac{a^2(2b+d)^2}{16b(b+d)^2} + \frac{\delta}{1-\delta}\frac{ba^2}{(2b+d)^2}.$$

Multiplying by $(1-\delta)$ on both sides of the inequality, we obtain

$$\frac{a^2}{4(b+d)} \geq \frac{a^2(2b+d)^2}{16b(b+d)^2}(1-\delta) + \delta\frac{ba^2}{(2b+d)^2}$$

and solving for δ yields the minimal discount factor that supports collusion in this industry,

$$\delta \geq \frac{(2b+d)^2}{8b^2+8bd+d^2} \equiv \delta_1.$$

Importantly, cutoff δ_1 increases in the parameter reflecting product differentiation, *d*, because the derivative

$$\frac{\partial\delta_1}{\partial d} = \frac{4bd(2b+d)}{(8b^2+8bd+d^2)^2}$$

is positive for all parameter values…"

- Page 313.
  - Second displayed equation should read

$$\frac{1}{1-\delta}\frac{a^2}{4(b+d)} \geq (1+\delta)\frac{a^2(2b+d)^2}{16b(b+d)^2} + \frac{\delta^2}{1-\delta}\frac{ba^2}{(2b+d)^2}.$$

  Multiplying by $(1-\delta)$ on both sides of the inequality, we obtain

$$\frac{a^2}{4(b+d)} \geq \frac{a^2(2b+d)^2}{16b(b+d)^2}(1-\delta^2) + \delta^2\frac{ba^2}{(2b+d)^2}$$

  - Fifth displayed equation should read

$$\delta \geq \frac{2b+d}{(8b^2+8bd+d^2)^{1/2}} \equiv \delta_2.$$

  where

$$\frac{\partial \delta_2}{\partial d} = \frac{2bd}{(8b^2 + 8bd + d^2)^{3/2}} > 0.$$

- Page 326:
  - Table 7.3. The 2/3 at the top left cell should not be underlined and bolded. The 1 in row Bf (right column) should be underlined instead.
  - First paragraph of the page should read "For player 1, his best response when player 2 bets…"
- Page 331. The legend of Table 7.9 should read "Bayesian normal form game with underlined best responses if $1/3 \leq p \leq 1/2$.
- Page 333. First line of the page should read "This is the probability of player 1 playing U when he knows that…"
- Page 360. Third displayed equation. Second term should read $dF(v)^{N-1} = (N-1)x^{N-2}dx$.

Chapter 9 – Perfect Bayesian Equilibrium and signaling games

- Page 435, in the second displayed equation, and the rest of the exercise, should read as follows:

$$1 - 2p_1^1 + p_2^1 + c_1 + 2\left(1 - \frac{1}{3}c_1\right)\frac{1}{3}\frac{1}{f'(f^{-1}(p_1^1)} = 0.$$

Simplifying, and solving for p_1^1, we find firm 1's best response function in the first-period game

$$p_1^1(p_2^1) = \frac{6 + c_1(9[f'(f^{-1}(p_1^1)] - 2) + 9[f'(f^{-1}(p_1^1)](1 + p_2^1)}{18[f'(f^{-1}(p_1^1)]}$$

Inserting this expression of p_1^1 into $p_2^1 = \frac{1+p_1^1}{2}$, we obtain the optimal second-period price for firm 1

$$p_2^1 = \frac{3 + c_1}{3} \mp \frac{2(3 - c_1)}{27[f'(f^{-1}(p_1^1)]}$$

Plugging this result into the best response function $p_1^1(p_2^1)$, yields the optimal first-period price for firm 1

$$p_1^1 = 1 + \frac{2c_1}{3} \mp \frac{4(3 - c_1)}{27f'(f^{-1}(p_1^1)]}$$

As suggested in the exercise, let us now assume that there exists a linear function $p_1^1 = f(c_1)$, that generates the previous strategy profile. That is:

$$p_1^1 = A_O + A_1c_1 = f(c_1)$$

where A_O and A_1 are positive constants. Intuitively, a firm with zero-unit costs, $c_1 = 0$, would charge a first-period price of A_0, while a marginal increase in its unit costs, c_1, would entail a corresponding increase in prices of A_1. Figure 9.52 illustrates this pricing function for firm 1. (Note that this is a separating strategy profile, as firm 1 charges a different first-period price depending on its unit cost as long as $A_1 > 0$. A pooling strategy profile would exist if $A_1 = 0$.)

Setting it equal to the expression for p_1^1 we found above and letting $A_1 = \frac{1}{f'(f^{-1}(p_1^1)]}$, we obtain:

$$A_O + A_1.c_1 = 1 + \frac{2c_1}{3} + A_1\frac{4(3 - c_1)}{27}$$

since A_1 measures the slope of the pricing function (see Fig. 9.52), thus implying $A_1 = \frac{1}{f'(f^{-1}(p_1^1)]}$.

Rearranging the above expression, we find

$$27A_O + 31A_1c_1 = 3(9 + 4A_1 + 6c_1)$$

or, after solving for A_1, we obtain

$$A_1 = \frac{9(3 - 3A_o + 2c_1)}{31c_1 - 12}$$

In addition, when firm 1's costs are nil, $c_1 = 0$, the above expression becomes,

$$A_0 = 1 + \frac{4}{9}A_1.$$

Inserting this equation into the expression for A_1found above, $A_1 = \frac{9(3-3A_o+2c_1)}{31c_1-12}$, yields

$$A_1 = \frac{9\left(3 - 3\left(1 + \frac{4}{9}A_1\right) + 2c_1\right)}{31c_1 - 12}$$

Solving for A_1, we obtain $A_1 = \frac{18}{31} \approx 0.58$. Therefore, the intercept of the pricing function, A_0, becomes:

$$A_0 = \frac{39}{31} \approx 1.26.$$

Hence, the pricing function p_1^1 of firm 1, $p_1^1 = A_O + A_1c_1$, becomes:

$$p_1^1 = \frac{18}{31} + \frac{39}{31}c_1.$$

Chapter 10: Cheap talk games

- Page 457, first sentence in part (b) should read "Figure 10.5 depicts the pooling strategy profile (m_1, m_1') in which…"